A14 • Exploring equations in parametric form

Mathematical goals

To enable learners to:

- find stationary points when a function is given in parametric form;
- determine the nature of stationary points when the function is given in parametric form;
- find the intercepts when a function is given in parametric form.

Starting points

Learners should be able to find stationary points and their nature when equations are given in Cartesian form. Learners should understand the parametric form of an equation in the sense of being able to find pairs of coordinates for given values of the parameter. They should also know how to find $\frac{dy}{dx}$ from an equation given in parametric form.

Materials required

OHT 1 – *Graphs 1*

OHT 2 – *Graphs 2*

These could be shown on an interactive whiteboard.

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Graphs* (2 pages);
- Card set B – *Equations*;
- Sheet 1 – *Where are the errors?*;
- at least one large sheet of paper for making a poster;
- felt tip pen;
- glue stick.

Time needed

At least 1 hour 15 minutes.

Suggested approach

Beginning the session

Show OHT 1 – *Graphs 1* on the board, hand-drawn on the board or via an OHP or a data projector. Ask learners to describe what is the same and what is different about these two graphs.

Then put OHT 2 – *Graphs* 2 on the board. This time ask learners, working in groups of two or more, to discuss the similarities and differences between the two graphs (one graph is the thick lines and the other is the thin lines). Ask for suggestions and write them briefly on the board. Then go through the list asking learners what sort of mathematics would be used to investigate that difference, e.g. "One graph is steeper than the other for a certain range of values of x" becomes "Investigate the gradient via $\frac{dy}{dx}$". Encourage learners to comment on the shapes of the graphs and to suggest what happens when x is large, for example.

If necessary, check, using mini-whiteboards, that learners remember how to differentiate equations in parametric form.

Working in groups

Ask learners to work in pairs. Explain that they are going to be given five equations in parametric form and five graphs. Their task is to match each graph with its equation and to give as much justification as they can for their decision. Encourage them to use some of the criteria that they suggested earlier. Explain that they should use their knowledge of dealing with Cartesian equations and work out how to apply it to parametric equations. All working should be shown on the large sheets of paper. The pairs of graphs and equations should be stuck alongside the working.

Give Card set A – *Graphs* and Card set B – *Equations* to each pair of learners, plus a large sheet of paper, felt tip pen and glue stick.

The task could be set at different levels of challenge. 'Normal' challenge could be graphs B, C, D and E along with their equations; 'medium' challenge could be all five graphs and five equations; 'high' challenge could be graphs B, C, D and E with all the equations and learners have to sketch the missing graph.

As learners investigate their functions they may get confused between t, x and y. Some may try to identify the stationary point at a value of t instead of x. It is helpful to go round and ask learners to guide you through what they are doing and why they have come to the conclusion that they did. It is also a good opportunity for learners to see that:

- not all curves have stationary points and the algebra can show this;
- important properties about the shape of a curve can be found by looking at t approaching zero or infinity.

Whole group discussion

When all pairs have matched equations to graphs with some justification, share the findings. Ask each pair of learners to identify something about one of their graphs that surprised them and how the algebra showed them what was actually happening. Encourage those who have used non-standard reasons for justifying their matching to share these reasons.

Reviewing and extending learning

Give each pair of learners Sheet 1 – *Where are the errors?*. Ask them to mark the answers and identify any errors. When learners have finished, discuss the errors and clarify the misconceptions.

What learners might do next

Find equations of tangents and normals using equations in parametric form.

Further ideas

Matching equations and graphs and justifying decisions can be used for all types of functions. It allows learners to answer at different levels.

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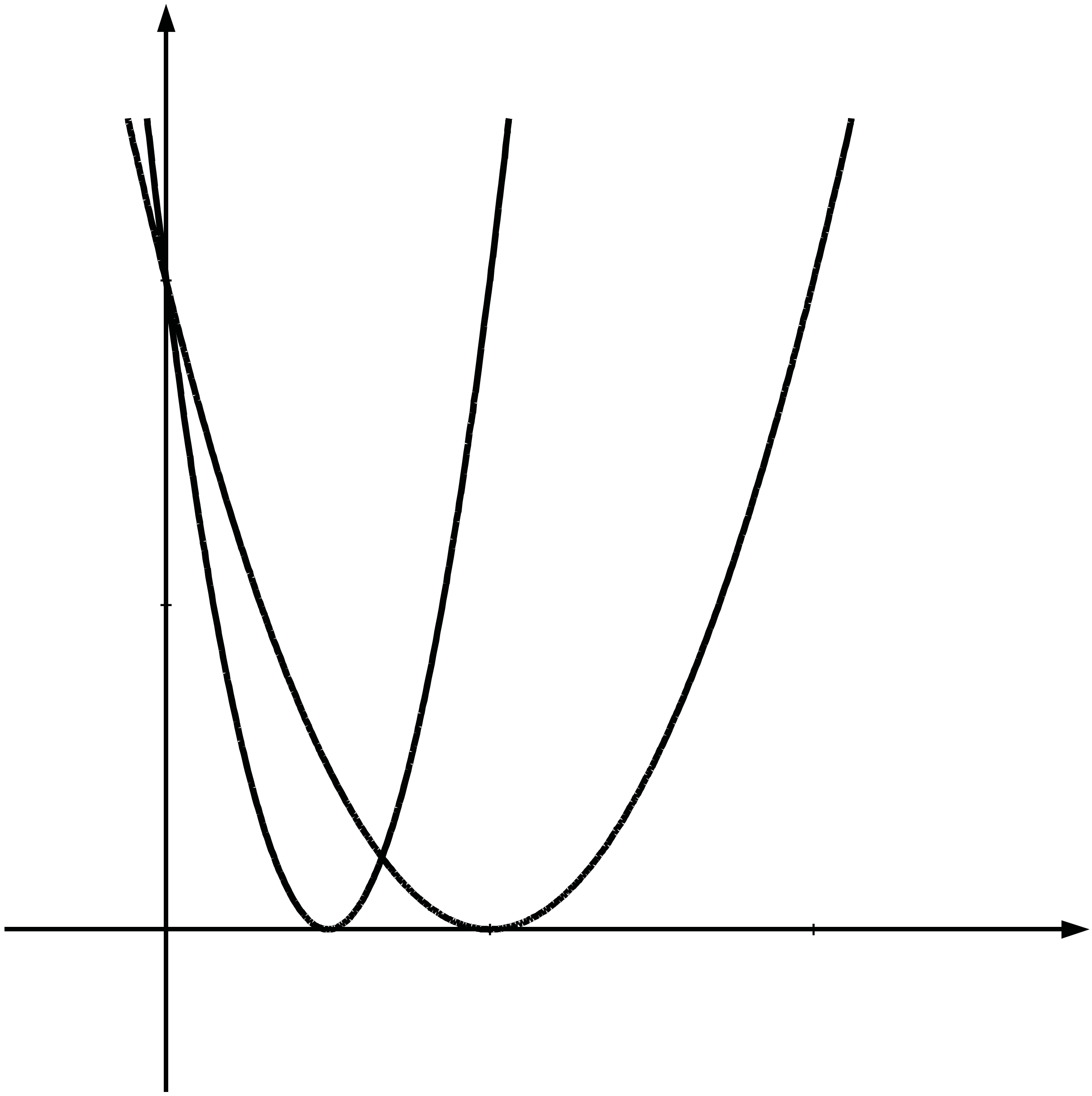
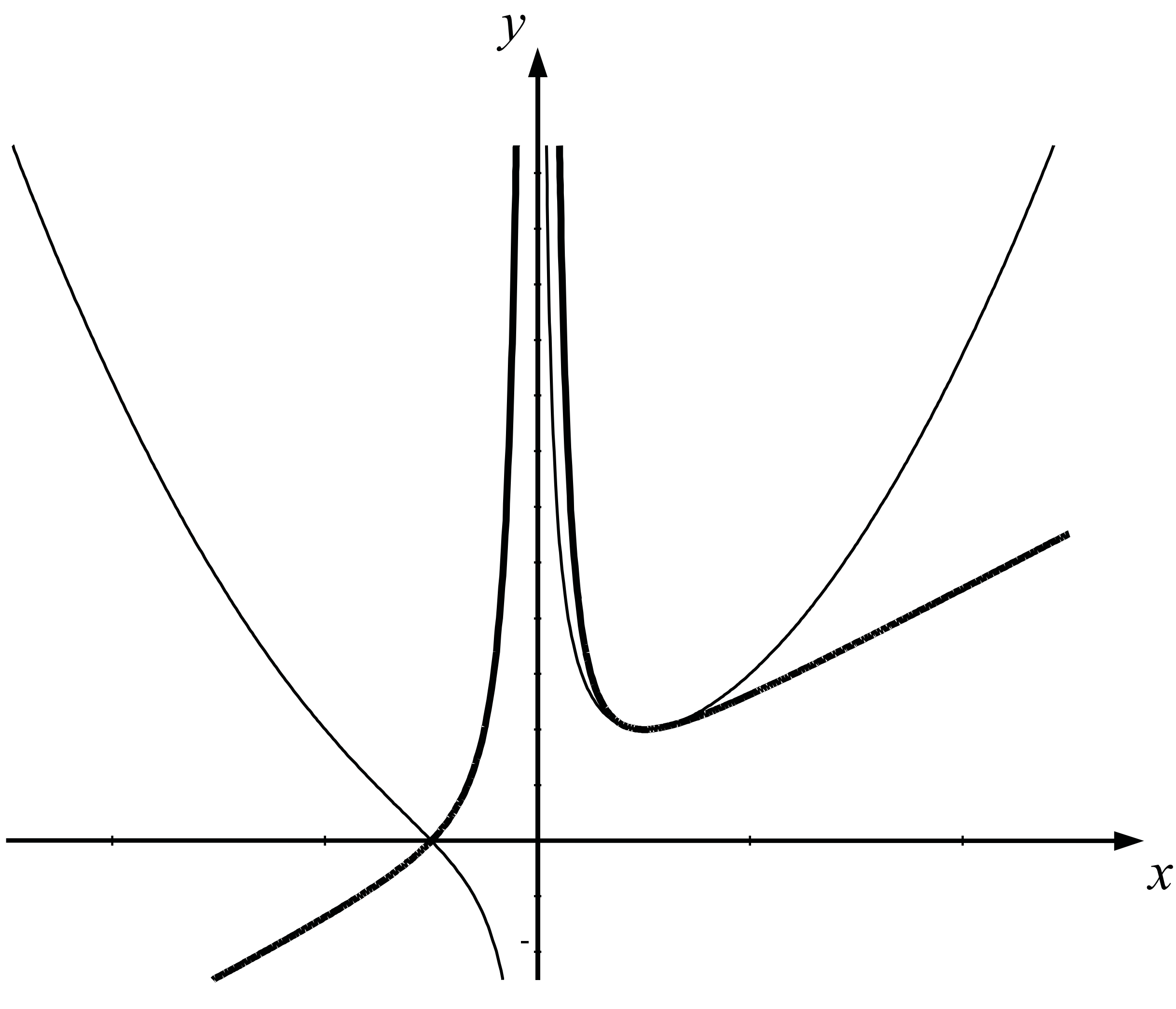
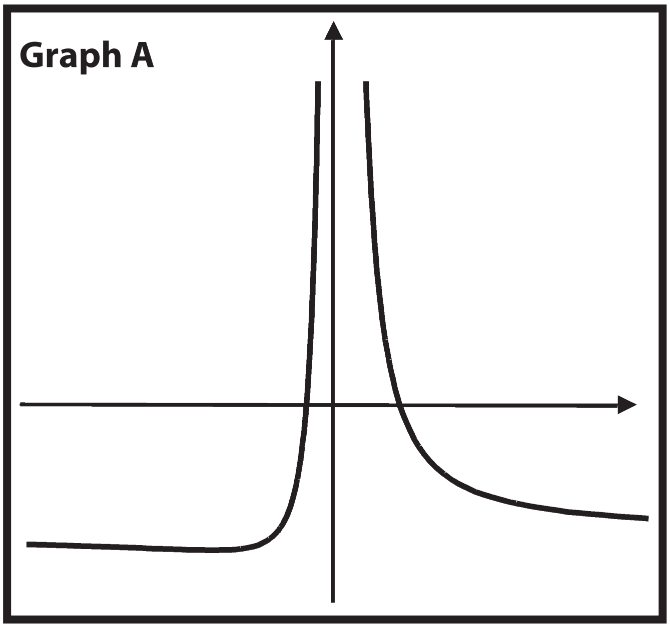
A14 OHT 1 – *Graphs 1*

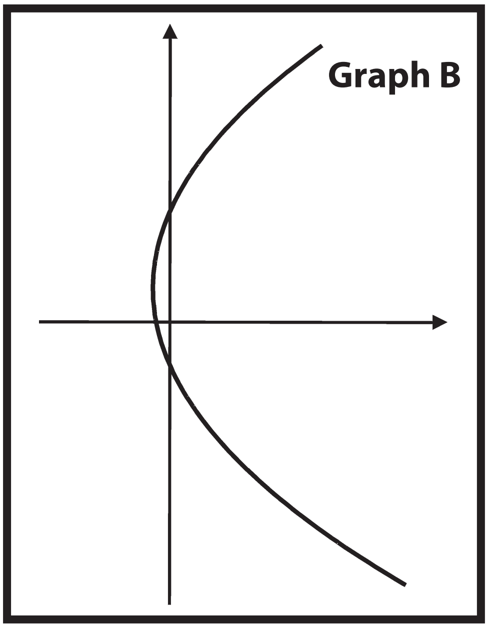


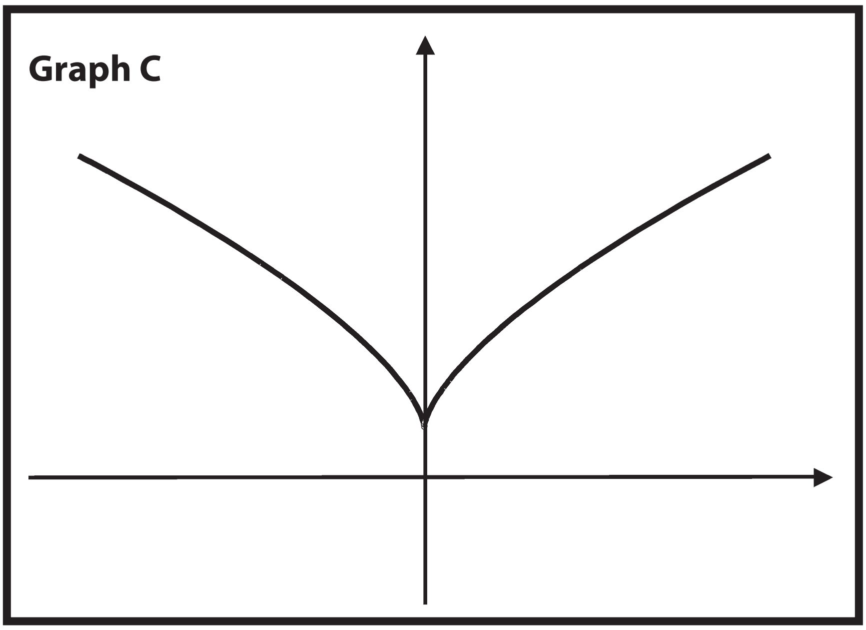
A14 OHT 2 – *Graphs 2*



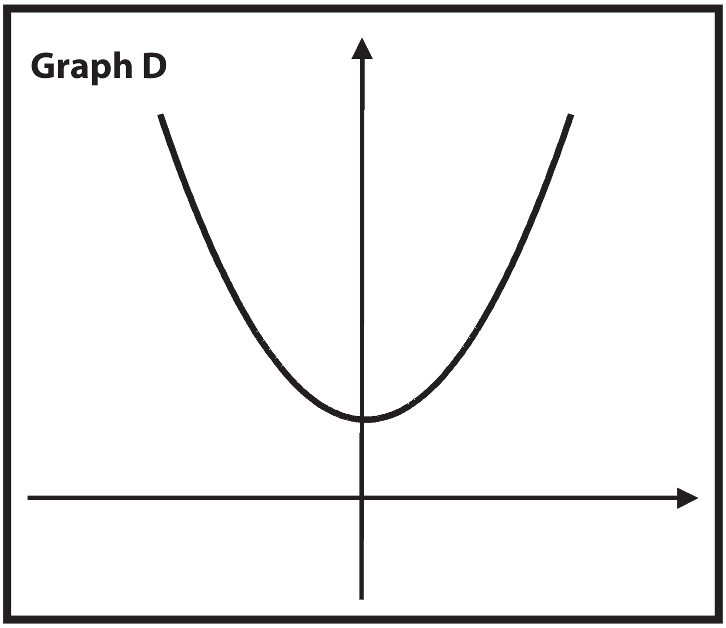
A14 Card set A – *Graphs and equations*

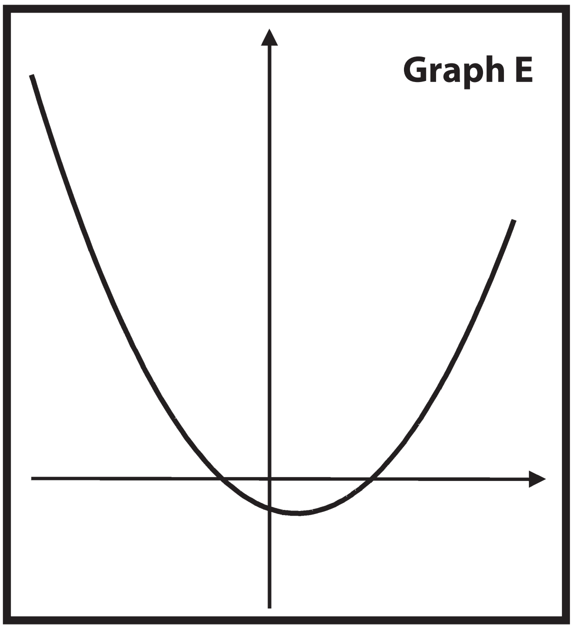






A14 Card set A – *Graphs and equations* (continued)





A14 Card set B – *Equations*

$x = 3t + 2$	$y = t^2 + t$
$x = t^3$	$y = t^2 + 1$
$x = \dfrac{1}{t}$	$y = t^2 + t - 2$
$x = 0.5t$	$y = t^2 + 1$
$x = t + t^2$	$y = 3t + 2$

A14 Sheet 1 – *Where are the errors?*

Questions

1. Find the intercepts of the graph whose equation is: $x = t^2 - 1, \quad y = t - 6$

2. Find the gradient of the curve: $x = t^3, \quad y = 2t^2 - 1$
   at the point where $t = 3$.

3. Find the gradient of the curve: $x = \dfrac{12}{t^4}, \quad y = 3t + 1$
   at the point (3,7).

4. Find any stationary points on the curve: $x = 2t^2 - 1, \quad y = t^3 + 1$

Answers

1.

$x = t^2 - 1$
$x = 0 \Rightarrow t^2 - 1 = 0$
$\Rightarrow t^2 = 1$
$\Rightarrow t = 1$
$\Rightarrow y = -5$

$y = t - 6$
$y = 0 \Rightarrow t - 6 = 0$
$\Rightarrow t = 6$
$\Rightarrow x = 35$

2.

$$\frac{dx}{dt} = 3t^2 \qquad \frac{dy}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{3t^2}{4t} = \frac{3 \times 3^2}{4 \times 3} = \frac{27}{12} = 2.25$$

3.

$y = 3t + 1 = 7 \qquad \Rightarrow \qquad t = 2$

$$\frac{dy}{dt} = 3$$

$$\frac{dx}{dt} = 4 \times \frac{12}{t^3}$$

$$\text{At } t = 2, \frac{dx}{dt} = 4 \times \frac{12}{8} = 6$$

$$\frac{dy}{dx} = \frac{3}{6} = 0.5$$

4.

$$\frac{dy}{dt} = 3t^2 \qquad \frac{dx}{dt} = 4t$$

$$\frac{dy}{dx} = \frac{3t^2}{4t} = \frac{3t}{4} = 0 \qquad \Rightarrow \qquad t = 0$$

$\therefore$ *Stationary point is at (0,1)*