S8 • Using binomial probabilities

Mathematical goals

To enable learners to:

- calculate binomial probabilities;
- calculate cumulative binomial probabilities.

To develop learners' understanding of:

- the context in which it is appropriate to use binomial probabilities;
- the symmetrical nature of the formula for a binomial probability, e.g. P(2 right out of 13) = P(11 wrong out of 13);
- alternative strategies for calculating cumulative binomial probabilities,
  e.g. P(at least 3 successes out of 10 trials) = 1 – P(fewer than 3 successes).

Starting points

Learners should understand what is meant by independent events and how to calculate probabilities of a series of independent events using $P(A \text{ and } B) = P(A) \times P(B)$.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Events*;
- Card set B – *Probabilities*.

Time needed

At least 1 hour.

Suggested approach

Beginning the session

Discuss what independent events are and how to calculate the probability of successive independent events.

Whole group discussion (1)

Use successively larger tree diagrams to establish the formula for binomial probabilities. Introduce the notation $\binom{12}{9}$ or $^{12}C_9$ and check that learners can use their calculators to evaluate it. Emphasise the criteria that must be true in order for the binomial to be applied. Ask learners to come up with some repeated events that are clearly not suitable for use with binomial probabilities e.g. the probability that it rains at midday and on successive days.

Give each learner a binomial probability such as $^{15}C_7\ 0.8^7 \times 0.2^8$ and ask them to write a question for which that is the answer. Collect in the questions and read them out one at a time. Ask learners to write the answer to each question on their whiteboards and compare these with the original answer. Discuss any differences and, in particular, any questions in which the information is not clear enough in order to arrive at the answer. Check that the events given in each question are appropriate for use with binomial probabilities.

Working in groups

Arrange learners in pairs and give each pair Card set A – *Events*. Set the scenario:

> "The probability of winning a game is always 0.6 and there are 8 games left to play."

Ask learners to match any events on the cards that are essentially the same event, e.g. P(lose at least 5) and P(win fewer than 4). Ask pairs of learners to compare with other pairs how many matchings they have until there is a consensus in the group as a whole.

Encourage learners who find the language difficult to draw a line and highlight the events that the card refers to e.g.

Losses	8	7	6	5	4	3	2	1	0
Wins	0	1	2	3	4	5	6	7	8

This example represents P(win 3 or more) or P(lose 5 or less).

If learners find this easy, ask them to add some more phrases for the events in each group, e.g. P(win 2 or less) can be placed with P(win fewer than 3).

Give out Card set B – *Probabilities*. Ask learners to match the probabilities with their sets of events from Card set A.

Whole group discussion (2)

Where there is more than one *Probabilities* card that matches a set of events, discuss the reason for this. Emphasise that it does not matter whether you consider the number of wins or the number of losses; the formulae give the same value.

Ask learners to find two events that, together, cover all possibilities, e.g. P(win more than 3) and P(win 3 or less). Discuss the possibility of calculating P(win more than 3) as 1 – P(win 3 or less) as a more efficient strategy.

Ask learners to identify other events with long probabilities that can be reduced in this way. Encourage learners to find the cards that contain these long probabilities and, for each one, write the alternative calculation on the card.

Reviewing and extending learning

Building on Whole group discussion (1), give each pair of learners a cumulative binomial probability and ask them to write a suitable question for which it is the answer. As before, collect in the questions and read them out one at a time. Learners write down the answer to the question on their mini-whiteboards. These are then compared with the original answer. Discuss any differences and in particular any questions in which the information is not clear enough in order to give the answer. Check that the events given in each question are appropriate for use with cumulative binomial probabilities.

What learners might do next

Use cumulative binomial tables for a larger number of repeated events.

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S8 Card set A – *Events*

P(win fewer than 3)	P(win no more than 3)
P(win at least 3)	P(lose more than 5)
P(win exactly 3)	P(win more than 3)
P(lose no more than 5)	P(lose at least 5)
P(lose exactly 5)	P(win fewer than 4)
P(lose fewer than 5)	P(lose more than 4)
P(lose 5 or less)	P(win 3 or more)
P(lose 5 or more)	P(win 3 or less)

S8 Card set B – *Probabilities*

${}^8C_5 0.4^5 \times 0.6^3 + {}^8C_4 0.4^4 \times 0.6^4 + {}^8C_3 0.4^3 \times 0.6^5 + {}^8C_2 0.4^2 \times 0.6^6 + {}^8C_1 0.4 \times 0.6^7 + 0.6^8$
${}^8C_2 0.6^2 \times 0.4^6 + {}^8C_1 0.6 \times 0.4^7 + 0.4^8$
${}^8C_6 0.4^6 \times 0.6^2 + {}^8C_7 0.4^7 \times 0.6 + 0.4^8$
${}^8C_3 0.6^3 \times 0.4^5 + {}^8C_4 0.6^4 \times 0.4^4 + {}^8C_5 0.6^5 \times 0.4^3 + {}^8C_6 0.6^6 \times 0.4^2 + {}^8C_7 0.6^7 \times 0.4 + 0.6^8$
${}^8C_4 0.4^4 \times 0.6^4 + {}^8C_3 0.4^3 \times 0.6^5 + {}^8C_2 0.4^2 \times 0.6^6 + {}^8C_1 0.4 \times 0.6^7 + 0.6^8$
${}^8C_3 0.6^3 \times 0.4^5 + {}^8C_2 0.6^2 \times 0.4^6 + {}^8C_1 0.6 \times 0.4^7 + 0.4^8$
${}^8C_5 0.4^5 \times 0.6^3 + {}^8C_6 0.4^6 \times 0.6^2 + {}^8C_7 0.4^7 \times 0.6 + 0.4^8$
${}^8C_4 0.6^4 \times 0.4^4 + {}^8C_5 0.6^5 \times 0.4^3 + {}^8C_6 0.6^6 \times 0.4^2 + {}^8C_7 0.6^7 \times 0.4 + 0.6^8$
${}^8C_3 0.6^3 \times 0.4^5$
${}^8C_5 0.4^5 \times 0.6^3$