A10 • Connecting perpendicular lines

Mathematical goals

To help learners to:

- identify perpendicular gradients;
- identify, from their equations, lines that are perpendicular;
- relate their learning about perpendicular lines to their previous learning about straight lines;
- explain the reasons why lines are parallel and perpendicular.

Starting points

Learners should:

- have some knowledge of equations of straight lines;
- be able to identify parallel lines;
- be able to use gradient triangles.

Materials required

For each learner you will need:

- mini-whiteboard;
- (possibly) a copy of Sheet 2 – *Perpendicular bisectors*.

For each small group of learners you will need:

- several sheets of squared paper;
- a protractor;
- Sheet 1 – *Properties, enlarged onto A3 paper*;
- Card set A – *Equations*;

optional

- graphic calculators.

Time needed

At least 1 hour 30 minutes.

Suggested approach

Beginning the session

Use mini-whiteboards to check that learners can remember how to calculate the gradient of a line between two given points.

Working in groups (1)

Ask learners to work in pairs. Give each pair some squared paper. Ask them to draw a line that has gradient 2. Next, ask them to draw a line that is perpendicular to this (using a protractor if necessary) and find its gradient. Suggest that they try other starting gradients and, working together, find the connection between a gradient and its perpendicular gradient.

Whole group discussion

Discuss the findings and check learners' understanding of gradients and perpendicular gradients by using mini-whiteboards and open questions such as:

> Give me an example of a line that has gradient 4.
>
> Give me an example of a line that is perpendicular to $y = 3x - 2$.
>
> Show me the equations of two lines that are perpendicular.

Working in groups (2)

Give each pair a copy of Sheet 1 – *Properties* (size A3) and Card set A – *Equations*. Ask them to match two equations to each property and add a property for the two that are left over.

The missing property is just passing through the point of intersection of the two lines. It is included so that learners cannot match the last two properties by default (i.e. there are only two cards left so they must fit into the last property).

The equations have been chosen to highlight the possible misconception that the number in front of x is the gradient and the y intercept is the number at the end. Therefore, while learners are working on matching, it is useful to ask them to explain their reasoning and to have graphic calculators or a computer with a graphical drawing package available to help sort out any problems.

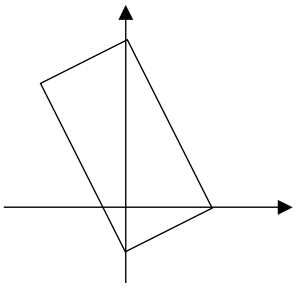
If a pair finish early, give them some blank cards to write another equation for each property and ask them to write a justification as to why they match.

Reviewing and extending learning

Discuss the matchings by asking questions such as:

> Why were these two not parallel?
>
> How do you know that these are perpendicular?
>
> These two equations have both got a 4 at the end. Why do they not have the same y intercept?

Ask learners to find possible equations to make the shape:



Ask learners to generalise their findings using the equation $ax + by + c = 0$.

What learners might do next

Use Sheet 2 – *Perpendicular bisectors* to follow through a skeleton solution. Learners should write an explanation beside each step of the solution.

Further ideas

The activity of matching cards to properties can be used for any type of function.

It can also be used for properties of number, shape and space, and data handling.

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A10 Card set A – *Equations*

$y = 4x + 4$	$4y = x + 3$
$y = 8x - 3$	$y + 4x + 6 = 0$
$3y = 2x - 8$	$y + 6x = 11$
$y + 8x = 6$	$2y + 8 = 3x$
$2y + x = 4$	$2y = 8x + 3$
$y = 6x - 4$	$y + x + 8 = 0$

These lines are parallel.	**These lines are perpendicular.**
These lines have the same y intercept.	**These lines have the same x intercept.**
These lines both go through the point (1, 5).	**These lines …..**

A10 Sheet 2 – *Perpendicular bisectors*

Question: Find the perpendicular bisector of the line joining the points (–2, 11) and (4, –7).

Follow the solution and explain what is happening at each stage.

Solution	Explanation
$-\frac{18}{6} = -3$	
$-3 \rightarrow \frac{1}{3}$	
$y = \frac{1}{3}x + c$	
$(1, 2)$	
$2 = \frac{1}{3} \times 1 + c$	
$c = \frac{5}{3}$	
$y = \frac{1}{3}x + \frac{5}{3}$	
$3y = x + 5$	