EDEXCEL STUDENT CONFERENCE 2006

A2 MATHEMATICS

STUDENT NOTES

South: Thursday 23rd March 2006, London

EXAMINATION HINTS

Before the examination

- 🕮 Obtain a copy of the formulae book – and use it!
- 🕮 Write a list of and LEARN any formulae not in the formulae book
- 🕮 Learn basic definitions
- 🕮 Make sure you know how to use your calculator!
- 🕮 Practise all the past papers - TO TIME!

At the start of the examination

- ✏ Read the instructions on the front of the question paper and/or answer booklet
- ✏ Open your formulae book at the relevant page

During the examination

- 🕑 Read the WHOLE question before you start your answer
- 🕛 Start each question on a new page (traditionally marked papers) or
- 🕐 Make sure you write your answer within the space given for the question (on-line marked papers)
- 🕕 Draw clear well-labelled diagrams
- 🕒 Look for clues or key words given in the question
- 🕖 Show ALL your working - including intermediate stages
- 🕘 Write down formulae before substituting numbers
- 🕔 Make sure you finish a ‘prove’ or a ‘show’ question – quote the end result
- 🕙 Don’t fudge your answers (particularly if the answer is given)!
- 🕚 Don’t round your answers prematurely
- 🕛 Make sure you give your final answers to the required/appropriate degree of accuracy
- 🕖 Check details at the end of every question (e.g. particular form, exact answer)
- 🕑 Take note of the part marks given in the question
- 🕐 If your solution is becoming very lengthy, check the original details given in the question
- 🕓 If the question says “hence” make sure you use the previous parts in your answer
- 🕚 Don’t write in pencil (except for diagrams) or red ink
- 🕒 Write legibly!
- 🕘 Keep going through the paper – go back over questions at the end if time

At the end of the examination

- 🗐 If you have used supplementary paper, fill in all the boxes at the top of every page

C3 KEY POINTS

C3 Algebra and functions

Simplification of rational expressions (uses factorising and finding common denominators)
Domain and range of functions
Inverse function, $f^{-1}(x)$ $[\,ff^{-1}(x) = f^{-1}f(x) = x]$
Knowledge and use of: domain of f = range of f^{-1}; range of f = domain of f^{-1}
Composite functions e.g. $fg(x)$
The modulus function
Use of transformations (as in C1) with functions used in C3

Transformation	Description
$y = f(x) + a$ $\quad a > 0$	Translation of $y = f(x)$ through $\begin{pmatrix} 0 \\ a \end{pmatrix}$
$y = f(x + a)$ $\quad a > 0$	Translation of $y = f(x)$ through $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
$y = af(x)$ $\quad a > 0$	Stretch of $y = f(x)$ parallel to y-axis with scale factor a
$y = f(ax)$ $\quad a > 0$	Stretch of $y = f(x)$ parallel to x-axis with scale factor $\frac{1}{a}$
$y = \lvert f(x)\rvert$	For $y \geq 0$, sketch $y = f(x)$ For $y < 0$, reflect $y = f(x)$ in the x-axis
$y = f(\lvert x\rvert)$	For $x \geq 0$, sketch $y = f(x)$ For $x < 0$, reflect $[y = f(x)$ for $x > 0]$ in the y-axis
Also useful	
$y = -f(x)$	Reflection of $y = f(x)$ in the x-axis (line $y = 0$)
$y = f(-x)$	Reflection of $y = f(x)$ in the y-axis (line $x = 0$)

C3 Trigonometry

$\sec x = \dfrac{1}{\cos x}$ $\quad$ $\operatorname{cosec} x = \dfrac{1}{\sin x}$ $\quad$ $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$; $\quad 1 + \tan^2 x = \sec^2 x$; $\quad 1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\qquad$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin 2x = 2\sin x \cos x$; $\quad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$; $\quad \tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Graphs of inverse trig. functions

$-\,{}^{\pi}/_{2} \le \arcsin x \le {}^{\pi}/_{2}$ $\quad 0 \le \arccos x \le \pi$ $\quad -\,{}^{\pi}/_{2} < \arctan x < {}^{\pi}/_{2}$

Expressing $a\cos\theta + b\sin\theta$ in the form $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$ and applications (e.g. solving equations, maxima, minima)

C3 Exponentials and logarithms

Graphs of $y = e^x$ and $y = \ln x$ and use of transformations to sketch e.g. $y = e^{3x} + 2$

Solutions to equations using e^x and $\ln x$ (e.g. $e^{2x+1} = 3$)

$e^{x\ln a} = a^x$

C3 Differentiation

$$\frac{d(e^x)}{dx} = e^x \qquad \frac{d(e^{kx})}{dx} = e^{kx} \qquad \frac{d(\ln x)}{dx} = \frac{1}{x} \qquad \frac{d(\ln kx)}{dx} = \frac{1}{x}$$

$$\frac{d(\sin kx)}{dx} = k\cos kx \qquad \frac{d(\cos kx)}{dx} = -k\sin kx \qquad \frac{d(\tan kx)}{dx} = k\sec^2 kx$$

Differentiation of other trig. functions: see formulae book

Chain rule $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$ $\qquad \dfrac{dy}{dx} = \dfrac{1}{\left(\dfrac{dy}{dx}\right)}$

Product rule $\dfrac{d(uv)}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ $\qquad$ Quotient rule $\dfrac{d(\frac{u}{v})}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

C3 Numerical methods

For a continuous function, a change in sign of $f(x)$ in the interval $(a, b) \Rightarrow$ a root of $f(x) = 0$ in the interval (a, b)

Accuracy of roots by choosing an interval (e.g. 1.47 to 2 d.pl. test f(1.465) and f(1.475) for change of sign)

Iterative methods: rearranging equations in the form $x_{n+1} = f(x_n)$ and using repeated iterations

C4 KEY POINTS

C4 Algebra and functions

Partial fractions: Methods for dealing with degree of numerator ≥ degree of denominator, partial fractions of the form

$$\frac{2x+3}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1} \text{ and } \frac{5-x}{(x+2)(x-3)^2} \equiv \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

C4 Coordinate geometry

Changing equations of curves between Cartesian and parametric form

Use of $\int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$ to find area under a curve

C4 Sequences are series

Expansion of $(ax + b)^n$ for any rational n and for $|x| < \frac{b}{a}$, using

$$(1 + x)^n = 1 + nx + {}^nC_2x^2 + \ldots + {}^nC_rx^r + \ldots + x^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!} + \ldots + x^n$$

where $^nC_r = \dfrac{n!}{r!(n-r)!}$

C4 Differentiation

Implicit and parametric differentiation including applications to tangents and normals

Exponential growth and decay

$$\frac{\mathrm{d}(a^x)}{\mathrm{d}x} = a^x \ln a$$

Formation of differential equations

C4 Integration

$\int \mathrm{e}^x\,\mathrm{d}x = \mathrm{e}^x + c \qquad \int e^{kx}\,\mathrm{d}x = \frac{1}{k}e^{kx} + c$

$$\int \frac{1}{x}\,\mathrm{d}x = \ln|x| + c \qquad \int \frac{1}{ax}\,\mathrm{d}x = \int \frac{1}{a}.\frac{1}{x}\,\mathrm{d}x = \frac{1}{a}\ln|x| + c_1 \quad \text{or} \quad \int \frac{1}{ax}\,\mathrm{d}x = \frac{1}{a}\ln|ax| + c_2$$

$$\int \cos kx\,\mathrm{d}x = \frac{1}{k}\sin kx + c \qquad \int \sin kx\,\mathrm{d}x = -\frac{1}{k}\cos kx + c \qquad \int \sec^2 kx\,\mathrm{d}x = \frac{1}{k}\tan kx + c$$

Use of $\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$ and $\displaystyle\int \mathrm{f}'(x)[\mathrm{f}(x)]^n\,\mathrm{d}x = \frac{[\mathrm{f}(x)]^{n+1}}{n+1} + c$

Integration of other trig. functions: see formulae book

Volume: use of $\int \pi y^2\,\mathrm{d}x$ when rotating about x-axis

Integration by substitution

Integration by parts

Use of partial fractions in integration
Differential equations: first order separable variables

e.g. $\mathrm{f}(x)\mathrm{g}(y)\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{h}(x)\mathrm{k}(y) \quad \Rightarrow \quad \displaystyle\int \frac{\mathrm{g}(y)}{\mathrm{k}(y)}\,\mathrm{d}y = \int \frac{\mathrm{h}(x)}{\mathrm{f}(x)}\,\mathrm{d}x$

Trapezium rule applied to C3 and C4 functions

$\displaystyle\int_a^b \mathrm{f}(x)\,\mathrm{d}x \approx \tfrac{1}{2}h[y_0 + y_n + 2(y_1 + \ldots + y_{n-1})]$ where $y_i = \mathrm{f}(a + ih)$ and $h = \dfrac{b-a}{n}$

C4 Vectors

If $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, $|\mathbf{a}| = \sqrt{(x^2 + y^2 + z^2)}$

If $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the unit vector in the direction of **a** is $[(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \div \sqrt{(x^2 + y^2 + z^2)}]$

Scalar product:

If $\mathbf{OP} = \mathbf{p} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $\mathbf{OQ} = \mathbf{q} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and $\angle POQ = \theta$, then

$\mathbf{p.q} = |\mathbf{p}||\mathbf{q}|\cos\theta$ and $\mathbf{p.q} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})\,.\,(a\mathbf{i} + b\mathbf{j} + c\mathbf{k}) = xa + by + cz$

If OP and OQ are perpendicular, $\mathbf{p.q} = 0$

Vector equation of line where **a** is the position vector of a point on the line and **m** is a vector parallel to the line:

$\mathbf{r} = \mathbf{a} + \lambda\mathbf{m}$

Vector equation of line where **a** and **b** are the position vectors of points on the line:

$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$