

# C3 • Matching functions and derivatives

## Mathematical goals

To enable learners to:

- practise differentiating quadratic functions;
- practise finding the values of a function and its derivative at specific points;
- distinguish between  $f(x)$  and  $f'(x)$ ;
- relate values of  $f(x)$  and  $f'(x)$  to the graph of  $y = f(x)$ ;

and to reflect on and discuss these processes.

## Starting points

Learners should have some understanding of:

- function notation;
- differentiation of quadratic functions.

## Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Functions*.

## Time needed

At least 30 minutes.

## Suggested approach **Beginning the session**

Explain to learners that they will be given a set of 20 cards relating to five different functions. Learners have to make five groups of cards, so that each group refers to the same function.

### Working in groups

Ask learners to work in pairs. Give each pair a set of cards.

Some learners may be unclear about the difference between, for example,  $f(2)$  and  $f'(2)$ . If this is the case, invite other learners to explain.

Learners who are likely to struggle may be given a smaller set of cards and asked to create only three or four groups.

Learners who are likely to find the task easy may be given a set of cards with some of the  $f(x) = \dots$  cards missing. Learners have to create these cards to make complete groups.

Learners who finish quickly may be asked to make supplementary cards to add to each group, or they may enjoy devising their own groups at a more challenging level. These may then be given to other learners.

When most pairs have finished and every pair has created at least three groups of cards, ask learners, either in their pairs or in groups of four, to discuss what each card tells them about the graph of that function.

Ask each pair to explain to the whole group why they matched one of their cards to a particular function and what it tells them about the graph.

### Reviewing and extending learning

Using the mini-whiteboards, ask learners questions such as the following:

If  $f(x) = 3x^2 - 2x - 1$ , then show me the value of  $f(0)$ ,  $f(3)$ ,  $f'(0)$ ,  $f'(3)$ .

Show me a function where  $f(2) = 1$ .

Show me a function where  $f'(2) = 1$ .

If  $f'(x) = 2x - 3$ , then show me a possible function  $f(x)$ .

Sketch me a graph of a function where  $f(0) = 10$ .

Then sketch one where  $f(3) = 0$ .

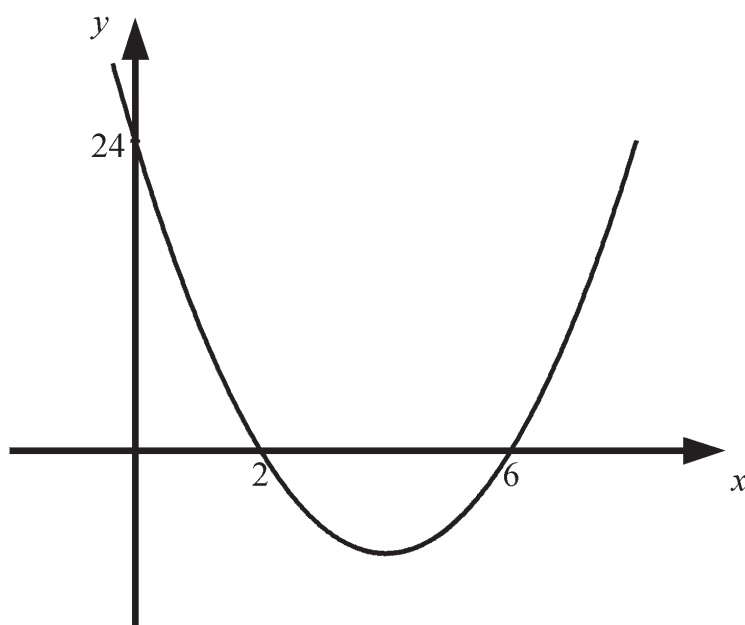
Sketch a graph of a function where  $f'(0) = 0$ .

Sketch a graph of a function where  $f'(2) = 0$ .

Sketch a graph of a function where  $f'(0) = 3$ .

### What learners might do next

Ask learners to write an explanation of the difference between  $f(4)$  and  $f'(4)$  for a function of their own choice.



Ask learners to describe as much as they can about  $f(x)$  and  $f'(x)$  at various points on this graph.

This session could be developed into work with stationary points.

### Further ideas

This idea could be used with other types of functions, e.g. cubic functions, or functions with fractional or negative indices.

This session could be adapted to practise substitution skills using surds, negative numbers or fractions.

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C3 Card set A – Functions

$f(x) = 3x^2 - 3x + 1$	$f(3) = -3$
$f(x) = x^2 - 12$	$f(0) = 5$
$f(x) = 2x - x^2 + 3$	$f(-1) = 0$
$f(x) = 4x^2 - 6x + 5$	$f(1) = 3$
$f(x) = 4x^2 - 3x + 2$	$f(2) = 7$
$f'(x) = 8x - 6$	$f'(x) = 2x$
$f'(x) = 6x - 3$	$f'(x) = 2 - 2x$
$f'(x) = 8x - 3$	$f'(2) = 10$
$f'(4) = 21$	$f'(1) = 5$
$f'(-1) = -2$	$f'(3) = -4$