A1 • Interpreting algebraic expressions

Mathematical goals

To help learners to:

- translate between words, symbols, tables, and area representations of algebraic expressions;
- recognise the order of operations;
- recognise equivalent expressions;
- understand the distributive laws of multiplication and division over addition (expansion of brackets).

Starting points

This unit develops the ideas presented in **N5 Understanding the laws of arithmetic**. Learners will need to recall how to find the area of simple compound shapes made from rectangles and simple indices, and to understand the difference between 3×2 and 3^2.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- Card set A – *Algebraic expressions*;
- Card set B – *Explanations in words*;
- Card set C – *Tables of numbers*;
- Card set D – *Areas of shapes*;
- glue stick;
- felt tip pens;
- large sheet of paper for making a poster.

Time needed

From 1 to 2 hours. If your sessions are normally shorter than this, you can split this session into two.

Suggested approach

Beginning the session

Hold a short question and answer session, using mini-whiteboards, to revise how the order of operations is represented algebraically.

Show me an algebraic expression that means:

Multiply n by 3, then add 4.
Add 4 to n, then multiply your answer by 3.
Add 2 to n, then divide your answer by 4.
Multiply n by n, then multiply your answer by 4.
Multiply n by 6, then square your answer.

Working in groups (1)

Arrange learners in pairs or threes.

Give each group Card set A – *Algebraic expressions* and Card set B – *Explanations in words*. Ask learners to take turns at matching cards and explaining their reasons for each matching. Point out that some cards are missing. Learners will need to make these extra cards themselves.

The activity is designed to help learners interpret the symbols and realise that the symbolism defines the order of operations. Some learners may notice that some expressions are equivalent, e.g. $2(n + 3)$ and $2n + 6$. Do not comment on this at this stage.

Now give learners Card set C – *Tables of numbers* and ask them to match these to Card sets A and B. Some tables have numbers missing. Learners will need to work these out.

Learners will soon notice that there are fewer tables than algebraic expressions. This is because some tables match more than one expression. Allow learners time to discover this for themselves.

This activity is designed to encourage learners to substitute into expressions and thus, again, to interpret their meaning. At this stage, they will begin to notice that several expressions are equivalent, but they may not realise why.

For learners who finish quickly, ask them to find out if the pairs of expressions always give the same answer, even when fractions or decimals are substituted. Push them further to explain why these pairs match for all numbers.

Encourage those who struggle to substitute numbers for letters in the algebraic expressions.

Reviewing and extending learning (1)

Ask learners to suggest reasons why different expressions always appear to give the same answer. (You don't need to provide answers yourself at this stage).

Can you generate additional examples of your own?

Volunteers may like to offer suggestions on the board.

Leave 'open' the question of why expressions are equivalent. The next part of the session will take these ideas further, so they do not need to be resolved at this point.

(If you run this over two sessions, this would be a good point to break off.)

Working in groups (2)

Begin this part of the session by considering just the *Algebraic expressions* cards (Set A). Ask learners to try to remember which expressions went together/were equivalent. If they have forgotten, they should try substituting some numbers of their own for n. This will encourage them to interpret the expressions without the help of the *Explanations in words* cards (Set B).

Now hand out Card set D – *Areas of shapes*. Ask learners to match these with the cards in Set A – *Algebraic expressions*. When learners reach agreement, they should paste their cards onto a large sheet of paper, to make a poster. They should write on the poster why the areas show that different algebraic expressions are equivalent. These posters may then be displayed for the final whole group discussion.

Reviewing and extending learning (2)

Hold a whole group discussion to review what has been learned during this session. Ask pairs of learners to present their posters.

Use mini-whiteboards and questioning to begin to generalise the learning.

Draw me an area that shows this expression: $3(x + 4)$.
Write me a different expression that gives the same area.

Draw me an area that shows this expression: $(4y)^2$.
Write me a different expression that gives the same area.

Draw me an area that shows this expression: $(z + 5)^2$.
Write me a different expression that gives the same area.

Draw me an area that shows this expression: $\frac{w + 6}{2}$.

Write me a different expression that gives the same area.

What rules have you found for rearranging expressions?

What learners might do next

Ask learners if they can draw diagrams that might show a set of expressions containing subtractions, such as the following:

$2(x - 3)$

$(x + 3)(x - 3)$

$x^2 - 9$

$(x - 3)^2$

$x^2 - 6x + 9$

Learners might contribute their own questions as a homework activity and evaluate the work of other learners as a follow-up.

Further ideas

This activity uses multiple representations to deepen understanding of algebra. This type of activity may be used in any topic where a range of representations is used. Examples in this pack include:

SS6 Representing 3D shapes;

SS7 Transforming shapes;

S5 Interpreting bar charts, pie charts, box and whisker plots;

S6 Interpreting frequency graphs, cumulative frequency graphs, box and whisker plots.

A1 Card set A – *Algebraic expressions*

E1 $\dfrac{n+6}{2}$	E2 $3n^2$
E3 $2n+12$	E4 $2n+6$
E5 $2(n+3)$	E6 $\dfrac{n}{2}+6$
E7 $(3n)^2$	E8 $(n+6)^2$
E9 $n^2+12n+36$	E10 $\dfrac{n}{2}+3$
E11 n^2+6	E12 n^2+6^2
E13	E14

A1 Card set B – *Explanations in words*

W1 **Multiply n by two, then add six.**	W2 **Multiply n by three, then square the answer.**
W3 **Add six to n, then multiply by two.**	W4 **Add six to n, then divide by two.**
W5 **Add three to n, then multiply by two.**	W6 **Add six to n, then square the answer.**
W7 **Multiply n by two, then add twelve.**	W8 **Divide n by two, then add six.**
W9 **Square n, then add six.**	W10 **Square n, then multiply by nine.**
W11	W12
W13	W14

A1 Card set C – *Tables of numbers*

T1

n	1	2	3	4
Ans	14	16	18	20

T2

n	1	2	3	4
Ans			81	144

T3

n	1	2	3	4
Ans		10	15	22

T4

n	1	2	3	4
Ans	3		27	48

T5

n	1	2	3	4
Ans			81	100

T6

n	1	2	3	4
Ans		10	12	14

T7

n	1	2	3	4
Ans		4		5

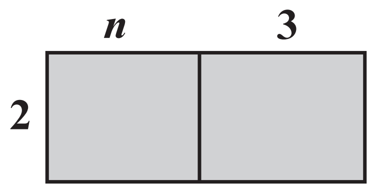
T8

n	1	2	3	4
Ans	6.5	7	7.5	8

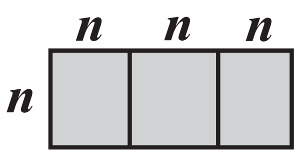
A1 Card set D – *Areas of shapes*

A1

A2



A3



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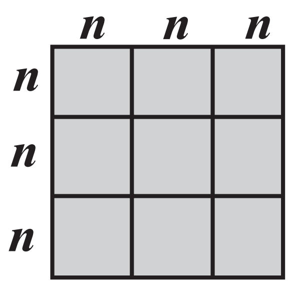
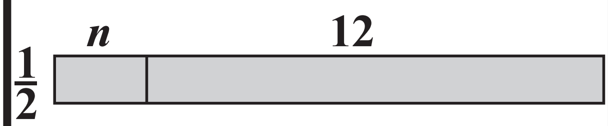


A5

A6



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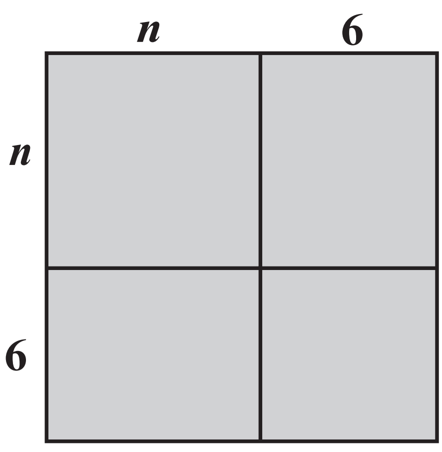

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