A12 • Exploring trigonometrical graphs

Mathematical goals

To give learners practice in:

- recognising translations, stretches and reflections of trigonometrical graphs from their equations;
- sketching trigonometrical graphs.

To introduce learners to:

- the period and amplitude of a trigonometrical graph.

Starting points

Learners should be familiar with the graphs of $y = \sin x$ and $y = \cos x$.

Learners should have previously met the basic transformations of functions, i.e. one-way stretches, translations and reflections, but not necessarily in the context of trigonometrical graphs.

Materials required

For each learner you will need:

- mini-whiteboard;
- Sheet 2 – *Trigonometrical graphs*.

For each small group of learners you will need:

- Card set A – *Trigonometrical equations*;
- some blank paper or card;
- Sheet 1 – *Properties of trigonometrical graphs, enlarged to A3 size*;

and optionally either:

- a computer with graph-drawing software;

or

- graphic calculators.

Time needed

At least 45 minutes.

Suggested approach

Beginning the session

Remind learners what the graphs of $y = \sin x$ and $y = \cos x$ look like and discuss the important features.

Working in groups (1)

Ask learners to work in pairs; give out Card set A – *Trigonometrical equations* to each pair of learners. Ask them to sort the cards into two sets, deciding for themselves the criteria for their sets. Then ask them to sort their largest set into two subsets, again choosing their own criteria. Ask them to repeat this once more for their largest set. They will now have four sets of cards. Their criteria might be as simple as 'Has a 2 in front' but that is fine.

Give out some blank paper or card. Ask learners to write a description of each of their sets and add an equation of their own to each set.

Whole group discussion (1)

Share all the criteria that learners have come up with and translate them into mathematical language if necessary; for example, "they have a 3 in front" can become "a one-way stretch in the direction of the y-axis with scale factor 3"; "A number in the bracket with x" can become "a translation of . . . in the direction of the x-axis".

If learners are not familiar with the link between transformations of a graph and its equation, use a computer with graph-drawing software or graphical calculators to justify the descriptions.

If learners need more practice with transformations and trigonometrical graphs, use mini-whiteboards to ask questions such as:

> Give me the equation of $y = \sin x$ after it has been reflected in the x-axis.
>
> Give me the equation of $y = \cos x$ after it has been stretched with a scale factor of 2 in the direction of the x-axis.
>
> $y = \sin x$ is transformed into $y = 4 \sin x$. What was the transformation involved?

Define 'period' and 'amplitude' and link these with stretches. Check learners' understanding of how the equation gives information about period and amplitude by asking them to find cards that fit certain criteria such as:

> Find me the equations of all the graphs that have an amplitude 3.

Find me the equations of two graphs that have the same period.

Find me the equation of a graph that has period 180°.

etc.

Working in groups (2)

Give out Sheet 1 – *Properties of trigonometrical graphs (enlarged)* to each pair of learners and ask them to find cards from Card set A to fit as many boxes as possible. When they have found a possible card they should place it in the right box on Sheet 1, covering up the property. They should aim to cover up as many boxes as possible.

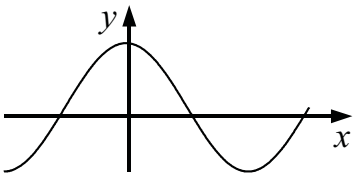
If learners finish early, ask them to find out if any of their remaining cards from set A fit into a box on Sheet 1. If there are some they cannot fit anywhere, ask them to extend the grid and write properties for those cards. If they have any boxes for which they cannot find a card they should write one for it.

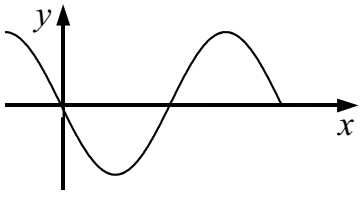
Whole group discussion (2)

It is likely that different pairs of learners will have placed different cards on a particular box. Choose one of the boxes and write down all the different cards that have been placed on it. Discuss as a whole group whether they are all appropriate and, if so, why. Repeat this for a few other boxes.

Give out mini-whiteboards and ask questions such as:

- Sketch me the graph of $y = 3\sin x$ or $y = \sin 2x$ or $y = \cos(x + 90)$
- Give me possible equations for





etc.

Reviewing and extending learning

Give out Sheet 2 – *Trigonometrical graphs* to each learner. Ask them to decide which equation goes with which graph. When they have done this, they should label the x and y axes accordingly.

Learners could then generalise their work by sketching and labelling graphs such as $y = a \sin x$ and $y = \cos bx$.

What learners might do next

Learners could consider how the axes of trigonometrical graphs would be labelled if the equations were using radians.

Further ideas

Sorting cards according to criteria chosen either by learners or by the teacher can be used for other functions such as quadratic or linear functions or circles. It can also be used for properties such as number types or shapes.

A12 Card set A – *Trigonometrical equations*

$y = 2 \sin x$	$y = \sin(x + 90)$	$y = \cos x - 1$
$y = -\cos x$	$y = \sin x + 1$	$y = \cos(x + 90)$
$y = \sin(x - 90)$	$y = \cos(x + 180)$	$y = \cos 3x$
$y = \sin \frac{1}{2} x$	$y = \sin 2x$	$y = 3 \cos x$
$y = 2 \cos x$	$y = -\sin x$	$y = \cos(x - 180)$
$y = \frac{1}{2} \cos x$	$y = 3 \sin x$	$y = \cos 2x$

A12 Sheet 1 – *Properties of trigonometrical graphs*

Period 180° **Passes through (0, 0)**	**Amplitude 3** **Passes through (0, 0)**
Period 360° **Passes through (0, 1)**	**Amplitude 1** **Passes through (0, –1)**
Period 180° **Passes through (0, 1)**	**Amplitude 1** **Passes through (90, 0)**
Period 360° **Passes through (0, –1)**	**Amplitude 3** **Passes through (–90, 0)**
Period 360° **Passes through (0, 0)**	**Amplitude 2** **Passes through (0, 0)**
Period 720° **Passes through (0, 0)**	**Amplitude 2** **Passes through (90, 0)**

A12 Sheet 2 – *Trigonometrical graphs*

Match the equations to the graph and label the axes appropriately.

$y = -\sin x$ $y = 3 \sin x$ $y = \cos(x + 90)$ $y = \cos 2x$