C4 • Differentiating and integrating fractional and negative powers

Mathematical goals

To enable learners to:

- convert functions into an appropriate form for differentiating or integrating;
- differentiate negative and fractional powers of x;
- integrate negative and fractional powers of x.

Starting points

Learners should be able to differentiate and integrate polynomial functions and have some knowledge of fractional and negative indices.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Functions*;
- Card set B – *Functions in index form*;
- Card set C – *Differentiated functions*;
- Card set D – *Integrated functions*;
- Card set E – *Differentiated functions in square root and fractional form*;
- Card set F – *Integrated functions in square root and fractional form*.

Each set should be photocopied on to different coloured card.

Time needed

At least 1 hour.

Suggested approach

Beginning the session

Use mini-whiteboards to briefly revise differentiation and integration of polynomials, fractional and negative indices. Ask questions such as:

For $y = x^4 - 3x^2 + 2x - 7$, find $\frac{dy}{dx}$.

$\int (x^3 + 4x^2 - 5x + 1)\, dx$

Write $\frac{1}{2}$ as a power of 2.

Write $\frac{1}{3^2}$ as a power of 3.

Write $\sqrt{x}$ as a power of x.

Write $\frac{1}{x^4}$ as a power of x.

How else can $\frac{1}{2x^3}$ be written?

It is likely that some learners will write the last one as $2x^{-3}$. If this happens, discuss this answer in detail, possibly demonstrating that it is not equivalent by substituting $x = 1$.

Leave any jottings on the board for learners to refer to as the session progresses.

Working in groups

Ask learners to work in pairs. Give each pair Card set A – *Functions*, Card set B – *Functions in index form*, Card set C – *Differentiated functions* and Card set D – *Integrated functions*. Ask learners to start with Set A and match them to Set B in order to prepare the functions for differentiating and integrating.

Next, ask them to match Sets C and D to Set B by applying the same principles to fractional and negative indices that they use for polynomial functions of x.

Learners may struggle to decide what happens to the constant (in this case the 2). Refer them back to the work done earlier in the session and left on the board.

If learners are not sure if their matchings are correct they can check their integration by differentiating and vice versa. Pairs of learners can also compare their matchings with other pairs and discuss any differences or problems until they come to a consensus.

Learners who complete the task quickly can be given Card set E – *Differentiated functions in square root and fractional form* and Card set F – *Integrated functions in square root and fractional form* to match with their integrals and differentials.

Reviewing and extending the learning

When all learners have matched sets A, B, C and D, ask each pair to explain one of their matchings to the rest of the group as a whole. Check understanding using mini-whiteboards and asking questions similar to those on the cards.

Ask learners to generalise their findings to find integrals and differentials such as:

$$\int \frac{a}{x^n}\,\mathrm{d}x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{a}{x^n}\right) \qquad \int \sqrt[n]{x}\,\mathrm{d}x \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt[n]{x}\right)$$

What learners might do next

Find stationary points and solve equations involving fractional and negative indices.

Find areas under a curve.

Further ideas

This approach could be used for differentiation and integration of any type of function.

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C4 Card set A – *Functions*

$y = 2\sqrt{x}$	$y = \dfrac{1}{2x^3}$
$y = \dfrac{2}{x^2}$	$y = \sqrt{x}$
$y = \dfrac{2}{\sqrt{x}}$	$y = \dfrac{1}{2\sqrt{x}}$
$y = \dfrac{1}{2x^2}$	$y = \dfrac{1}{\sqrt{x}}$
$y = \dfrac{2}{x^3}$	$y = \dfrac{1}{x^2}$

C4 Card set B – *Functions in index form*

$y = 2x^{-2}$	$y = 2x^{-\frac{1}{2}}$
$y = \frac{1}{2}x^{-2}$	$y = 2x^{\frac{1}{2}}$
$y = x^{\frac{1}{2}}$	$y = 2x^{-3}$
$y = x^{-2}$	$y = \frac{1}{2}x^{-\frac{1}{2}}$
$y = x^{-\frac{1}{2}}$	$y = \frac{1}{2}x^{-3}$

C4 Card set C – *Differentiated functions*

$\frac{dy}{dx} = -4x^{-3}$	$\frac{dy}{dx} = -\frac{1}{4}x^{-\frac{3}{2}}$
$\frac{dy}{dx} = -6x^{-4}$	$\frac{dy}{dx} = x^{-\frac{1}{2}}$
$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$	$\frac{dy}{dx} = -x^{-3}$
$\frac{dy}{dx} = -x^{-\frac{3}{2}}$	$\frac{dy}{dx} = -\frac{3}{2}x^{-4}$
$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$	$\frac{dy}{dx} = -2x^{-3}$

C4 Card set D – *Integrated functions*

$\int y\,\mathrm{d}x = \frac{4}{3}x^{\frac{3}{2}} + c$	$\int y\,\mathrm{d}x = -\frac{1}{2}x^{-1} + c$
$\int y\,\mathrm{d}x = -x^{-2} + c$	$\int y\,\mathrm{d}x = -\frac{1}{4}x^{-2} + c$
$\int y\,\mathrm{d}x = \frac{2}{3}x^{\frac{3}{2}} + c$	$\int y\,\mathrm{d}x = -2x^{-1} + c$
$\int y\,\mathrm{d}x = 2x^{\frac{1}{2}} + c$	$\int y\,\mathrm{d}x = 4x^{\frac{1}{2}} + c$
$\int y\,\mathrm{d}x = -x^{-1} + c$	$\int y\,\mathrm{d}x = x^{\frac{1}{2}} + c$

C4 Card set E – *Differentiated functions in square root and fractional form*

$\frac{dy}{dx} = \frac{1}{\sqrt{x}}$	$\frac{dy}{dx} = -\frac{6}{x^4}$
$\frac{dy}{dx} = \frac{-3}{2x^4}$	$\frac{dy}{dx} = -\frac{2}{x^3}$
$\frac{dy}{dx} = -\frac{1}{4\sqrt{x^3}}$	$\frac{dy}{dx} = -\frac{4}{x^3}$
$\frac{dy}{dx} = -\frac{1}{\sqrt{x^3}}$	$\frac{dy}{dx} = -\frac{1}{x^3}$
$\frac{dy}{dx} = -\frac{1}{2\sqrt{x^3}}$	$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

C4 Card set F – ***Integrated functions in square root and fractional form***

$\int y\,\mathrm{d}x = -\dfrac{2}{x} + c$	$\int y\,\mathrm{d}x = -\dfrac{1}{x} + c$
$\int y\,\mathrm{d}x = -\dfrac{1}{2x} + c$	$\int y\,\mathrm{d}x = \sqrt{x} + c$
$\int y\,\mathrm{d}x = \dfrac{2}{3}\sqrt{x^3} + c$	$\int y\,\mathrm{d}x = \dfrac{4}{3}\sqrt{x^3} + c$
$\int y\,\mathrm{d}x = 2\sqrt{x} + c$	$\int y\,\mathrm{d}x = -\dfrac{1}{4x^2} + c$
$\int y\,\mathrm{d}x = -\dfrac{1}{x^2} + c$	$\int y\,\mathrm{d}x = 4\sqrt{x} + c$