S2 • Evaluating probability statements

Mathematical goals

To help learners to:

- discuss and clarify some common misconceptions about probability.

This involves discussing the concepts of:

- equally likely events;
- randomness;
- sample sizes.

Learners also learn to reason and explain.

Starting points

This session assumes that learners have encountered probability before. It aims to draw on their prior knowledge and develop it through discussion. It does not assume that they are already competent.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *True, false or unsure?*

Time needed

Between 30 minutes and 1 hour. The issues raised will not all be resolved in this time and will therefore need to be followed up in later sessions.

Suggested approach

Beginning the session

Using mini-whiteboards and questioning, remind learners of some of the basic concepts of probability. For example, ask learners to show you answers to the following:

Estimate the probability that:

- you will be hit by lightning this afternoon;
- you will get a tail with one toss of a coin;
- you will get a four with one roll of a die;
- you will sleep tonight.

Describe an event, different from those already mentioned, that has a probability of:

- zero;
- one;
- one half;
- more than one half, but less than one;
- less than one half, but greater than zero.

Working in groups

Give each pair of learners Card set A – *True, false or unsure?* Explain that these cards are intended to reveal some common misconceptions about probability.

Ask learners to take each card in turn and:

- decide whether it is a true statement or a false statement;
- write down reasons to support their decision;
- if they are unsure, explain how to find out whether it is true or not. For example, is there a simple experiment (simulation) or diagram that might help them decide?

As they do this, listen carefully to their reasoning and note down misconceptions that arise for later discussion with the whole group. When two pairs have reached agreement, ask them to join together and try to reach agreement as a group of four.

Whole group discussion

Ask each group of learners to choose one card they are certain is true and to explain to the rest of the group why they are certain. Repeat this with the statements that learners believe are false.

Finally, as a whole group, tackle the statements that learners are not so sure about.

Try to draw out the following points, preferably after learners have had the opportunity to do this in their own words.

- Statements B and H are true. For B it is enough to notice that there are two ways of obtaining a total of 3 (1,2 and 2,1), whereas there is only one way of obtaining a score of 2. For H, it is enough to notice that there are more learners than days of the week.

The remaining statements offer examples of common misconceptions.

- 'Special' events are less likely than 'more representative' events.

Statements A and C are indicative of this misconception. In both cases the outcomes are equally likely. Some learners remember trying to begin a game by rolling a six and it appeared to take a long time. The special status of the six has thus become associated with it being 'hard to get'. Others may think that they increase their chances in a lottery or raffle by spreading out their choices rather than by clustering them together. In fact this makes no difference.

- All outcomes are assumed to be equally likely.

Statements D and E are typical examples. The different outcomes are simply counted without considering that some are much more likely than others. For D, there are in fact four equally likely outcomes: HH, HT, TH, TT. Clearly, the probabilities for E will change whether the opposing team is Arsenal or Notts County.

- Later random events 'compensate' for earlier ones.

This is also known as the gambler's fallacy. Statements G and I are indicative of this. Statement G, for example, implies that the coin has some sort of 'memory' and later tosses will compensate for earlier ones. People often use the phrase 'the law of averages' in this way.

- Sample size is irrelevant.

Statement J provides an example of this subtle misconception. The argument typically runs that, if the probability of one head in two coin tosses is $\frac{1}{2}$, then the probability of n heads in $2n$ coin tosses is also $\frac{1}{2}$. In fact the probability of three out of six coin tosses being heads is $\frac{20}{64}$ or just under $\frac{1}{3}$. This may be calculated from Pascal's triangle.

- Probabilities give the proportion of outcomes that will occur.

Statement F would be correct if we replaced the word 'certain' with the words 'most likely'. Probabilities do not say for certain what will happen, they only give an indication of the likelihood of something happening. The only time we can be certain of something is when the probability is 0 or 1.

Learners who struggle with these ideas may like to do some simple practical probability experiments using coins and dice.

Reviewing and extending learning

Ask learners to suggest further examples that illustrate the misconceptions shown above.

What learners might do next

Session **S3 Playing probability computer games** may be used to follow up and deepen the ideas. This will make links between theoretical probabilities and experimental outcomes.

Further ideas

The idea of evaluating statements through discussion may be used at any level and in any topic where misconceptions are prevalent. Examples in this pack include:

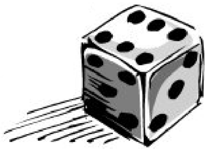
N2 Evaluating statements about number operations;

SS4 Evaluating statements about length and area.

S2 Card set A – *True, false or unsure?*

A

When you roll a fair six-sided die, it is harder to roll a six than a four.



B

Scoring a total of three with two dice is twice as likely as scoring a total of two.

C

In a lottery, the six numbers
3, 12, 26, 37, 44, 45
are more likely to come up than the six numbers 1, 2, 3, 4, 5, 6.

D

When two coins are tossed there are three possible outcomes: two heads, one head or no heads. The probability of two heads is therefore $\frac{1}{3}$.

E

There are three outcomes in a football match: win, lose or draw. The probability of winning is therefore $\frac{1}{3}$.

F

In a 'true or false?' quiz with ten questions, you are certain to get five right if you just guess.

G

If you toss a fair coin five times and get five heads in a row, the next time you toss the coin it is more likely to show a tail than a head.

H

In a group of ten learners, the probability of two learners being born on the same day of the week is 1.

I

If a family has already got four boys, then the next baby is more likely to be a girl than a boy.

J

The probability of getting exactly three heads in six coin tosses is $\frac{1}{2}$.