A - A* Algebra

MR BARTON'S SOLUTIONS

① $\boxed{x}$ $\frac{-4+10}{2} = \frac{6}{2} = 3$

$\boxed{y}$ $\frac{6+-8}{2} = \frac{-2}{2} = -1$ → Midpoint $= (3, -1, 8)$

$\boxed{z}$ $\frac{10+6}{2} = \frac{16}{2} = 8$

② $x^2 + 6x - 2$

$= (x+3)^2 - 11$

$(x+3)(x+3)$

$= x^2 + 6x + 9$

③ $x^2 - 4x - 8 = 0$

$a = 1$

$b = -4$

$c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2a}$$

$$x = \frac{4 \pm \sqrt{48}}{2}$$

$x = \frac{4 + \sqrt{48}}{2}$ → $x = 5.4641\ldots$

$x = \frac{4 - \sqrt{48}}{2}$ → $x = -1.46410\ldots$

4 ① $y = x^2 - 1$

② $y = 5 - x$

NOTE: I assumed it was supposed to be '5 - x'.

sub ② into ①

$\rightarrow \quad x^2 - 1 = 5 - x$

$$\begin{aligned} +x \quad & x^2 + x - 1 = 5 \\ -5 \quad & x^2 + x - 6 = 0 \end{aligned}$$

$(x + 3)(x - 2) = 0$

$x + 3 = 0$

$\rightarrow \boxed{x = -3}$

$y = 5 - x$

$\rightarrow y = 5 - (-3)$

$\rightarrow \boxed{y = 8}$

$x - 2 = 0$

$\rightarrow \boxed{x = 2}$

$y = 5 - x$

$\rightarrow y = 5 - 2$

$\rightarrow \boxed{y = 3}$

5 a) $4x + 20$

$= 4(x + 5)$

b) $3y^2 + 12y$

$= 3y(y + 4)$

c) $x^2 + 4y - 21$

$= (x + 7)(x - 3)$

⑥ $y = 2x + 5$

a) Parallel $\rightarrow$ gradient $= 2$

$y = mx + c$ 		gradient $= 2$

$x = 3$

$\rightarrow$ $4 = \cancel{2x}\ 2(3) + c$ 		$y = 4$

$4 = 6 + c$

$\rightarrow$ $c = -2$

Equation: $y = 2x - 2$

⑦ $\dfrac{4}{a + a^2} \times \dfrac{a^3 - a}{ab} = \dfrac{4a^3 - 4a}{a^2 b + a^3 b}$

$= \dfrac{4a(a^2 - 1)}{a^2 b(1 + a)}$

Difference of 2 squares $\longrightarrow$ $= \dfrac{4a(a+1)(a-1)}{a^2 b(a+1)}$

$\div (a+1)$ $= \dfrac{4a(a-1)}{a^2 b}$

$\div a$ $\dfrac{4(a-1)}{ab}$

(8) $$y = \frac{x + 2a}{x - a}$$

$\times (x-a)$ $\quad y(x-a) = x + 2a$

$\quad xy - ay = x + 2a$

$-x$ $\quad xy - x - ay = 2a$

$+ay$ $\quad xy - x = 2a + ay$

FACT $\quad x(y - 1) = 2a + ay$

$\div (y-1)$ $$x = \frac{2a + ay}{y - 1}$$

Number

(1) a) $4^0 = 1$

b) $125^{\frac{2}{3}} = \left[\sqrt[3]{125}\right]^2$

$= 5^2 = 25$

c) $64^{\frac{1}{2}} = \sqrt{64} = 8$

d) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(2)

$$\begin{tikzcd} \underline{\text{Old}} & \underline{\text{New}} \\ ? \arrow[r, bend right, "\times 0.55"'] & £16.12 \arrow[l, bend right, "\div 0.55"'] \end{tikzcd}$$

→ Old =

$16.12 \div 0.55$

$= £29.31$

OR

£16.12 = 55%

£0.293... = 1%

£29.31 = 100%

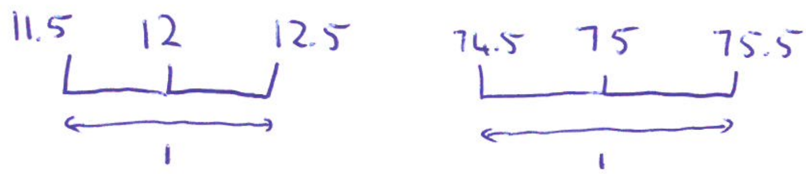
(3) $\boxed{Y1}$ $2500 \times 0.9 = 2250$

$\boxed{Y2}$ $2250 \times 0.95 = 2137.50$

$\boxed{Y3}$ $2137.50 \times 0.95 = 2030.625$

$\boxed{Y4}$ $2030.625 \times 0.95 = £1929.09$

(4) I am going to assume lengths rounded to nearest whole number/integer



MAX Area = $12.5 \times 75.5 = 943.75 \text{ cm}^2$

MIN Area = $11.5 \times 74.5 = 856.75 \text{ cm}^2$

(5) a) $0.77777\ldots$

$$
\begin{aligned}
10x &= 7.7777\ldots \\
- \quad x &= 0.7777\ldots \\
\hline
9x &= 7 \\
\rightarrow x &= \frac{7}{9}
\end{aligned}
$$

b) $0.758\ 758\ 758\ldots$

$$
\begin{aligned}
1000x &= 758.758\ 758\ 758 \\
- \quad x &= 0.758\ 758\ 758\ldots \\
\hline
999x &= 758 \\
\rightarrow x &= \frac{758}{999}
\end{aligned}
$$

c) $0.54242424\cancel{5}2\ldots$

$$
\begin{aligned}
1000x &= 542.424222\ldots \\
- \quad 10x &= 5.424242\ldots \\
\hline
990x &= 537 \\
\rightarrow x &= \frac{537}{990}
\end{aligned}
$$

(6) a) $\sqrt{24} = \sqrt{4} \times \sqrt{6}$

$= 2\sqrt{6}$

b) $\sqrt{5} \times \sqrt{7} = \sqrt{35}$

c) $(\sqrt{3} + 4)(\sqrt{3} - 2)$

$= \sqrt{9} - 2\sqrt{3} + 4\sqrt{3} - 8$

$= 3 + 2\sqrt{3} - 8$

$= -5 + 2\sqrt{3}$

(7) $T \propto m$

$\boxed{T = km} \longrightarrow T = 2.4m$

when $m = 400$

$600 = k(250)$

$\rightarrow k = \frac{600}{250}$

$= 2.4$

$\rightarrow T = 2.4(400)$

$= 960$ seconds

(8) a) $\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$

b) $\frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{6+2\sqrt{5}}{9+3\sqrt{5}-3\sqrt{5}-5}$

$= \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$

Geometry

(1) $V = \frac{1}{3}\pi r^2 h$

Large cone: $V = \frac{1}{3}\pi(15)^2 \times 40$

$= 3000\pi$

$\frac{40}{20}$

$\downarrow$

Length Scale Factor between Large + Small cone $= \times 2$

$\rightarrow$ radius of small cone $= \frac{\cancel{30}^{15}}{2} = 7.5$

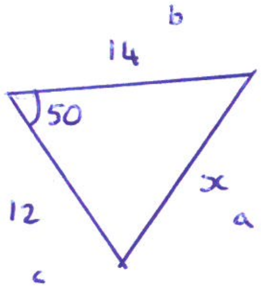
Small cone: $V = \frac{1}{3}\pi(7.5)^2 \times 20$

$= 375\pi$

Frustrum $= 3000\pi - 375\pi = 2625\pi \text{ cm}^2$

$= 8246.68\ldots \text{ cm}^2$

(2)



'Angle Sandwich' $\rightarrow$ cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$x^2 = 14^2 + 12^2 - 2(14)(12)\cos(50)$

$x^2 = 124.023\ldots$

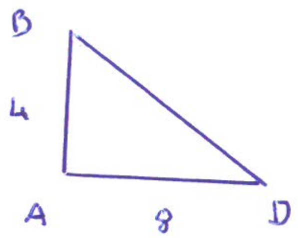
$x = 11.1365\ldots \text{ cm}$

Area $= \frac{1}{2}ab\sin(c)$

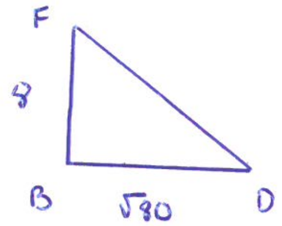
$= \frac{1}{2}(14)(12)\sin(50)$

$= 64.3477\ldots \text{ cm}^2$

3. Long Way



$$DB = \sqrt{8^2 + 4^2}$$
$$= \sqrt{80}$$



$$FD^2 = \sqrt{(\sqrt{80})^2 + 8^2}$$
$$= \sqrt{144}$$
$$= 12 \text{ cm}$$

Quick Way

$$FD = \sqrt{4^2 + 8^2 + 8^2}$$
$$= \sqrt{144} = 12 \text{ cm}$$

4. Length scale factor $= \frac{50}{40} = 1.25$

→ Area scale factor $= 1.25^2$

→ Paper needed for small box $= 3.27 \div 1.25^2$

$= 2.0928 \text{ m}^2$

5. $OAD = 90^\circ$ (tangent meets radius at 90°)

$AOD = 54^\circ$ (angles in a triangle add to 180°)

$ABC = 27^\circ$ (angle at the centre is double angle at circumference)

⑥ $A = \frac{\theta}{360} \times \pi r^2$

$\theta = 360 - 72$
$= 288$

$r = 9$
$d = 18$

$\rightarrow A = \frac{288}{360} \times \pi \times 9^2$

$= \frac{324}{5}\pi = 203.57\ldots \text{ cm}^2$

Arc length $= \frac{\theta}{360} \times \pi d$

$= \frac{288}{360} \times \pi \times 18$

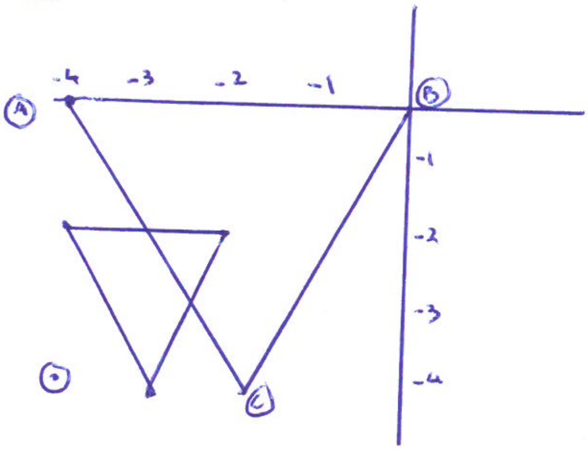
$= \frac{72}{5}\pi = 45.238\ldots \text{ cm}$

Perimeter $=$ Arc Length $+ 2 \times$ radius

$= 45.238\ldots + 9 + 9$

$= 63.2389\ldots$ cm

⑦



Ⓐ $(-4, 0) \rightarrow (-4, -2)$

Ⓑ $(0, 0) \rightarrow (-2, -2)$

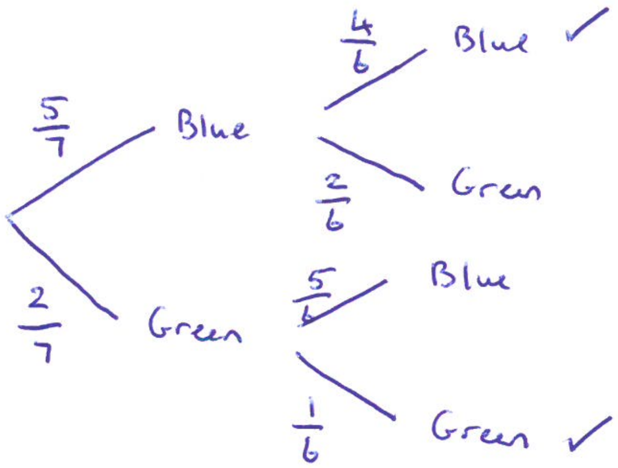
Ⓒ $(-2, -4) \rightarrow (-3, -4)$

Data

① 1, No option for £0

2, No option for £15+

3, Overlapping groups

② 

$P(B,B) = \frac{5}{7} \times \frac{4}{6} = \frac{20}{42}$

$P(G,G) = \frac{2}{7} \times \frac{1}{6} = \frac{2}{42}$

P(same colour)

ADD THEM

$\frac{20}{42} + \frac{2}{42}$

$\frac{22}{42}$

③

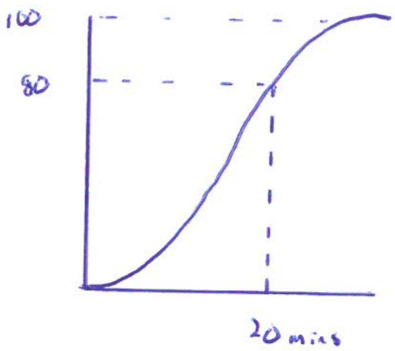
Height	Freq	Midpoint	MP × Freq
$0 < h \le 10$	9	5	45
$10 < h \le 20$	7	15	105
$20 < h \le 40$	8	30	240
$40 < h \le 50$	6	45	270
	30		660

Mean $= \frac{660}{30} = 22$ cm

④ Club A's median (52) is higher than club B's (50), which means on average players at club A are older

Club A's interquartile range (30) is greater than Club B's (21), which means there is a greater spread of ages.

⑤

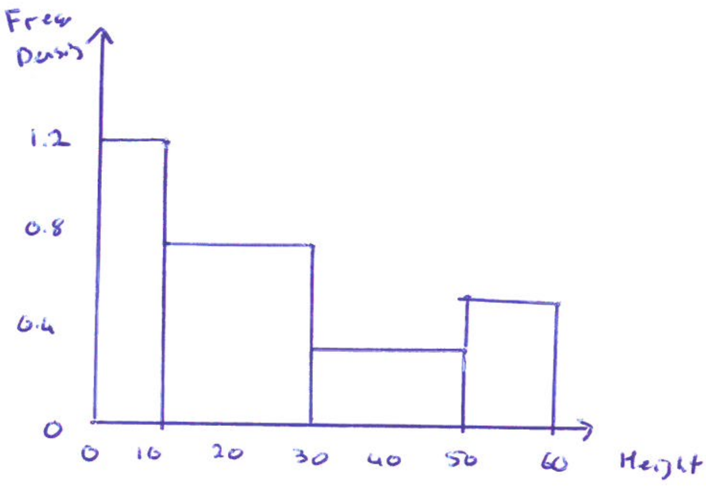


100 - 80 = 20 people faster than 20 minutes

⑥

Height	Freq	Width	Freq Density
$0 < h \leq 10$	12	10	1.2
$10 < h \leq 30$	14	20	0.7
$30 < h \leq 50$	8	20	0.4
$50 < h \leq 60$	6	10	0.6

$$\text{Freq Density} = \frac{\text{Freq}}{\text{width}}$$



(1)

~~coin~~ dice

coin	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

$$P(\text{Head} + \text{Even}) = \frac{3}{12} = \frac{1}{4}$$