A8 • Developing an exam question: generalising patterns

Mathematical goals

To help learners to:

- use past examination papers creatively;
- explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether a particular case can be generalised further;
- understand the importance of counter-examples;
- develop the ability to generalise from geometric patterns;
- devise and explore their own questions in this context.

Starting points

Most learners will be familiar with looking for growth patterns in geometric patterns and many will be able to express these patterns verbally. However, it may be that they will find it more difficult to express the nth term algebraically.

Materials required

For each small group of learners you will need:

- Sheet 1 – *Growing cross patterns*;
- Sheet 2 – *Template for growth patterns*;
- a supply of squared or triangular dotted or lined paper.

Time needed

About 1 hour.

Suggested approach

Beginning the session

Ask learners to work in pairs on the GCSE examination question in Sheet 1 – *Growing cross patterns*. When everyone has had time to have a go at this, ask them to gather round for a whole group discussion on the approaches used.

Whole group discussion (1)

(i) Answering the question

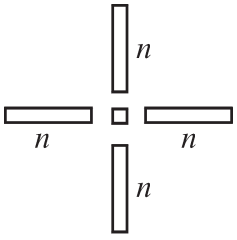
Ask learners to describe their methods for tackling part (a) of the question.

> Chris, why do you say that there are 21 squares in diagram 6?
> (It goes up in fours: 5, 9, 13, 17, 21.)
>
> Jack, tell me why you thought the answer was 26 squares.
> (You just double the number of squares that are in diagram 3.)
>
> What did you do, Sam?
> (I drew the diagram for six and then counted the squares.)

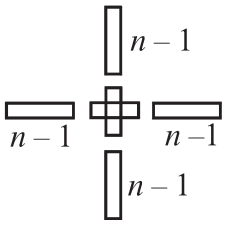
Ask learners to explain the advantages and disadvantages of each method.

> Chris's method works because there are four arms to the diagram and each time you add a square on at the end of each arm.
>
> Jack's method doesn't work. If you double the number for the first diagram you don't get the number for the second diagram.
>
> Sam's method will work, but it takes a long time.

Similarly, invite learners to suggest answers to parts (b) and (c) of the question and to explain their reasoning. Where several different methods are suggested, ask learners to say why they are correct (or not) and to justify why different answers may be equivalent.



> You have one square in the middle then each arm is n squares long. So the formula is $1 + 4n$.
>
> I started with the 5 squares in the middle then added 4 arms on with $n - 1$ squares on each. $5 + 4(n - 1)$.



> How can you tell that it is diagram 31 that has 125 squares?
>
> If $4n + 1 = 125$, then $4n = 124$ and $n = 31$.
>
> The first diagram uses 5 squares, which leaves 120 squares. $120 \div 4 = 30$, so it must be the 31st diagram.

(ii) Generating more questions

There are many other questions an examiner might ask. Ask learners to suggest some of these and write them at the bottom of Sheet 1. In doing this, they should not change the diagrams in any way, but simply ask new questions about the existing diagrams.

Can you have a cross diagram with 500 squares? How do you know?

The first cross is 3 squares long. How long is the nth cross?

The first diagram has a perimeter of 12. What is the perimeter of the 4th diagram? The 100th diagram? The nth diagram?

Is it possible to draw a cross diagram with a perimeter of 100? How can you be sure?

The first diagram can be made from 16 matchsticks. How many matchsticks would it take to make the 6th diagram? The 20th diagram? The nth diagram?

Is it possible to make a cross diagram with 100 matchsticks? How can you be sure?

Working in groups (1)

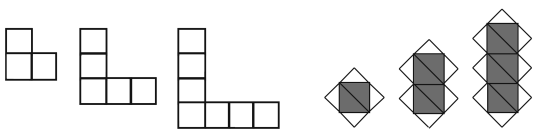
Ask learners to choose one of these questions that they think they can answer and encourage them to work on it in pairs. Learners may like to compare their different ideas by writing them on overhead transparencies or on the board.

Whole group discussion (2)

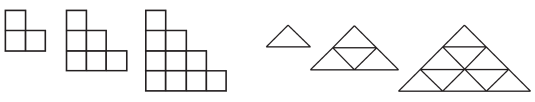
(iii) Developing the situation

Hand out copies of Sheet 2 – *Template for growth patterns*. Explain to learners that they will work in pairs or threes to write their own GCSE question using this template.

Discuss with them how they might do this. They will need to draw a clear simple starting diagram, use a consistent rule for growth, and think of a suitable way of measuring the growth (squares, perimeters, areas, matchsticks etc.).



Many possibilities may be suggested, e.g. using different shapes and colours.



Linear and quadratic sequences may be developed.

Working in groups (2)

Learners should then work in pairs or threes and write new questions together with solutions (on the back of the sheet). Encourage learners to ask questions that they consider challenging but that are within their capabilities.

If you wish, you may hand out squared or triangular dotted paper for learners to use when trying to devise more imaginative questions.

The new questions should be passed around the groups to be answered by other learners. Where learners have difficulties in answering questions, the question writers should explain what they intended and act as a teacher, helping each other to answer the questions.

Alternatively, some of the new questions may be photocopied for future sessions or for homework.

Reviewing and extending learning

Finally, hold a whole group discussion on what has been learned, drawing out any common misconceptions. You should include a discussion of the level of difficulty of the new questions.

What learners might do next

If learners wish to explore generalisations of patterns further, using a computer, there is a useful program at www.fi.uu.nl

Ask learners to choose another question from an exam paper and follow the process adopted in this session, i.e.

(i) Answer the question.

(ii) Ask new questions about the same situation, and answer them.

(iii) Change the situation and make a new question.

Further ideas

This method for developing exam questions may be used in any topic. Examples in this pack include:

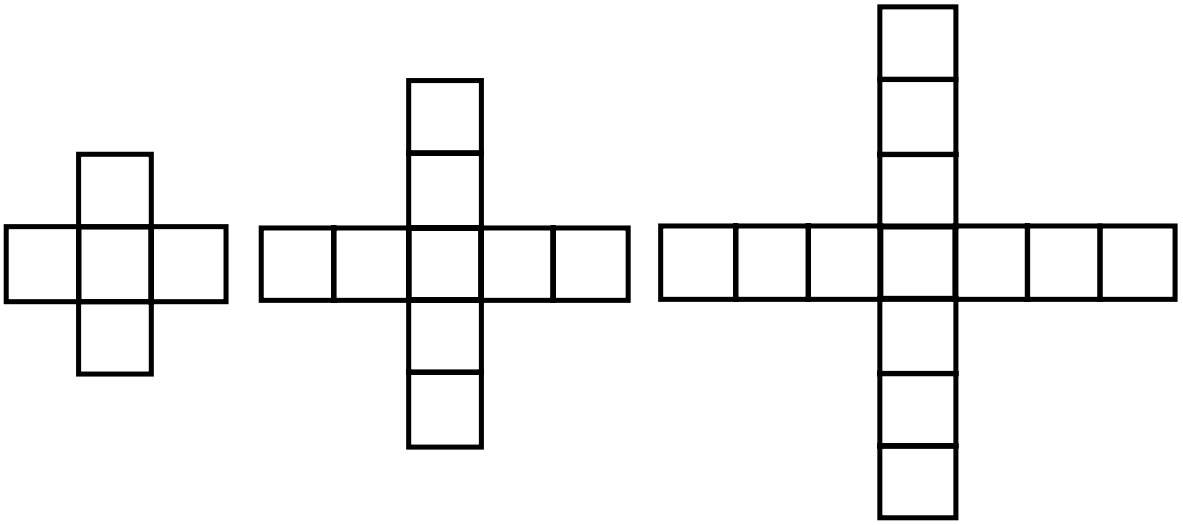
N10 Developing an exam question: number;

SS8 Developing an exam question: transformations;

S7 Developing an exam question: probability.

A8 Sheet 1 – *Growing cross patterns*

Some cross patterns are made of squares.



(a) How many squares will be in diagram 6?

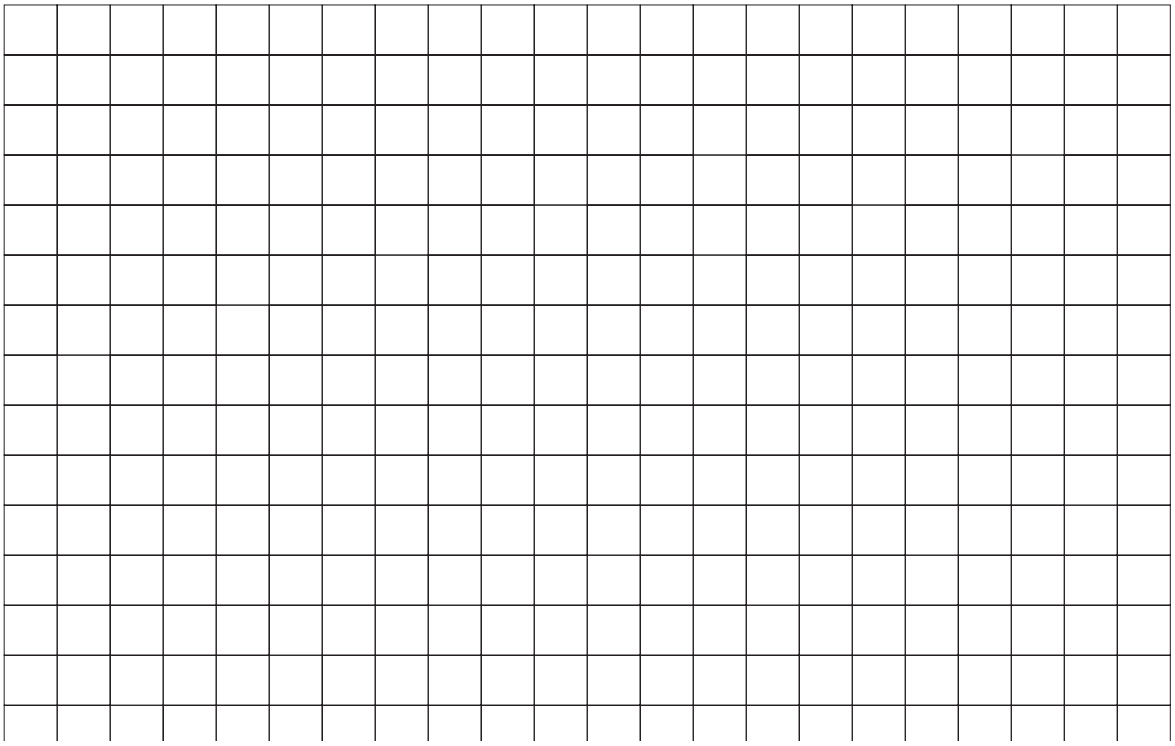
(b) Write down an expression for the number of squares in diagram n.

(c) Which diagram will have 125 squares?

Write down some other questions that may be asked about this situation.

A8 Sheet 2 – *Template for growth patterns*

A pattern is made of. . . . . .



(a) How many . . . . . will be in diagram number . . . . . ?

(b) Write down an expression for the number of . . . . . in diagram n.

(c) Which diagram will have . . . . . .

(d) . . . . . .

. . . . . .

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