S7 • Developing an exam question: probability

Mathematical goals

To help learners to:

- use past examination papers creatively;

- analyse the demands made by examination questions;

- understand and use estimates or measures of probability from theoretical models;

- list all outcomes for two successive events in a systematic way;

- identify mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1.

Starting points

Most learners will have attempted to answer probability questions such as these before, but they may have misconceptions. This session aims to expose learners' misconceptions and to enrich their understanding of the probability concepts and skills likely to be demanded in GCSE examination questions.

Materials required

- An OHT of Sheet 1 – *Spinners*.

For each learner you will need:

- Sheet 1 – *Spinners*;

- Sheet 2 – *Template for spinners question*.

Time needed

This session will take about an hour.

Suggested approach

Beginning the session

Ask learners to work in pairs on the GCSE examination question in Sheet 1 – *Spinners*. When everyone has had time to have a go at this, hold a whole group discussion on the approaches used.

Whole group discussion

(i) Completing the question

Place the OHT of Sheet 1 – *Spinners* on the overhead projector.

In order to answer this question, learners will need to recall that, when we have equally likely outcomes:

The probability of an event $= \dfrac{\text{the number of ways the event can happen}}{\text{the total number of equally likely outcomes}}$

Collect suggestions for correct answers to question 1 and write these numbers into the table on the OHT.

		Spinner A		
		1	**3**	**5**
Spinner B	**2**	3	5	7
	4	5	7	9
	5	6	8	10

After they have agreed on the correct values to be written in all the cells in the table, invite learners to suggest answers to question 2, explaining their reasoning. Discuss any disagreements.

> The probability that Sam wins a prize is $\dfrac{1}{3}$ because there are three players.
>
> The probability of Sam winning is $\dfrac{4}{9}$ because four of the nine numbers are 5 or 7.

Finally, consider the concept of mutually exclusive outcomes adding to 1.

> Why don't the three probabilities add up to 1?
> (Because, if the total is 6, Max and Amy will both win.)
> Yes, that is another way of saying that both events are not mutually exclusive. Max winning a prize does not exclude the possibility that Amy will also win a prize.

Whole group discussion

(ii) Generating further questions – same situation

There are many other questions an examiner might ask about this situation. Ask learners to suggest alternatives. In doing this, they should not seek to change the situation in any way, but simply ask new questions about the given situation.

For example, question 1 could have been:

> Draw a tree diagram to show the outcomes.

Question 2 could ask other probability questions such as:

> What is the probability that nobody wins a prize?
> What is the probability that only one person wins a prize?
> What is the probability that two people win a prize?

We could ask further questions where you can add probabilities:

> What is the probability of getting a multiple of 5 or a multiple of 4?

and where you cannot:

> What is the probability of getting a multiple of 5 or a multiple of 2?

or harder questions where you multiply probabilities:

> What is the probability that Amy wins two games in a row?

Working in groups (1)

Ask learners to choose questions they think they can answer and encourage them to work on it in pairs and try to agree on the correct answers. Learners may like to write and compare their different answers on the board.

Whole group discussion

(iii) Developing the situation

Hand out Sheet 2 – *Template for spinners questions*.

Ask learners to write their own GCSE question using this template.

Discuss with them how they might do this. They will need to draw two fair spinners, decide on the kinds of numbers they want on the spinners (e.g. negative numbers), how they may be combined (adding, multiplying, etc.), and what will happen for various different outcomes. They can choose whether to ask for a table or a tree diagram, and can ask any further questions.

If some learners feel more ambitious, they may like to try to develop spinners that are unfair, or use more than two spinners.

Working in groups (2)

The new questions should be passed around the group to be answered by other learners. Where learners have difficulty answering questions, the question-writers should explain what they intended and act as a teacher, helping other learners to answer the questions.

Alternatively, some of the new questions can be photocopied for further sessions or for homework.

Reviewing and extending learning

Finally, hold a whole group discussion about what has been learned, drawing out any continuing misconceptions. You should include a discussion of the level of difficulty of the new questions.

What learners might do next

Ask learners to choose another question from an exam paper and follow the process adopted in this session.

(i)   Answer the question;

(ii)   Ask new questions about the same situation (and answer them);

(iii)   Change the situation and write a new question.

Further ideas

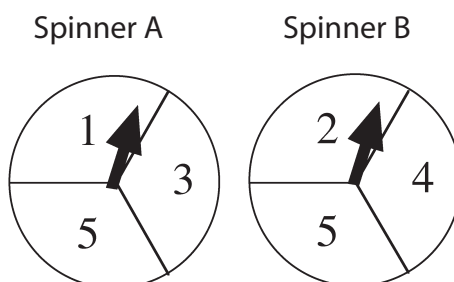This method for developing exam questions can be used in any topic. Examples in this pack include:

N10   Developing an exam question: number;

A8   Developing an exam question: generalising patterns;

SS8   Developing an exam question: transformations.

S7 Sheet 1 – *Spinners*

Two fair spinners are numbered 1, 3, 5 and 2, 4, 5 respectively.

Spinner A   Spinner B

You spin the spinners and add the numbers together.

- If the total is even, Amy wins a prize.
- If the total is a multiple of 3, Max wins a prize.
- If the total is 5 or 7, Sam wins a prize.

1. Draw a table to show all the equally likely outcomes:

		Spinner A		
		1	**3**	**5**
Spinner B	**2**			
	4			
	5			

2. Complete the table below:

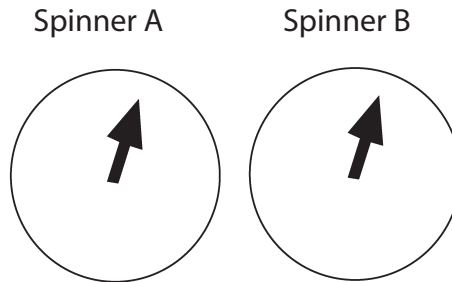| Name	Probability of winning a prize
Amy	$\dfrac{1}{3}$
Max	
Sam	

3. Explain what is wrong with the following statement:

    "The probability that someone will win is 1. This means that the probabilities in question 2 should add up to 1."

4. What further questions can you ask?

S7 Sheet 2 – *Template for spinners question*

Two fair spinners are numbered .... and .... respectively.

Spinner A   Spinner B

You spin the spinners and .... the numbers together.

If the result is .... then ....

If the result is .... then ....

If the result is .... then ....

1. Draw a .... to show all the equally likely outcomes:

2. ....

3. ....