**Core 4 Revision Sheet**

**Algebra and Functions**

**Remainder Theorem**

To find the remainder when a polynomial f(x) is divided by (ax + b) sub in f

*Eg, Find the remainder when f(x) = xᶟ + 2x² -5x + 3 is divided by (2x + 1)*

f = + 2² - 5 + 3 = 5 Therefore the remainder is **5 .**

**Factor Theorem**

If (ax – b) is a factor of the polynomial f(x) then f = 0

*Eg, Show (2x -1) is a factor of f(x) = 2xᶟ - x² -2x + 1. Hence, express f(x) as a product of three factors.*

f = 2ᶟ - - 2 + 1 = 0 therefore (2x -1) is a factor of f(x)

(2x – 1)(ax² + bx + c) = 2xᶟ - x² -2x + 1

(2x – 1)(x² - 1) - by inspection, expanding and comparing coefficients, or long division methods

**(2x – 1) (x + 1) (x – 1)**

**Partial Fractions**

*Express in partial fractions*

= + - multiply both sides by (x + 4)(x - 2)

7x + 16 = A(x - 2) + B(x + 4) - substitute values in for x

When x = 2 7(2) + 16 = 0A + 6B when x = -4 7(-4) + 16 = -6A

30 = 6B -12 = -6A

**B = 5** **A = 2**

**= +**

**Repeated factors**

*Express* *in partial fractions*

= + - multiply both sides by (x – 4)²

2x – 5 = A (x – 4) + B - sub in x = 4

2(4) – 5 = B **B = 3** **A = 2** (comparing x terms)

**= +**

**Improper Fractions**

*Express in partial fractions*

= A + + - multiply both sides by (x + 5)(x – 3)

x² + 7x – 14 = A(x + 5)(x - 3) + B(x – 3) + c(x + 5)

**A = 1** (comparing coefficients of x²) when x = -5 -24 = -8B **B = 3**  when x = 3 16 = 8C **C = 2**

**= 1 + +**

**Binomial Series**

= 1 + nx + + + ……. + - *this is in your formula book on pg 4.*

When n is not a positive integer, the expansion will not terminate and is only valid for -1 < x < 1, or |x| < 1

Therefore, you can only get an approximation of the expansion, usually up to xᶟ

*Eg, Obtain the expansion of up to and including the term in xᶟ, and state the range of x for which it is valid*

= 1 + (-3)(2x) + +

= **1 – 6x + 24x² - 80xᶟ |x| < 1**

**Expansion of**

Factorise the expression first by taking ‘a’ out as a factor

*Eg, Expand in ascending powers of x up to and including the term x². Hence evaluate , giving your answer to four decimal places*

=

=

=

=

=  **+ x + x² for** Hence, =

**|x| <**  =

3.97 = 4 – 3x

x = 0.01 (sub into expansion)

+ + ² = 0.50188554…… = **0.5019 (4d.p.)**

**Applications of partial fractions to series expansions**

Express as partial fractions first, and then carry out the binomial expansion.

*Eg, Express in partial fractions. Hence obtain the series expansion, giving all terms up to and including xᶟ, and state the values of x for which the expansion is valid.*

= +

= 1

= 1

= 1 – x + x² - xᶟ for |x| < 1

= 1

= 1

= 1 + 2x + 4x² + 8xᶟ for |x| < 1

Therefore, + = (1 – x + x² - xᶟ) + (1 + 2x + 4x² + 8xᶟ)

= **2 + x + 5x² + 7xᶟ valid for |x| < 1**

**Remember, if the range is not |x|< 1 for both, choose the smaller range so that it is valid for *both***

**Trigonometry**

**Compound Angles**

***Compound angle formulae are in your formula book on pg 5.***

sin (A + B) = sinA cosA + cosA sinB sin (A – B) = sinA cosB – cosA sinB

cos (A + B) = cosA cosB – sinA sinB cos (A – B) = cosA cosB + sinA sinB

tan (A B) = tan (A – B) =

**Double Angles**

sin 2x = 2sinxcosx

cos 2x = cos²x - sin²x OR cos 2x = 2cos²x – 1 OR cos 2x = 1 – 2sin²x

tan 2x =

**Triple-angle formulae**

sin 3x = 3 sinx – 4 sin³x

cos 3x = 4 cos³x – 3cosx

tan 3x =

*Both the double angle and triple-angle formulae are used in integration with powers of sin x and cos x*

***Eg, work out***

cos²x = (1 + cos2x) - rearrangement of cos 2x = 2cos²x – 1

4 cos²x = 4

= 2 (1 + cos2x)

= 2 + 2cos2x

=

=

= –

= + 1 – 0 **=**

**Harmonic Form**

*Expressing any function f() = a cos + b sin in the form*

***R sin ( or R cos (*** *where R > 0 and is acute is called expressing in* ***harmonic form****.*

*For f() = a cos + b sin*

R = tan = , for R cos ( **or** tan = , for R sin (

It allows you to solve equations in the form a cos + b sin = c, as well as finding maximum and minimum points of such functions.

***Eg, Express 12 sin + 5 cos in the form R sin () where R > 0 and , measured in degrees to 1 decimal place, is acute. Hence, solve the equation 12 sin + 5 cos = for 0 360, giving your answer to 1 d.p.***

Let 12 sin + 5 cos = R sin ( + ) R = , R = 13

tan  **=** ,  **=** 22.6

Therefore 12 sin + 5 cos = **13 sin ( + 22.6)**

So, 12 sin + 5 cos = becomes 13 sin ( + 22.6) =

sin ( + 22.6) =

sin ( + 22.6) =

+ 22.6 =

+ 22.6 = 30 , 150 = **7.4 , 127.4**  *Remember, the value for R gives the minimum or maximum value of the function. For example, when R = 5, the minimum value will be at -5 and the maximum value at +5. This is due to the fact the original function, sin x or cos x, has been multiplied by the constant term R.*

**Exponential Models**

This uses functions of the form y = A x where A, a, b, and c are constants and t is time.

***Eg, The value of a car depreciates in such a way that t years after it is new its value, £V, is given by the formula***

***V = 15000 x***

1. ***What is the initial value of the car?***

***After 2 years the value of the car is £8000.***

1. ***Calculate the value of the constant k to three decimal places.***
2. ***Calculate the time, to the nearest month, when the value of the car reaches £5000.***
3. Initial value, when t = 0,

V = 15000 x = **15000**

1. When V = 8000, and t = 2

8000 = 15000 x

= - take logs of both sides

) = -2k log (1.4)

= -2k

**k = 0.934 (3d.p)**

1. When V = 5000,

5000 = 15000 x

= - take logs of both sides

log = -0.934t log (1.4)

= -0.934t

t = 3.5 years or **3 years, 6 months**

**The Exponential Function**

This involves exponential functions to the base e, i.e. y = A, where A is a constant.

*Remember ln = x*

***Eg, A pan of water is heated and then allowed to cool. Its temperature, T, t minutes after cooling has begun, is given by the formula***

***T = 80***

1. ***What is the temperature of the water when it starts to cool?***
2. ***What is the temperature after 5 minutes?***
3. ***Find, to the nearest minute, the time taken for the water to reach a temperature of 12.***
4. When t = 0, T = 80

**T = 80**

1. When t = 5, T = 80

**T = 29.4**

1. When T = 12, 12 = 80

= - take logs of both sides

) = -0.2t

t = 9.486, therefore **9 minutes** (nearest min)

**Further Calculus**

**Implicit Functions**

These are functions that are not expressed in the form y = f(x), for example x² + 2xy + yᶟ = 5.

Differentiate using the chain rule and product rule.

***Eg, Differentiate xy² + 3x - 2y = 6*** *for xy², let u = x and v = y²*

2xy + y² + 3 - 2 = 0 = 1 = 2y = 2xy + y²

(2xy - 2) + 3 + y² = 0

(2xy - 2) = -3 – y²

= =

***Hence, find an equation to the normal to the curve at the point (2, 1)***

Gradient of tangent at x = 1

=  **= - =** -2

Therefore, gradient of normal is

Equation of normal is y – 1 = (x – 2)

2y – 2 = x – 2

**2y = x**

**Parametric Equations**

This is where x and y are defined in terms of a third variable t. You may be asked to find a Cartesian equation for parametric forms.

***Eg, 1. x = t – 4 y = 2t²***

t = x + 4 sub in for t in the equation for y y = 2(x + 4)²

***2. x = 3sin t, y = 2cos t***

sin t = cos t =

using the identity sin²t + cos²t = 1

+ = 1 **or** **4x² + 9y² = 1**

**Parametric Differentiation**

In order to find you must find and , then apply the chain rule.

***Eg,1. find an expression for in terms of t for x = 3, y = 2t² - 3***

x = 3 y = 2t² - 3

= 12tᶟ = 4t

= therefore, = x

= 4t x =

***2. Find the equation of the tangent to the curve x = 3t², y = 7 + 12t, at the point where t = 2.***

x = 3t² y = 7 + 12t

= 6t = = 12 = 12 x =

When t = 2, = = **1** When t = 2, x = 3(2)² = **12**, and y = 7 + 12(2) = **31**

**y – 31 = x – 12 y = x + 19**

**Using Partial Fractions**

Expressing functions as partial fractions can enable you to integrate them.

***Eg, Evaluate, giving your answer in the form ln where a and b are rational numbers.***

= - Express in partial fractions

Now integrate

(ln 3 – ln 4) – (ln 1 – ln 2)

ln – ln = ln = **ln**

**Differential Equations**

These are equations that include x and y as well as one or more derivatives of y with respect to x.

To solve these, you must separate the variables and integrate both sides with respect to x. Remember, you only need to add one constant value, c.

***Eg, = , y = 3 at x = -2. Find the particular solution to the differential equation.***

=

y = x + 1 separate the variables

= integrate both sides with respect to x

= + x + c

when y = 3, x = -2 sub in the values

= 2 – 2 + c

c = therefore, = + x +

y² = x² + 2x + 9

**y =**

**Modelling with differential equations**

You may be asked to solve differential equations within a context.

***Eg, At an air temperature of -T the thickness, x cm, of the ice on a pond at a time t minutes satisfies the equation***

***= , where x = 0 at t = 0***

1. ***Show that 7000x² = Tt***

14000x = T

=

= Tt + c

7000x² = + c

x = 0, when t = 0, therefore **c = 0**

**7000x² = (as required)**

1. **Given T = 10, when the ice starts forming, calculate how long it takes for the thickness of the ice to reach 2cm.**

7000x² =

7000 (2)² = 10t

28000 = 10t

T = **2800 seconds or 46 minutes, 40 seconds**

**Applications to exponential laws of growth and decay**

In general, if the rate of **growth** of x is proportional to x, then  **= kx**, where k is a positive constant.

If the rate of **decay** of x is proportional to x, then  **= -kx**, where k is a positive constant.

***Eg, The value of a car depreciates in such a way that when it is t years old, the rate of decrease in its value is proportional to the value, £V, of the car at that time. The car costs £12,000 when new.***

1. ***Show that V = 12000***

= -kV

= -k - separate the variables

dV = dt - integrate both sides

=

ln V – ln 12000 = -kt

ln = -kt

=

**V = 12000 (as required)**

***When the car is three years old its value has dropped to £4000.***

1. **Show *that k = ln 3***

t = 3, when V = 4000

4000 = 12000

=

=

3k = ln

ln 1 – ln 3)

k = 0 + ln 3 **k = ln 3 (as required)**

***The owner decides to sell the car when its value reaches reaches £2000.***

1. ***Calculate, to the nearest month, the age of the car at that time.***

V = 12000 when V = 2000:

2000 = 12000

=

= ln

- ln (3)t = ln

t =

t = 4.9 which is **4 years and 11 months** (nearest month)

**Vectors**

means to go from O to A is 5 right and 2 up. It is called a **column vector.**

is represented by the length as follows:

² = 5² + 3² = 34

=

**Addition and Subtraction of Vectors**

If given and , then to find :

= +

***Eg, 1. Given that = and = , find***

= +

= +

**=**

***2. Given that = and***

= +

= -

= -

**=**

**Scalar Multiplication**

If you multiply a vector by a scalar (magnitude), you get another vector. Eg, 3 x **a** = 3**a**

3**a** and **a** are parallel

***In general, if u = kv then u is parallel to v***

**Position vectors and direction vectors**

*The position vector of a point P with respect to a fixed origin O is the vector*

***Eg,If P has coordinate (1, 2, 3) then = p =***

***If Q has coordinate (3, 5, 2) then = q =***

***Therefore, = is the direction vector from P to Q.***

***Collinearity (lying on a straight line)***

Vectors can be used to show three points lie in a straight line.

***Eg, Points P, Q and R have position vectors , and , respectively.***

1. ***Find and***

= =

1. ***Deduce that P, Q and R are collinear and find the ratio PQ:QR***

` = 2 and = 2 which means they are parallel. They share point Q, making them collinear.

***PQ:QR = 1:2***

**Scalar product**

***The scalar product a.b of two vectors a and b is defined by a.b =*  cos where is the angle between the vectors.**

* *When two vectors* ***a*** *and* ***b*** *are perpendicular, = 90 and cos = 0. Therefore,* ***a.b*** *= 0*
* *When the angle between the two vectors* ***a*** *and* ***b*** *is acute, cos > 0 and* ***a.b*** *> 0*

If **a** = and **b** = , then **a.b** = + +

***Eg, find the angle between the vectors a = and b = , giving answers to one decimal place.***

***a.b =*** cos

**a.b =** . = (2 x 4) + (1 x -3) + (-2 x 12) = -19

**=**  = = 3

**=**  = = 13

Therefore, ***a.b =*** cos gives -19 = 3 x 13 cos

cos = and  **= 119.2**

**Perpendicular vectors**

* **If a.b = 0, then a and b are perpendicular vectors**
* **If a and b are perpendicular vectors, then a.b = 0**

***Eg, Given that the vectors andare perpendicular, find the value of the constant t.***

**a.b** = 0

= (2 x 1) + (t x -3) + (-4(t – 4)) = 0

2 – 3t – 4t + 16 = 0

7t = 18

t = = **2**

**Vector equation of a line**

**r = a + t (b – a) - where t is a scalar**

***Eg, Find a vector equation of the line passing through A( -4, 1, - 3) and B (6, -4, 12). Show that the point Q (-2, 0, 0) lies on the line AB.***

r = **a** + t (**b** – **a**)

r = + t

**r = + t = + t therefore, =**

compare against Q . If Q lies on the line then:

= + t

= x -> -2 = -4 + 2t t = 1

y -> 0 = 1 – t t = 1

z -> 0 = -3 + 3t t = 1

since t = 1 works for x, y and z, **Q lies on the line**.

**Parallel and skews lines**

If one vector is a scalar multiple of another the vectors are **parallel**.

The direction vectors and are parallel

In three dimensions, it is possible that lines are not parallel and also do not intersect. These are called **skew.** You can find the angle between such lines by finding the angle between the direction vectors.

***Eg, a) show that these two lines are skew:***

***= + and = + µ***

You need to show they do not intersect by finding the values of λ and µ using a pair of simultaneous equations and show they do not satisfy the third component.

x -> 2 + 5 = 5 + 12µ solving simultaneously gives λ = 3, µ = 1

y -> 1 - λ = 4 - 6µ

check these values in z -> 4 + 6(3) = 2 +3(1)

22 = 5 no, therefore they do not intersect and are skew.

1. **Calculate the angle between the two lines.**

You need to calculate the angle between the direction vectors **a** and **b**

**a.b** = = (5 x 12) + (-1 x -6) + (6 x 3) = 84

= =

**=** =

= = = **39.1**