SS3 • Dissecting a square

Mathematical goals

To enable learners to:

- express a part/whole diagram in fractions or percentages;
- convert a fraction to a percentage (using a calculator);
- calculate areas of rectangles, triangles, circles and parts of circles;
- add, subtract and multiply fractions.

These goals can be adapted for learners working at lower levels. For example, you may restrict the range of shapes to just rectangles.

Starting points

This session builds on learners' prior knowledge about fractions and percentages and about finding areas of rectangles, triangles and circles. Learners begin by completing a short discussion task that is designed to reveal their existing understandings and misunderstandings about fractions.

Materials required

For each small group of learners you will need:

- Sheet 1 – *Dissecting a square*;
- Sheet 2 – *Making up your own dissection* (two copies);
- Sheet 3 – *Further examples for discussion*;
- calculator;
- pencil, ruler and compasses.

For learners aiming at lower levels, just use Sheets 1 and 2.

The whole group discussions will be much easier if you make OHTs of some of the sheets. Alternatively, you can use an interactive whiteboard or data projector.

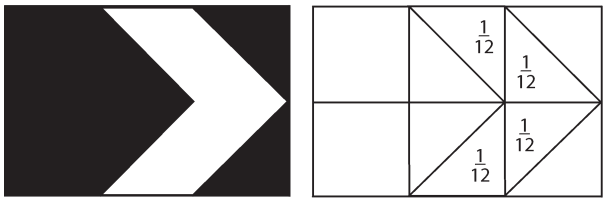
Time needed

This session will take from 1 to 2 hours.

Suggested approach

Beginning the session

Draw the following diagrams on the board. Explain that they show part of a road sign indicating a severe bend.



$\frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{4}{48}$

So $\frac{4}{48}$ of the sign will be white.

Ask learners to tell you what has gone wrong with the calculation. It is often helpful to ask them to explain their thinking to their neighbours before answering.

Now discuss the following mistake in a similar way:

> In a supermarket you see an item marked '20% off'.
> You decide to buy two. The person behind the checkout says:
> "20% off each item
> That will be 40% off the total price."

Working in groups

Ask learners to work in pairs. Hand out copies of Sheet 1 – *Dissecting a square*. Ask learners to calculate the fraction and percentage of the whole square that each piece represents.

If you think some learners will find this dissection too difficult, use Sheet 2 – *Making up your own dissection* to prepare an alternative using rectangular pieces.

You may need to remind learners how to convert a fraction to a percentage using a calculator by dividing the numerator by the denominator and multiplying by 100. They can then use the percentages to check their answers.

The purpose of this activity is to encourage reasoning. Thus learners may have some difficulty when writing piece E on Sheet 1 as a fraction, but they may reason, for example, that E can be calculated by subtracting a small triangle that is $\frac{1}{16}$ of the whole square from a right angled triangle that is $\frac{1}{8}$.

Look out for learners who make the same mistakes that were in the introductory discussion. For example, they might answer that piece A is $\frac{1}{4}$ but piece C is $\frac{3}{4}$, thus changing their view of the size of one unit.

Whole group discussion

Discuss different strategies for solving the problem. For example, for piece B:

> Divide the whole square into small squares like B and show that 16 Bs make up the whole. Therefore B = $\frac{1}{16}$.

or

> B has a length of $\frac{1}{4}$ and width of $\frac{1}{4}$ so altogether it has an area of $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$.

Similarly, ask learners to share different approaches for finding piece E.

> D + E + F = $\frac{1}{2} = \frac{8}{16}$;
>
> D = $\frac{5}{16}$;
>
> F = $\frac{1}{8} = \frac{2}{16}$;
>
> So E must be $\frac{1}{16}$.

or

> E has a base of $\frac{1}{4}$ and a height of $\frac{1}{2}$.
>
> Using area of triangle formula: E = $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{1}{16}$.

Discuss how fractions can be converted to percentages with a calculator and that these percentages can be added to check that they make 100.

Working in groups

Now hand out two copies of Sheet 2 – *Making up your own dissection* to each pair of learners. Encourage each learner to create a problem similar to Sheet 1 for a partner to solve. They should try

to make the problem quite challenging for the partner, but they must supply the correct answers on the back.

Learners aiming at lower levels may be asked to restrict their dissections to rectangles, and then triangles.

Learners may use a variety of shapes, e.g. squares, rectangles, triangles or circles. They should try to create problems according to their own level of confidence and competence.

A range of examples is given on Sheet 3 – *Further examples for discussion*.

Reviewing and extending learning

Finally, hold a whole group discussion sharing some of the problems produced by learners or from Sheet 3 – *Further examples for discussion*.

This may be facilitated using OHTs. These could be made by asking learners to trace some of their problems onto a blank acetate, or by photocopying the examples in Sheet 3 onto acetate.

What learners might do next

Learners could be given particular fraction calculations to illustrate. For example:

Draw me a dissection to show that:

$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$; $\frac{1}{5}+\frac{1}{4}+\frac{11}{20}=1$.

Draw me a dissection made entirely of triangles to show:

$\frac{1}{2}+\frac{2}{5}+\frac{1}{10}=1$.

Further ideas

This session invites learners to create and solve their own problems. This type of activity may be used throughout the mathematics curriculum. Examples in this pack include:

A2 Creating and solving equations;

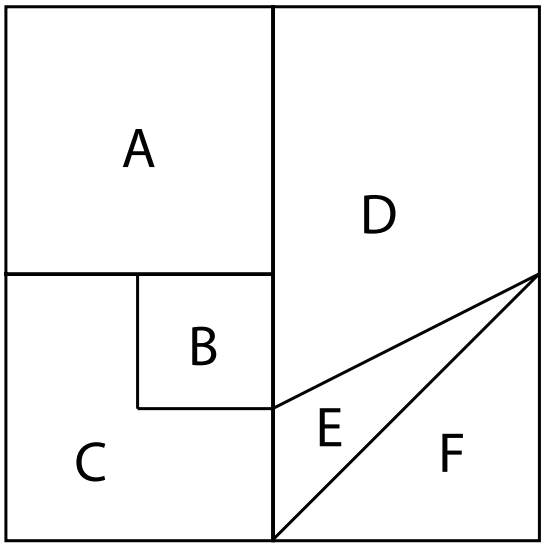
N10 Developing an exam question: number;

A8 Developing an exam question: generalising patterns;

SS8 Developing an exam question: transformations;

S7 Developing an exam question: probability.

SS3 Sheet 1 – *Dissecting a square*



Piece A is $\frac{1}{4}$ (or 25%) of the whole square.

1. What fraction and percentage of the whole square are the other pieces? Explain how you know.

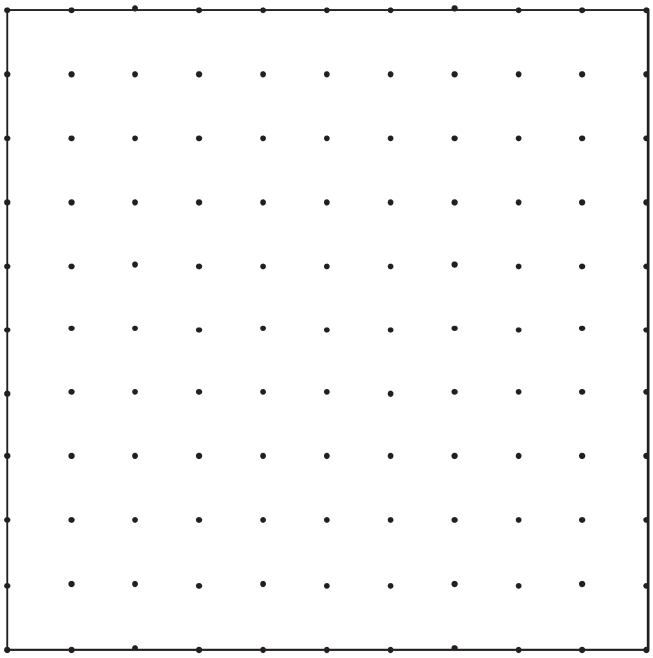
Piece	Fraction	Percentage	Reason
A			
B			
C			
D			
E			
F			

2. Check that your answers add up to a whole unit (or 100%).

SS3 Sheet 2 – *Making up your own dissection*

Divide this square into fewer than ten pieces and label them A, B etc. Try to make pieces with different sizes and shapes. On the back of this sheet, write down the size of each piece using fractions and percentages.

Give this sheet to your partner and see if they get the same answers.



Piece	Fraction	Percentage	Reason
A			
B			
C			
D			
E			
F			

SS3 Sheet 3 – *Further examples for discussion*