A11 • Factorising cubics

Mathematical goals

To enable learners to:

- associate x-intercepts with finding values of x such that $f(x) = 0$;
- sketch graphs of cubic functions;
- find linear factors of cubic functions;
- develop efficient strategies when factorising cubic functions.

Starting points

Learners should have some familiarity with quadratic graphs and factorising quadratic functions.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Factors*;
- Card set B – *True/false*.

Time needed

At least 1 hour 15 minutes.

Suggested approach

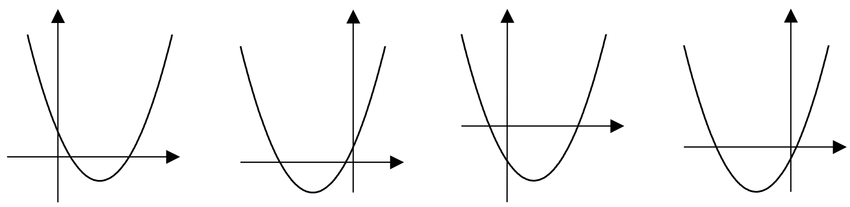
Beginning the session

Give out mini-whiteboards and ask learners to sketch a few straight lines such as $y = 4x - 8$ and $2y = 5x - 10$, marking the x and y intercepts. Through discussion, ensure that learners find the y intercept by putting x equal to zero and the x intercept(s) by putting y equal to zero.

Revise quadratic graphs by asking learners to sketch the graph of $y = (x - 3)(x + 4)$. Discuss how the x intercepts and y intercept were found.

Practise a few more examples.

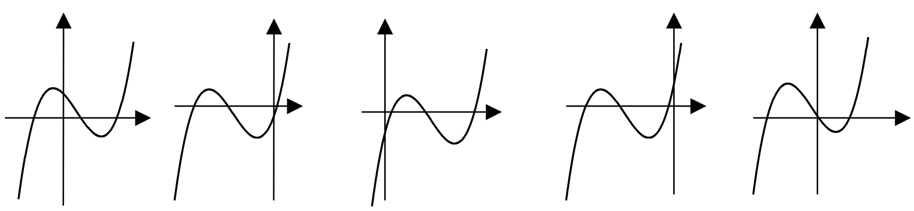
Then draw quadratic graphs on the board and ask for possible equations. Graphs could be:



Pay particular attention to the position of the intercepts.

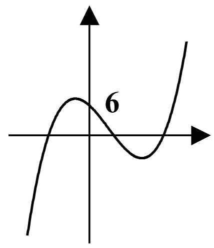
Then ask learners to sketch $y = (x - 1)(x - 2)(x - 3)$ on their whiteboards, using the same ideas (as they used with the quadratic functions) of finding the intercepts first. Check that everyone understands by asking learners to explain their graphs. Repeat for another two or three examples using a variety of + and – in the brackets.

Ask learners to sketch functions such as $y = (x - 2)(x^2 + 7x + 12)$. Discuss the usefulness of factorising. Then ask learners to suggest equations for graphs such as:



Use some of the suggestions to consider intercepts again. Then ask learners to suggest possible equations for graphs with a y intercept marked.

For example:



Repeat for a few more examples, discussing the need for the choice of brackets to give the correct y value when $x = 0$.

Ask learners what they can say about the sketch of $y = x^3 - 7x^2 + 4x + 12$ (e.g. y intercept is 12) and lead the discussion on to the need to be able to factorise cubic functions in order to sketch graphs. Ask for suggestions for factors and how they can be tested (values of x that make the brackets zero should also make y zero and making x equal to zero should give the same value of y in both the original equation and the factorised form).

Working in groups

Ask learners to work in pairs. Give Card set A – *Factors* to each pair and ask them to lay the cards out on the table. For a larger group of learners, provide two copies of Card set A, combined into a single set.

Write a cubic function on the board such as $y = x^3 - 2x^2 - 19x + 20$ and ask each pair/group to find as many sets of factors as they can (without checking any, other than giving the correct y intercept). Write a few sets on the board and ask for comments about whether they are appropriate ones to test.

Test one factor. Then ask the pairs if they want to change any of their set of three factors in the light of the result. Test another factor. Allow pairs to change factors again if they want to. Repeat until all the pairs have the correct set of three factors. Repeat for another cubic function such as $y = x^3 - 9x^2 + 26x - 24$.

Give each pair a cubic function that will factorise. Ask them to test values of x until they can come up with three factors. Suggest that it might be more convenient to use function notation to record their work. (This suggestion could have been made earlier.) When they have found all three factors, they should write them on the board. Give them another cubic function to work on, with the instruction that they should look at the completed ones to find strategies that make the process more efficient.

Learners who find the task difficult could be given functions that have positive roots.

Learners who find the task easy could be given a more challenging function for their second one, e.g. one with repeated roots, or passing through the origin.

Whole group discussion

Discuss the strategies that have been used and consider which are most efficient.

Reviewing and extending learning

Give each pair/group Card set B – *True/false*. They have to sort each pair of cards into 'True' and 'False' and be prepared to justify their answers. In whole group discussion, check that all learners understand the principles.

What learners might do next

This session could be followed by work on the remainder theorem and algebraic division.

A11 Card set A – *Factors* (page 1)

$(x-1)$	$(x-2)$	$(x-3)$
$(x+1)$	$(x+2)$	$(x+3)$
$(x-4)$	$(x-5)$	$(x-6)$
$(x+4)$	$(x+5)$	$(x+6)$
$(x-7)$	$(x-8)$	$(x-9)$
$(x+7)$	$(x+8)$	$(x+9)$
$(x-10)$	$(x+1)$	$(x+2)$
$(x+20)$	$(x+1)$	$(x+2)$
$(x-12)$	$(x+1)$	$(x-2)$
$(x-20)$	$(x+1)$	$(x-2)$

A11 Card set A – *Factors* (page 2)

$(x - 24)$	$(x - 1)$	$(x + 3)$
$(x + 24)$	$(x - 1)$	$(x + 6)$
$(x + 10)$	$(x - 1)$	$(x + 5)$
$(x + 12)$	$(x - 1)$	$(x + 4)$
$(x - 1)$	$(x - 2)$	$(x - 3)$
$(x + 1)$	$(x + 2)$	$(x + 3)$
$(x - 4)$	$(x - 5)$	$(x - 6)$
$(x + 4)$	$(x + 5)$	$(x + 6)$
$(x - 1)$	$(x - 2)$	$(x - 3)$
$(x + 1)$	$(x + 2)$	$(x + 3)$
$(x - 4)$	$(x - 5)$	$(x - 6)$

A11 Card set B – *True/false* (page 1)

A1

$f(2) = 0 \;\Rightarrow$
$(x + 2)$ is a factor of $f(x)$

A2

$f(2) = 0 \;\Rightarrow$
$(x - 2)$ is a factor of $f(x)$

B1

$f(-3) = 0 \;\Rightarrow$
$(x + 3)$ is a factor of $f(x)$

B2

$f(-3) = 0 \;\Rightarrow$
$(x - 3)$ is a factor of $f(x)$

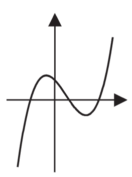
C1

$f(4) = 7 \;\Rightarrow$
$(x - 4)$ is a factor of $f(x)$

C2

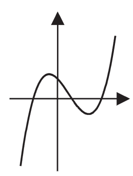
$f(4) = 7 \;\Rightarrow$
$(x - 4)$ is not a factor of $f(x)$

D1



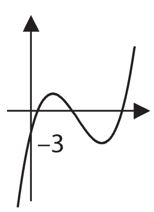
Possible equation for this graph is:
$f(x) = (x - 2)(x - 5)(x - 1)$

D2



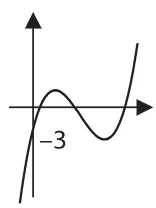
Possible equation for this graph is:
$f(x) = (x - 2)(x + 5)(x - 1)$

E1



Possible equation for this graph is:
$f(x) = x^3 + 6x^2 + 7x - 3$

E2



Possible equation for this graph is:
$f(x) = x^3 + 6x^2 + 7x + 3$

A11 Card set B – *True/false* (page 2)

F1 **$(x - 3)$ is a factor of** $f(x) = x^3 - x^2 - 3x - 2$	**F2** **$(x - 3)$ is not a factor of** $f(x) = x^3 + x^2 - 3x + 2$
G1 **If $f(x)$ is a cubic function and if $f(1) = 0$, $f(3) = 0$ and $f(0) = 12$ then $f(4) = 0$**	**G2** **If $f(x)$ is a cubic function and if $f(1) = 0$, $f(3) = 0$ and $f(0) = 12$ then $f(-4) = 0$**
H1 **If $f(x) = x^3 - 6x^2 - x + 6$ and $f(6) = 0$ then it would be a good idea to test $f(3)$**	**H2** **If $f(x) = x^3 - 6x^2 - x + 6$ and $f(6) = 0$ then it would be a silly idea to test $f(3)$**