A7 • Interpreting functions, graphs and tables

Mathematical goals

To enable learners to understand:

- the relationship between graphical, algebraic and tabular representations of functions;
- the nature of proportional, linear, quadratic and inverse functions;
- doubling and squaring, and the effect on positive and negative numbers.

Starting points

Learners should already be familiar with algebraic symbols such as those representing squares, square roots and fractions. This will be revised during the introduction. Most learners will already be familiar with making tabular representations of functions and drawing graphs, but they may not have considered the relations between different representations of functions.

Materials required

For each learner you will need:

- calculator;
- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Introductory activity*;
- Card set B – *Formulae*;
- Card set C – *Words*;
- Card set D – *Graphs*;
- Card set E – *Tables*;
- Card set F – *Variations*.

The computer program *Machines* may be used to enrich the work in this session. It is included on the DVD and is also available, with many others, at the Freudenthal Institute website www.fi.uu.nl (on the website it is called 'Algebra arrows', or 'Algebra pijlen'). It is used here with kind permission. Instructions for using this program are given on Sheet 1 – *Instructions for using the software: machines*.

Time needed

At least 1 hour.

Suggested approach

Beginning the session

The first part of the session aims to help learners recall the meaning of algebraic representations.

Arrange learners into pairs. Give each pair Card set A – *Introductory activity*.

Ask them to imagine that $n = 4$ and then try to place the cards in order of size. If they struggle, ask them to say in their own words what each symbol means. When they have placed the cards in order, ask them to repeat the task when n takes other values, for example $n = 9$, then $n = \frac{1}{4}$. Allow the use of calculators for this activity.

To enliven the activity, invite six learners out to the front and give each one a large version of a card. They must then place themselves in order, using suggested substitutions from the rest of the group. To make things more difficult, you could ask the group to nominate one standing learner and then find a substitution that will make this person stand at one end of the line.

Working in groups

Hand out Card sets B – *Formulae* and C – *Words* to each pair of learners. Ask learners to take it in turns to try to match a pair of cards and to explain why these two cards are equivalent. This process will help learners to articulate the meaning of the cards. Where there are blank cards, learners should complete these themselves.

It is helpful if learners are asked to place the cards side by side so that you can monitor their work as you move round the room. Some of the cards are more difficult in that they require the formula to be slightly rearranged.

When they have completed this, give learners Card sets D – *Graphs* and E – *Tables,* to be matched with those already on the table. These cards encourage learners to substitute numbers into each algebraic representation and to link it with a graphical representation.

If the table becomes too cluttered at this point, you may suggest that learners remove the *Words* cards.

If learners find the work difficult, suggest that they work with the easier cards first: say, $y = 2x$, $y = x + 2$, $y = 2$, $y = x - 2$, $y = \frac{x}{2}$.

Once they have correctly matched these cards, they may be ready to match the remaining cards.

Learners who complete the activity quickly may enjoy trying to match Card set F – *Variations* with the sets on the table. These invite learners to consider the effect on one variable as the other is changed.

Reviewing and extending learning

During the final part of the session, use mini-whiteboards and questioning to see if learners can begin to generalise.

Show me a graph of:

- $y = x + 3$;
- $y = -x + 3$;
- $y = \frac{3}{x}$.

Show me:

- an equation of a straight line graph that goes through the origin;
- . . . and another; and another; now a steeper line;
- an equation of a straight line graph that goes through (0,2);
- . . . and another; and another; now a steeper line;
- an equation of a line with a negative gradient;
- an equation of a parabola that goes through (0,3);
- and so on . . .

What learners might do next

Learners may use the computer program *Machines* to construct function machines and explore their graphs. Instructions are given on Sheet 1 – *Instructions for using the software: machines*.

Further ideas

This activity uses multiple representations to deepen understanding of functions. This type of activity may be used in any topic where a range of representations is used. Examples in this pack include:

A6 Interpreting distance–time graphs;

SS6 Representing 3D shapes.

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A7 Card set A – *Introductory activity*

n	n^2
$\sqrt{n}$	$8n$
$\frac{36}{n}$	$\frac{n}{2} + 1$

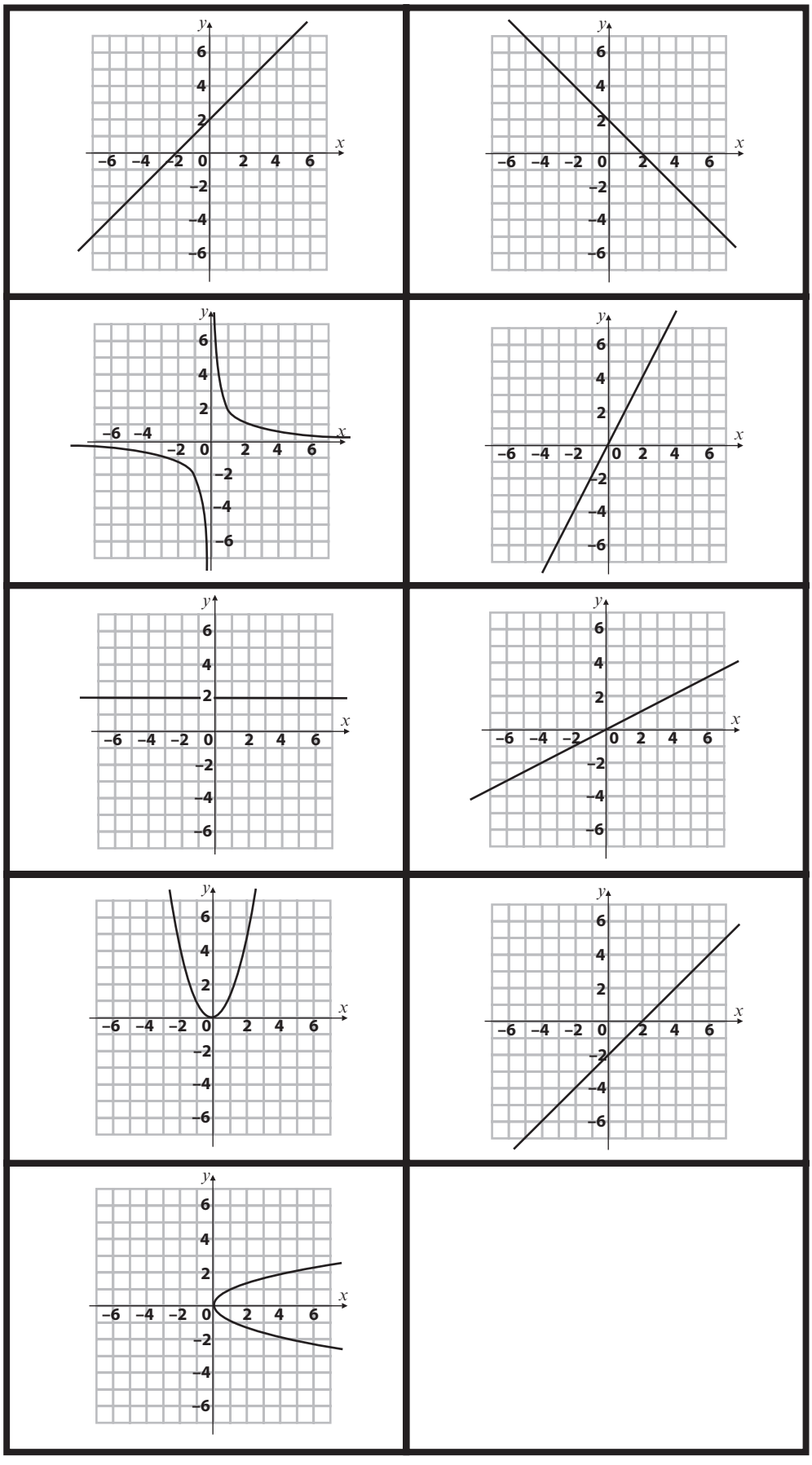
A7 Card set B – *Formulae*

$y = x^2$	$y = 2x$
$y^2 = x$	$y = x + 2$
$2y = x$	$y = 2$
$y = x - 2$	$xy = 2$
$y = \pm\sqrt{x}$	$x + y = 2$
$y = \frac{2}{x}$	$y = \frac{x}{2}$
$y = -x + 2$	$x = \pm\sqrt{y}$

A7 Card set C – *Words*

y is one half the size of x	x added to y is equal to 2
y is 2 more than x	x multiplied by y is equal to 2
y is 2 less than x	y is double the size of x
y is always equal to 2	x is the same as y multiplied by y
y is the same as 2 divided by x	y is the same as x multiplied by x
x is the square root of y	y is the same as x divided by 2

A7 Card set D – *Graphs*



A7 Card set E – *Tables*

x	–2	–1	0	1	2	3
y	–4	–3	–2	–1	0	1

x	–2	–1	0	1	2	3
y	4	3	2	1	0	–1

x	–2	–1	0	1	2	3
y	–1	–0.5	0	0.5	1	1.5

x	–2	–1	0	1	2	3
y	–4	–2	0	2	4	6

x	0	1	4	9	16
y	0	±1	±2	±3	±4

x	–2	–1	0	1	2	3
y	0	1	2	3	4	5

x	–2	–1	0	1	2	3
y	4	1	0	1	4	9

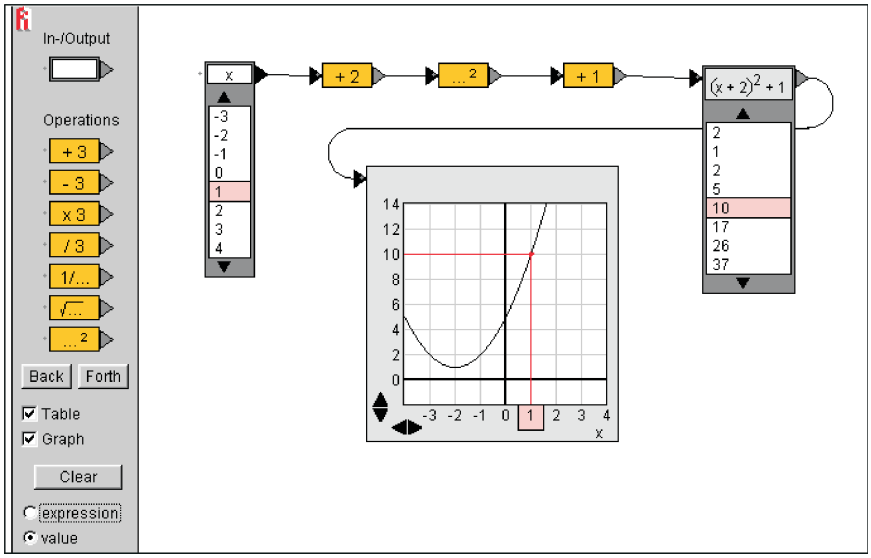
x	–2	–1	0	1	2	4
y	–1	–2	±∞	2	1	0.5

x	–2	–1	0	1	2	3
y	2	2	2	2	2	2

A7 Card set F – *Variations*

If you double x, y doubles.	If you double x, y halves.
If you add 1 to x, 2 is added to y.	If you add 1 to x, 1 is added to y.
If you add 1 to x, one half is added to y.	If you double x, y doesn't change.
If you double x, y is multiplied by 4.	If you multiply x by 4, y is multiplied by 2.

A7 Sheet 1 – *Instructions for using the software: machines*



This program provides an interactive way of creating function machines with one instance of the variable. It allows learners to explore connections between the machines, algebraic formulae, tables of numbers and graphs.

Running the program

As an example, suppose you want to make a machine for the function $y = (x + 2)^2 + 1$.

- Drag an input box onto the white screen area.
- The operations you want to do, in order are: +2, square, +1. At the moment, these are not all shown on the left hand side, so you must make them. Drag a yellow +3 box into the white screen. Now click on the +3 and change it to +2. Drag the arrow from the right hand side of the input box to the +2 yellow box. It should attach itself.
- Drag the yellow function box for squaring [....^{2}] onto the white screen and attach an arrow from the +2 function to this box. Finally repeat the process and create the +1 function.
- Now join this onto an output box. You now have finished making the machine.
- If you want to remove anything from the screen, just drag it off.

Try typing a number into the input box. Do you get the number you expect in the output box?

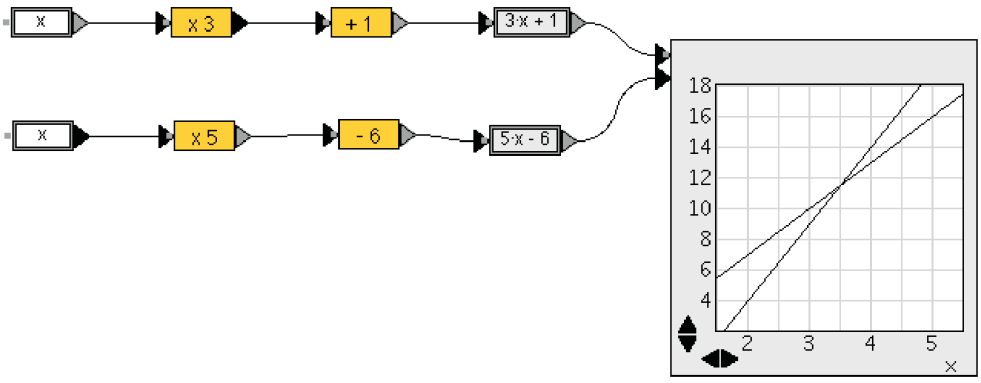
Try typing x into the input box. Do you get the correct formula in the output box?

With x in the input box, try clicking on the 'Table' button. What happens? Click on a number in the 'Table' box and see what happens now. You can scroll up and down the table boxes by clicking on the arrows at the top and bottom.

Click on the 'Graph' button and join the arrow from the output box onto the graph. Drag the inside of the graph around and adjust the scales by clicking on the arrows until you can see it clearly.

A7 Sheet 1 – *Instructions for using the software: machines* (continued)

You can of course have more than one function machine on the graph at the same time. This is useful when solving simultaneous equations. For example, you can solve $y = 3x + 1$ and $y = 5x - 6$ graphically:



If you click on the 'Back' button, the operations are all reversed. This is useful when showing inverse functions.

For example, the inverse function for $y = \sqrt{\frac{x+2}{2}}$ is $x = 2y^2 - 2$