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Core 2

Practice Examination Questions

Assessment Criteria by examination paper: Core 2

Algebra and Functions	I can divide by (x + a) or (x - a)	
	I know that (x – a) is a factor of f(x) the bracket provides a solution	
	I can factorise a cubic function.	
	I can find the remainder when dividing by (ax + b) or (ax – b).	
	I know the meaning of the word 'quotient'.	
Coordinate Geometry	I can find the equation of a circle given the centre and radius.	
	I can find the centre and radius given the equation of a circle.	
	I can write the equation of the circle in different ways.	
	I know the angle at the circumference in a right angle is 90°.	
	I know the perpendicular from the centre of the chord, passes through the centre.	
	I can use the perpendicularity of radius and tangents.	
Geometric Series	I can find the n^{th} term of a geometric sequence.	
	I can prove the formula to find the sum of the first n terms of a series.	
	I can find the sum of the first n numbers in a geometric series.	
	I can find the sum to infinity of a geometric series and know the conditions.	
	I can use the binomial expansion of $(a + bx)^n$.	
	I know the notations n! and $\binom{n}{r}$.	
Differentiation	I can find the second order derivative of a function.	
	I can sketch a curve with its stationary points identified.	
	I can indentify stationary points of a function.	
	I can identify maxima/minima/points of inflection.	
	I can decide whether a function is increasing (>0) or decreasing (<0) at a point.	
	I can apply differentiation to shape based problem.	

Integration	I can evaluate definite integrals.	
	I can find the area under a curve.	
	I can approximate the area under a curve using the trapezium rule.	
Logs	I can draw and identify graphs of the form $y = a^x$.	
	I can use the laws of logarithms to solve problems.	
	I can find solutions to equations of the form $a^x = b$.	
Trigonometry	I can use the sine and cosine rules.	
	I can find the area of a triangle in the form $\frac{1}{2}ab\sin C$.	
	I can find the arc length and area of a sector.	
	I can describe the trigonometric functions' graphs.	
	I can use the identity $\sin^2 x + \cos^2 x = 1$.	
	I can use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$	
	I can find solutions to simple trigonometric equations in an interval.	

This appendix lists formulae that candidates are expected to remember and that will not be included in formulae booklets. **(Plus those from core 1!)**

Laws of Logarithms

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$

$k \log_a x \equiv \log_a \left(x^k\right)$

Trigonometry

In the triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

area $= \frac{1}{2}ab\sin C$

Integration

area under a curve $= \int_a^b y\,\mathrm{d}x \quad (y \geq 0)$

Binomial Expansion

1. Find the first three terms, in ascending powers of x, of the binomial expansion of $(3 + 2x)^5$, giving each term in its simplest form.

(4)

2. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(1 + px)^9,$$

where p is a constant.

(2)

The first 3 terms are 1, $36x$ and qx^2, where q is a constant.

(*b*) Find the value of p and the value of q.

(4)

3. (*a*) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 - 2x)^5$. Give each term in its simplest form.

(4)

(*b*) If x is small, so that x^2 and higher powers can be ignored, show that

$$(1 + x)(1 - 2x)^5 \approx 1 - 9x.$$

(2)

4. (*a*) Find the first 4 terms of the expansion of $\left(1+\frac{x}{2}\right)^{10}$ in ascending powers of x, giving each term in its simplest form.

(4)

(*b*) Use your expansion to estimate the value of $(1.005)^{10}$, giving your answer to 5 decimal places.

(3)

5. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form.

(4)

6. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 - x)^6$$

and simplify each term.

(4)

7. Given that $\binom{40}{4} = \dfrac{40!}{4!b!}$,

(*a*) write down the value of *b*.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are *p* and *q* respectively.

(*b*) Find the value of $\dfrac{q}{p}$.

(3)

8. (*a*) Write down the first three terms, in ascending powers of *x*, of the binomial expansion of $(1 + px)^{12}$, where *p* is a non-zero constant.

(2)

Given that, in the expansion of $(1 + px)^{12}$, the coefficient of *x* is $(-q)$ and the coefficient of x^2 is $11q$,

(*b*) find the value of *p* and the value of *q*.

(4)

9. (*a*) Find the first 4 terms, in ascending powers of *x*, of the binomial expansion of $(1 + ax)^{10}$, where *a* is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^3 is double the coefficient of x^2,

(*b*) find the value of *a*.

(2)

10. (*a*) Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of

$$(2 + kx)^7$$

where *k* is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 is 6 times the coefficient of *x*,

(*b*) find the value of *k*.

(2)

11. (*a*) Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1+ ax)^7$, where a is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^2 in this expansion is 525,

(*b*) find the possible values of a.

(2)

12. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 + x)^6$, giving each term in its simplest form.

(4)

13. (*a*) Find the first four terms, in ascending powers of x, in the bionomial expansion of $(1 + kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(*b*) the value of k,

(2)

(*c*) the coefficient of x^3.

(1)

14. (*a*) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

(*b*) find the value of b.

(2)

Coordinate Geometry

1. The points A and B have coordinates (5, –1) and (13, 11) respectively.

(a) Find the coordinates of the mid-point of AB.

(2)

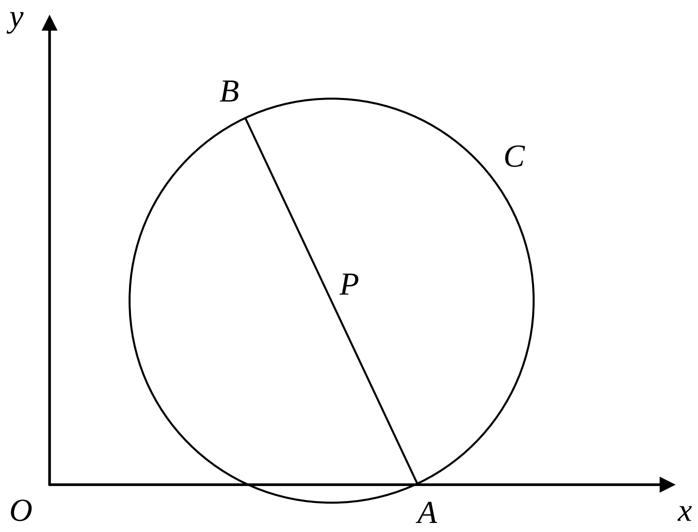
Given that AB is a diameter of the circle C,

(b) find an equation for C.

(4)

2.

Figure 1



In Figure 1, $A(4, 0)$ and $B(3, 5)$ are the end points of a diameter of the circle C.

Find

(a) the exact length of AB,

(2)

(b) the coordinates of the midpoint P of AB,

(2)

(c) an equation for the circle C.

(3)

3. The line joining points (–1, 4) and (3, 6) is a diameter of the circle C.

Find an equation for C.

(6)

4. A circle C has centre $M(6, 4)$ and radius 3.

(*a*) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$

(2)

Figure 3

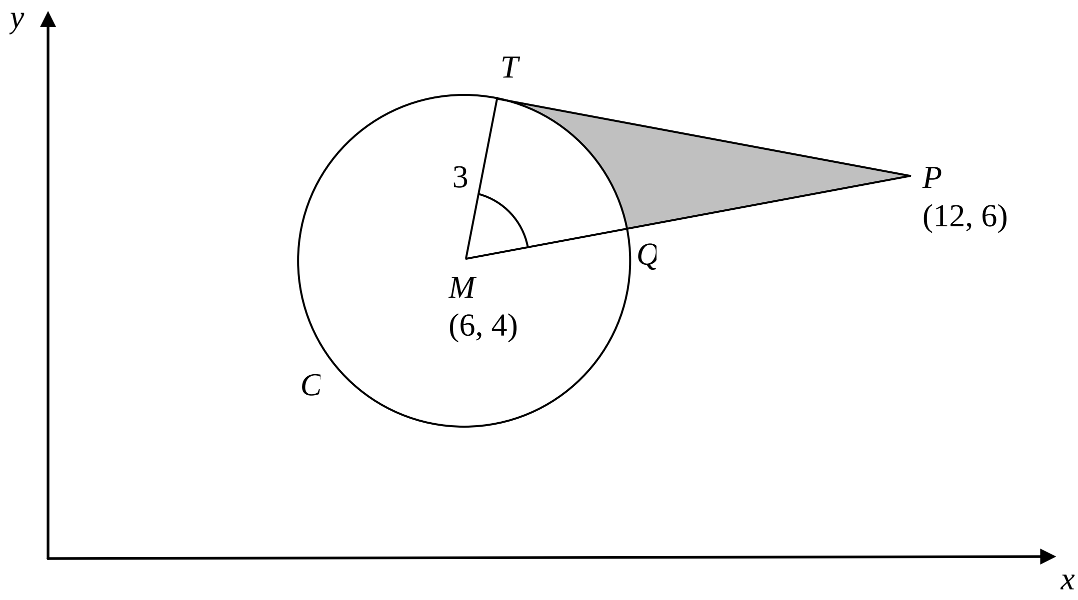


Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q.

(*b*) Show that the angle TMQ is 1.0766 radians to 4 decimal places.

(4)

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(*c*) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

5.

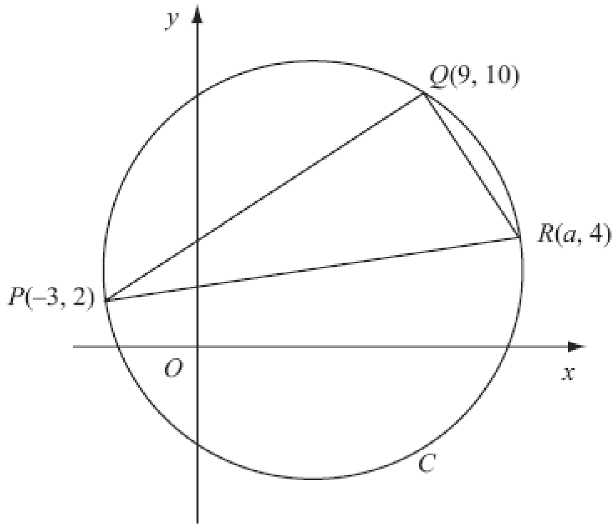


Figure 2

The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2.

Given that PR is a diameter of C,

(a) show that $a = 13$,

(3)

(b) find an equation for C.

(5)

6.

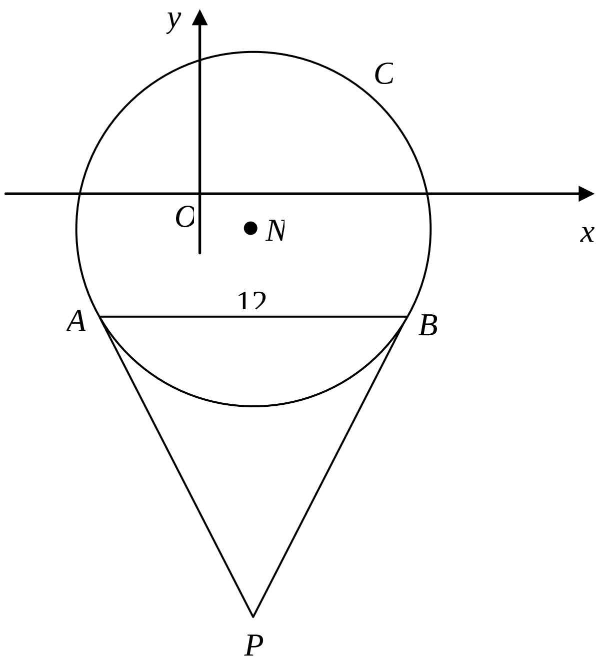


Figure 3

Figure 3 shows a sketch of the circle C with centre N and equation

$$(x-2)^2 + (y+1)^2 = \frac{169}{4}.$$

(*a*) Write down the coordinates of N.

(2)

(*b*) Find the radius of C.

(1)

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units as shown in Figure 3.

(*c*) Find the coordinates of A and the coordinates of B.

(5)

(*d*) Show that angle $ANB = 134.8°$, to the nearest 0.1 of a degree.

(2)

The tangents to C at the points A and B meet at the point P.

(*e*) Find the length AP, giving your answer to 3 significant figures.

(2)

7. The points A and B have coordinates (–2, 11) and (8, 1) respectively.

Given that AB is a diameter of the circle C,

(*a*) show that the centre of C has coordinates (3, 6),

(1)

(*b*) find an equation for C.

(4)

(*c*) Verify that the point (10, 7) lies on C.

(1)

(*d*) Find an equation of the tangent to C at the point (10, 7), giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

8. The circle C, with centre at the point A, has equation $x^2 + y^2 - 10x + 9 = 0$.

Find

(*a*) the coordinates of A,

(2)

(*b*) the radius of C,

(2)

(*c*) the coordinates of the points at which C crosses the x-axis.

(2)

Given that the line l with gradient $\frac{7}{2}$ is a tangent to C, and that l touches C at the point T,

(*d*) find an equation of the line which passes through A and T.

(3)

9. The circle C has centre (3, 1) and passes through the point $P(8, 3)$.

(*a*) Find an equation for C.

(4)

(*b*) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

10. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

(*a*) Find the centre and the radius of C.

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

(*b*) Show that PQ is a diameter of C.

(2)

The point R lies on the positive y-axis and the angle $PRQ = 90°$.

(*c*) Find the coordinates of R.

(4)

11. The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$.

(*a*) Find an equation for C.

(4)

The line l_1 is the tangent to C at the point B.

(*b*) Find an equation for l_1.

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of *AB*.

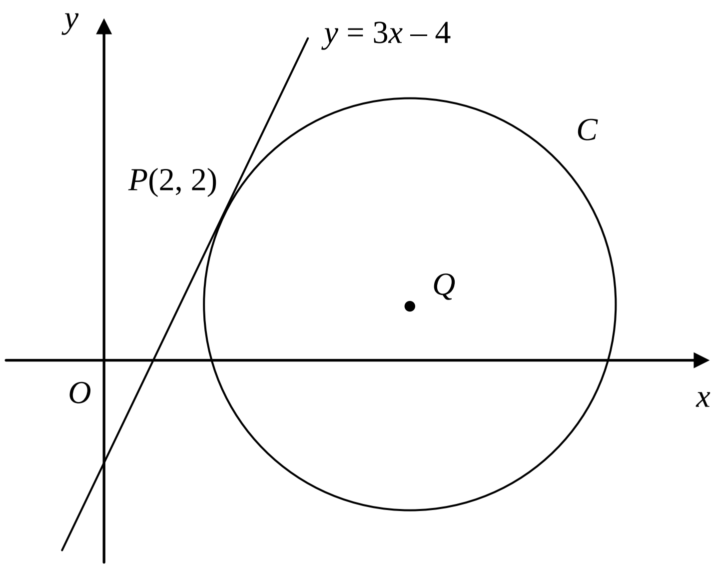
Given that l_2 intersects *C* at the points *P* and *Q*,

(*c*) find the length of *PQ*, giving your answer in its simplest surd form.

(3)

12.

Figure 1



The line $y = 3x - 4$ is a tangent to the circle *C*, touching *C* at the point P(2, 2), as shown in Figure 1.

The point *Q* is the centre of *C*.

(*a*) Find an equation of the straight line through *P* and *Q*.

(3)

Given that *Q* lies on the line $y = 1$,

(*b*) show that the *x*-coordinate of *Q* is 5,

(1)

(*c*) find an equation for *C*.

(4)

13.

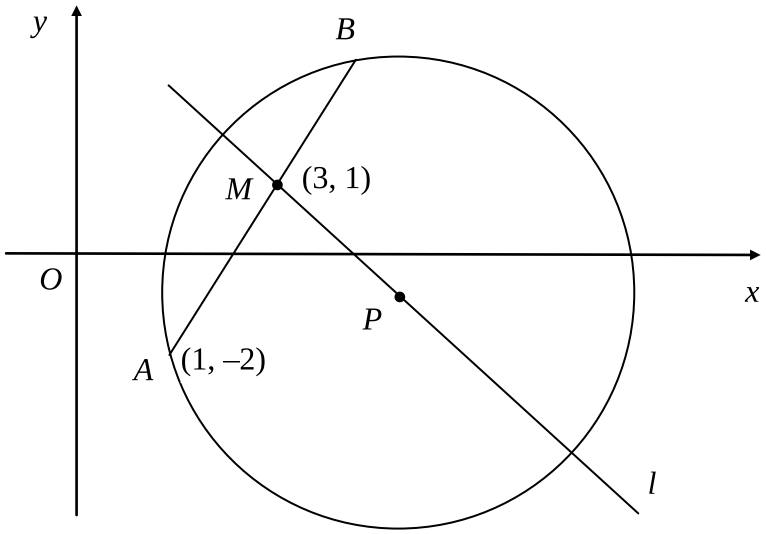


Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(*a*) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(*b*) use your answer to part (*a*) to show that the y-coordinate of P is -1,

(1)

(*c*) find an equation for the circle.

(4)

14. The circle C has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0.$$

Find

(*a*) the coordinates of the centre of C,

(2)

(*b*) the radius of C,

(2)

(*c*) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds.

(4)

Use of Logarithms and exponentials

1. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $3^x = 5$,

(3)

(b) $\log_2 (2x + 1) - \log_2 x = 2$.

(4)

2. Solve the equation $5^x = 17$, giving your answer to 3 significant figures.

(3)

3. Given that a and b are positive constants, solve the simultaneous equations

$$a = 3b,$$

$$\log_3 a + \log_3 b = 2.$$

Give your answers as exact numbers.

(6)

4. Given that $0 < x < 4$ and

$$\log_5 (4 - x) - 2 \log_5 x = 1,$$

find the value of x.

(6)

5. (a) Find the positive value of x such that

$$\log_x 64 = 2.$$

(2)

(b) Solve for x

$$\log_2 (11 - 6x) = 2 \log_2 (x - 1) + 3.$$

(6)

6. (*a*) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes.

(2)

(*b*) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0,$$

giving your answers to 2 decimal places where appropriate.

(6)

7. Solve

(*a*) $5^x = 8$, giving your answer to 3 significant figures,

(3)

(*b*) $\log_2 (x + 1) - \log_2 x = \log_2 7$.

(3)

8. (*a*) Find, to 3 significant figures, the value of x for which $5^x = 7$.

(2)

(*b*) Solve the equation $5^{2x} - 12(5^x) + 35 = 0$.

(4)

9. (*a*) Find the value of y such that

$$\log_2 y = -3.$$

(2)

(*b*) Find the values of x such that

$$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x.$$

(5)

10. (*a*) Given that

$$2 \log_3 (x - 5) - \log_3 (2x - 13) = 1,$$

show that $x^2 - 16x + 64 = 0$.

(5)

(*b*) Hence, or otherwise, solve $2 \log_3 (x - 5) - \log_3 (2x - 13) = 1$.

(2)

11. (i) Write down the value of $\log_6 36$.

(1)

(ii) Express $2 \log_a 3 + \log_a 11$ as a single logarithm to base a.

(3)

12. (*a*) Find, to 3 significant figures, the value of x for which $8^x = 0.8$.

(2)

(*b*) Solve the equation

$$2 \log_3 x - \log_3 7x = 1.$$

(4)

13. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(*a*) $5^x = 10$,

(2)

(*b*) $\log_3 (x - 2) = -1$.

(2)

Trigonometry

1. **Figure 1**

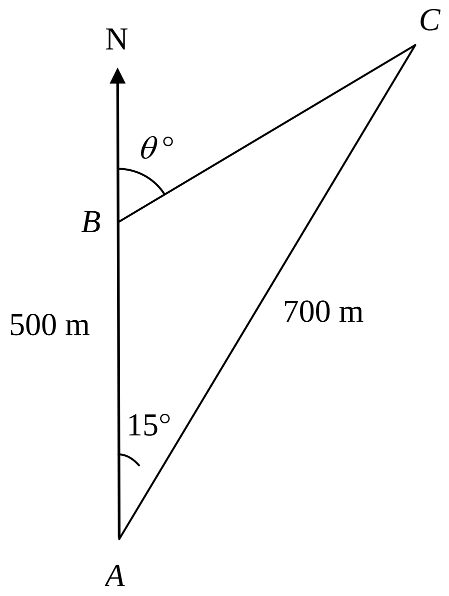


Figure 1 shows 3 yachts A, B and C which are assumed to be in the same horizontal plane. Yacht B is 500 m due north of yacht A and yacht C is 700 m from A. The bearing of C from A is 015°.

(*a*) Calculate the distance between yacht B and yacht C, in metres to 3 significant figures.

(3)

The bearing of yacht C from yacht B is $\theta°$, as shown in Figure 1.

(*b*) Calculate the value of θ.

(4)

2. In the triangle ABC, AB = 11 cm, BC = 7 cm and CA = 8 cm.

(*a*) Find the size of angle C, giving your answer in radians to 3 significant figures.

(3)

(*b*) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures.

(3)

3.

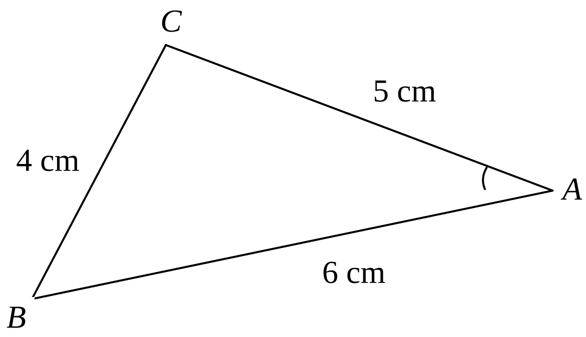


Figure 1

Figure 1 shows the triangle ABC, with AB = 6 cm, BC = 4 cm and CA = 5 cm.

(a) Show that $\cos A = \frac{3}{4}$.

(3)

(b) Hence, or otherwise, find the exact value of $\sin A$.

(2)

Trigonometric Identities

1. (*a*) Show that the equation

$$5 \cos^2 x = 3(1 + \sin x)$$

can be written as

$$5 \sin^2 x + 3 \sin x - 2 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$, the equation

$$5 \cos^2 x = 3(1 + \sin x),$$

giving your answers to 1 decimal place where appropriate.

(5)

2. (*a*) Find all the values of θ, to 1 decimal place, in the interval $0° \leq \theta < 360°$ for which

$$5 \sin (\theta + 30°) = 3.$$

(4)

(*b*) Find all the values of θ, to 1 decimal place, in the interval $0° \leq \theta < 360°$ for which

$$\tan^2 \theta = 4.$$

(5)

3. Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2 \cos^2 x + 1 = 5 \sin x,$$

giving each solution in terms of π.

(6)

4. (*a*) Show that the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1$$

can be written as

$$5 \sin^2 \theta = 3.$$

(2)

(*b*) Hence solve, for $0° \leq \theta < 360°$, the equation

$$3 \sin^2 \theta - 2 \cos^2 \theta = 1,$$

giving your answer to 1 decimal place.

(7)

5. (*a*) Show that the equation

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

can be written as

$$4 \cos^2 x - 9 \cos x + 2 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 720°$,

$$4 \sin^2 x + 9 \cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

6. (*a*) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(2)

(*b*) Solve, for $0 \leq x < 360°$,

$$2 \sin^2 x + 5 \sin x - 3 = 0.$$

(4)

7. (*a*) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0.$$

(2)

(*b*) Hence solve, for $0 \leq x < 360°$,

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

8. Solve, for $0 \leq x \leq 180°$, the equation

(*a*) $\sin(x + 10°) = \frac{\sqrt{3}}{2}$,

(4)

(*b*) $\cos 2x = -0.9$, giving your answers to 1 decimal place.

(4)

9. Solve, for $0 \leq x < 360°$,

(*a*) $\sin(x - 20°) = \frac{1}{\sqrt{2}}$,

(4)

(*b*) $\cos 3x = -\frac{1}{2}$.

(6)

10. (i) Solve, for $-180° \leq \theta < 180°$,

$$(1 + \tan\theta)(5\sin\theta - 2) = 0.$$

(4)

(ii) Solve, for $0 \leq x < 360°$,

$$4\sin x = 3\tan x.$$

(6)

11. (*a*) Given that $5\sin\theta = 2\cos\theta$, find the value of $\tan\theta$.

(1)

(*b*) Solve, for $0 \leq x < 360°$,

$$5\sin 2x = 2\cos 2x,$$

giving your answers to 1 decimal place.

(5)

12. (*a*) Given that $\sin\theta = 5\cos\theta$, find the value of $\tan\theta$.

(1)

(*b*) Hence, or otherwise, find the values of θ in the interval $0 \leq \theta < 360°$ for which

$$\sin\theta = 5\cos\theta,$$

giving your answers to 1 decimal place.

(3)

13. (*a*) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

(2)

(*b*) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

(*c*) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places.

(5)

14. (*a*) Solve for $0 \leq x < 360°$, giving your answers in degrees to 1 decimal place,

$$3 \sin (x + 45°) = 2.$$

(4)

(*b*) Find, for $0 \leq x < 2\pi$, all the solutions of

$$2 \sin^2 x + 2 = 7\cos x,$$

giving your answers in radians.

You must show clearly how you obtained your answers.

(6)

Factors and Remainders of Functions

1. $$f(x) = x^3 - 2x^2 + ax + b, \text{ where } a \text{ and } b \text{ are constants.}$$

When $f(x)$ is divided by $(x - 2)$, the remainder is 1.

When $f(x)$ is divided by $(x + 1)$, the remainder is 28.

(*a*) Find the value of a and the value of b.

(6)

(*b*) Show that $(x - 3)$ is a factor of $f(x)$.

(2)

2. $$f(x) = 2x^3 + x^2 - 5x + c, \text{ where } c \text{ is a constant.}$$

Given that $f(1) = 0$,

(*a*) find the value of c,

(2)

(*b*) factorise $f(x)$ completely,

(4)

(*c*) find the remainder when $f(x)$ is divided by $(2x - 3)$.

(2)

3. $$f(x) = x^3 + 4x^2 + x - 6.$$

(*a*) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(*b*) Factorise $f(x)$ completely.

(4)

(*c*) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0.$$

(1)

4. (a) Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(i) $x - 3$,

(ii) $x + 2$.

(3)

(*b*) Hence, or otherwise, find all the solutions to the equation

$$x^3 - 2x^2 - 4x + 8 = 0.$$

(4)

5. $$\mathrm{f}(x) = x^4 + 5x^3 + ax + b,$$

where a and b are constants.

The remainder when $\mathrm{f}(x)$ is divided by $(x - 2)$ is equal to the remainder when $\mathrm{f}(x)$ is divided by $(x + 1)$.

(*a*) Find the value of a.

(5)

Given that $(x + 3)$ is a factor of $\mathrm{f}(x)$,

(*b*) find the value of b.

(3)

6. $$\mathrm{f}(x) = 2x^3 + ax^2 + bx - 6,$$

where a and b are constants.

When $\mathrm{f}(x)$ is divided by $(2x - 1)$ the remainder is -5.

When $\mathrm{f}(x)$ is divided by $(x + 2)$ there is no remainder.

(*a*) Find the value of a and the value of b.

(6)

(*b*) Factorise $\mathrm{f}(x)$ completely.

(3)

7. (*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + x^2 - 25x + 12$.

(2)

(*b*) Factorise $2x^3 + x^2 - 25x + 12$ completely.

(4)

8.

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

(*a*) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$.

(2)

(*b*) Factorise $f(x)$ completely.

(4)

9.

$$f(x) = (3x - 2)(x - k) - 8$$

where k is a constant.

(*a*) Write down the value of $f(k)$.

(1)

When $f(x)$ is divided by $(x - 2)$ the remainder is 4.

(*b*) Find the value of k.

(2)

(*c*) Factorise $f(x)$ completely.

(3)

10.

$$f(x) = 3x^3 - 5x^2 - 58x + 40.$$

(*a*) Find the remainder when $f(x)$ is divided by $(x - 3)$.

(2)

Given that $(x - 5)$ is a factor of $f(x)$,

(*b*) find all the solutions of $f(x) = 0$.

(5)

11. $$f(x) = 2x^3 + 3x^2 - 29x - 60.$$

(*a*) Find the remainder when f(x) is divided by (x + 2).

(2)

(*b*) Use the factor theorem to show that (x + 3) is a factor of f(x).

(2)

(*c*) Factorise f(x) completely.

(4)

12. $$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(*a*) Find the remainder when f(x) is divided by (x – 2).

(2)

Given that (x + 2) is a factor of f(x),

(*b*) factorise f(x) completely.

(4)

13. $$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(*a*) Find the remainder when f(x) is divided by (x – 1).

(2)

(*b*) Use the factor theorem to show that (x + 1) is a factor of f(x).

(2)

(*c*) Factorise f(x) completely.

(4)

Geometric Series

1. The second and fourth terms of a geometric series are 7.2 and 5.832 respectively.

The common ratio of the series is positive.

For this series, find

(*a*) the common ratio,

(2)

(*b*) the first term,

(2)

(*c*) the sum of the first 50 terms, giving your answer to 3 decimal places,

(2)

(*d*) the difference between the sum to infinity and the sum of the first 50 terms, giving your answer to 3 decimal places.

(2)

2. The first term of a geometric series is 120. The sum to infinity of the series is 480.

(*a*) Show that the common ratio, r, is $\frac{3}{4}$.

(3)

(*b*) Find, to 2 decimal places, the difference between the 5th and 6th terms.

(2)

(*c*) Calculate the sum of the first 7 terms.

(2)

The sum of the first n terms of the series is greater than 300.

(*d*) Calculate the smallest possible value of n.

(4)

3. A geometric series is $a + ar + ar^2 + \ldots$

(*a*) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(*b*) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(*c*) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + ...$$

(3)

(*d*) State the condition for an infinite geometric series with common ratio r to be convergent.
(1)

4. The fourth term of a geometric series is 10 and the sixth term of the series is 80.

For this series, find

(*a*) the common ratio,
(2)

(*b*) the first term,
(2)

(*c*) the sum of the first 20 terms, giving your answer to the nearest whole number.
(2)

5. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(*a*) Show that $k^2 - 7k - 60 = 0$.
(4)

(*b*) Hence show that $k = 12$.
(2)

(*c*) Find the common ratio of this series.
(2)

(*d*) Find the sum to infinity of this series.
(2)

6. A car was purchased for £18 000 on 1st January.

On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(*a*) Show that the value of the car exactly 3 years after it was purchased is £9216.
(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(*b*) Find the value of n.
(3)

An insurance company has a scheme to cover the cost of maintenance of the car. The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88.

(*c*) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.
(2)

(*d*) Find the total cost of the insurance scheme for the first 15 years.
(3)

7. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(*a*) the common ratio of the series,
(3)

(*b*) the first term of the series,
(2)

(*c*) the sum to infinity of the series.
(2)

8. (*a*) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$
(4)

Mr King will be paid a salary of £35 000 in the year 2005. Mr King's contract promises a 4% increase in salary every year, the first increase being given in 2006, so that his annual salaries form a geometric sequence.

(*b*) Find, to the nearest £100, Mr King's salary in the year 2008.
(2)

Mr King will receive a salary each year from 2005 until he retires at the end of 2024.

(*c*) Find, to the nearest £1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.
(4)

9. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(*a*) the 20th term of the series, to 3 decimal places,

(2)

(b) the sum to infinity of the series.

(2)

Given that the sum to k terms of the series is greater than 24.95,

(c) show that $k > \dfrac{\log 0.002}{\log 0.8}$,

(4)

(d) find the smallest possible value of k.

(1)

10. The third term of a geometric sequence is 324 and the sixth term is 96.

(a) Show that the common ratio of the sequence is $\dfrac{2}{3}$.

(2)

(b) Find the first term of the sequence.

(2)

(c) Find the sum of the first 15 terms of the sequence.

(3)

(d) Find the sum to infinity of the sequence.

(2)

11. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(b) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N - 1) \log 1.03 > \log 1.6$$

(3)

(d) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)

12. A geometric series has first term a and common ratio r. The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2 - 25r + 4 = 0$.

(4)

(b) Find the two possible values of r.

(2)

(c) Find the corresponding two possible values of a.

(2)

(d) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24.

(2)

13. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \dfrac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(*d*) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

14. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(*a*) the common ratio,

(2)

(*b*) the first term,

(2)

(*c*) the sum to infinity,

(2)

(*d*) the smallest value of n for which the sum of the first n terms of the series exceeds 1000.

(4)

Radian Measure

1. **Figure 1**

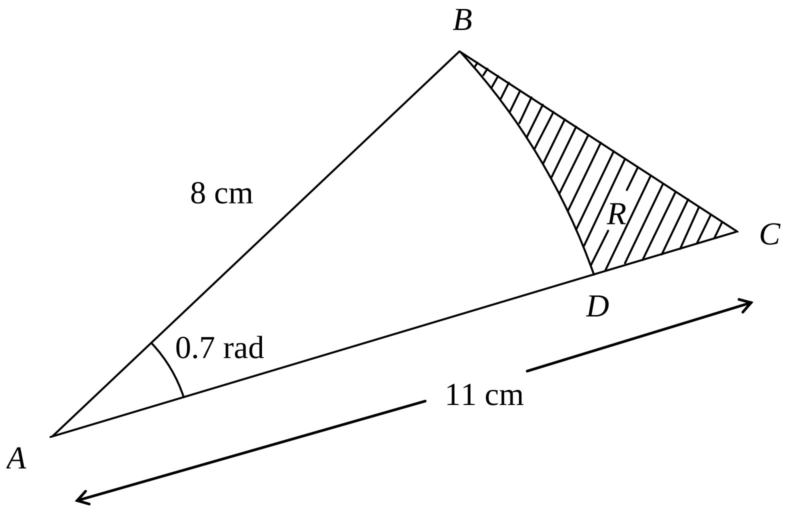


Figure 1 shows the triangle ABC, with AB = 8 cm, AC = 11 cm and $\angle BAC$ = 0.7 radians. The arc BD, where D lies on AC, is an arc of a circle with centre A and radius 8 cm. The region R, shown shaded in Figure 1, is bounded by the straight lines BC and CD and the arc BD.

Find

(*a*) the length of the arc BD,

(2)

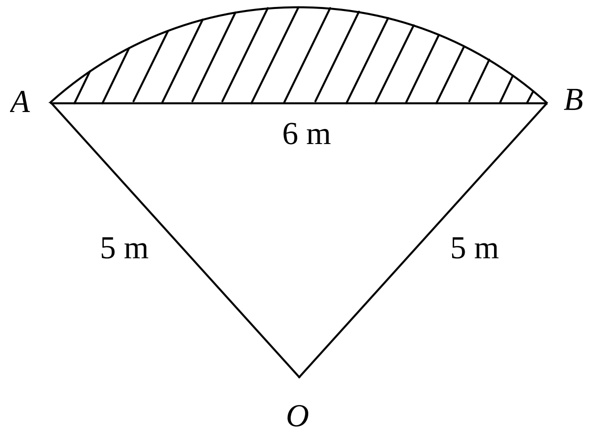
(*b*) the perimeter of R, giving your answer to 3 significant figures,

(4)

(*c*) the area of R, giving your answer to 3 significant figures.

(5)

2. **Figure 2**



In Figure 2 OAB is a sector of a circle, radius 5 m. The chord AB is 6 m long.

(*a*) Show that $\cos A\hat{O}B = \dfrac{7}{25}$.

(2)

(*b*) Hence find the angle $A\hat{O}B$ in radians, giving your answer to 3 decimal places.

(1)

(*c*) Calculate the area of the sector *OAB*.

(2)

(*d*) Hence calculate the shaded area.

(3)

3.

Figure 2

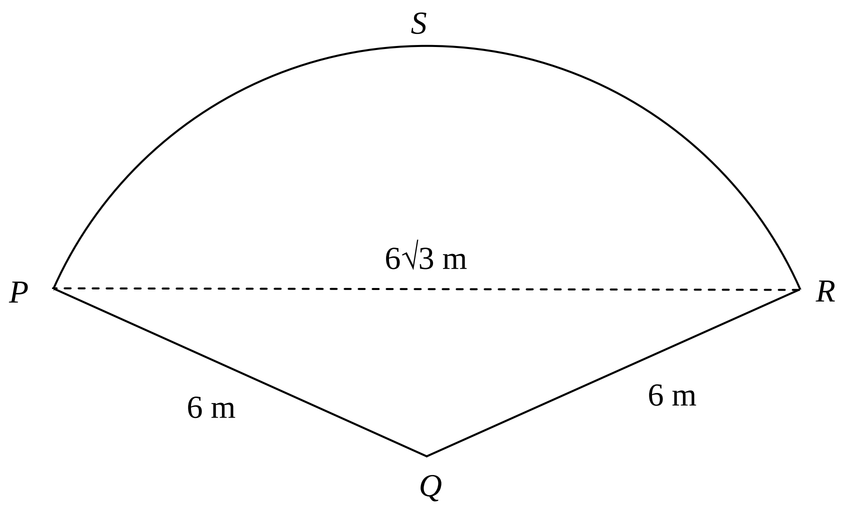


Figure 2 shows a plan of a patio. The patio *PQRS* is in the shape of a sector of a circle with centre *Q* and radius 6 m.

Given that the length of the straight line *PR* is $6\sqrt{3}$ m,

(*a*) find the exact size of angle *PQR* in radians.

(3)

(*b*) Show that the area of the patio *PQRS* is $12\pi\,\text{m}^2$.

(2)

(*c*) Find the exact area of the triangle *PQR*.

(2)

(*d*) Find, in m^2 to 1 decimal place, the area of the segment *PRS*.

(2)

(*e*) Find, in m to 1 decimal place, the perimeter of the patio *PQRS*.

(2)

4.

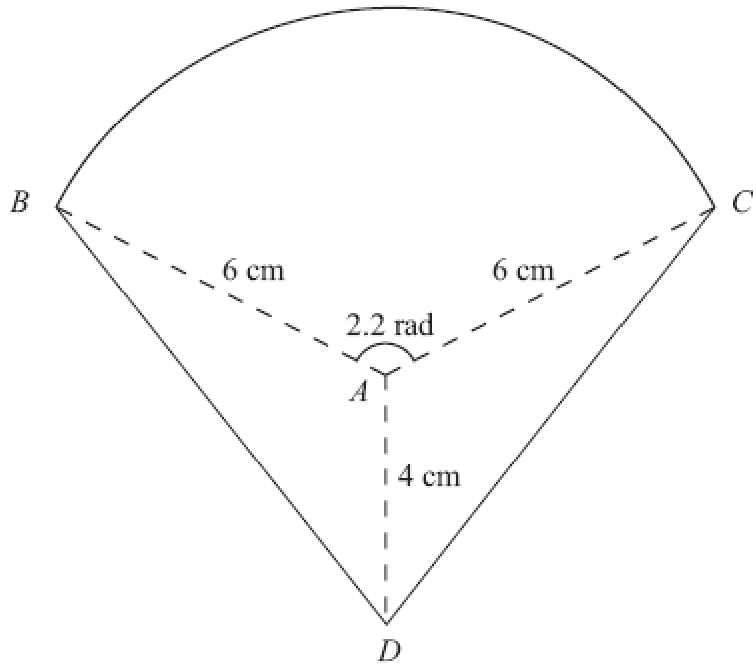


Figure 3

The shape *BCD* shown in Figure 3 is a design for a logo.

The straight lines *DB* and *DC* are equal in length. The curve *BC* is an arc of a circle with centre *A* and radius 6 cm. The size of $\angle BAC$ is 2.2 radians and $AD = 4$ cm.

Find

(*a*) the area of the sector *BAC*, in cm^2,

(2)

(*b*) the size of $\angle DAC$, in radians to 3 significant figures,

(2)

(*c*) the complete area of the logo design, to the nearest cm^2.

(4)

4.

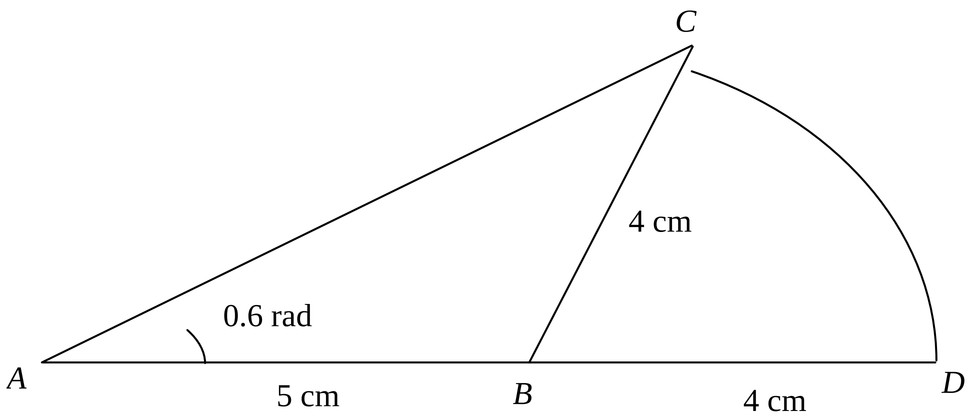


Figure 1

An emblem, as shown in Figure 1, consists of a triangle ABC joined to a sector CBD of a circle with radius 4 cm and centre B. The points A, B and D lie on a straight line with AB = 5 cm and BD = 4 cm. Angle BAC = 0.6 radians and AC is the longest side of the triangle ABC.

(*a*) Show that angle ABC = 1.76 radians, correct to three significant figures.

(4)

(*b*) Find the area of the emblem.

(3)

5.

$$f(x) = x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When f(x) is divided by (x – 1), the remainder is 7.

(*a*) Show that $a + b = 3$.

(2)

When f(x) is divided by (x + 2), the remainder is –8.

(*b*) Find the value of a and the value of b.

(5)

6. In the triangle ABC, AB = 8 cm, AC = 7 cm, $\angle ABC$ = 0.5 radians and $\angle ACB = x$ radians.

(*a*) Use the sine rule to find the value of sin x, giving your answer to 3 decimal places.

(3)

Given that there are two possible values of x,

(*b*) find these values of x, giving your answers to 2 decimal places.

(3)

7.

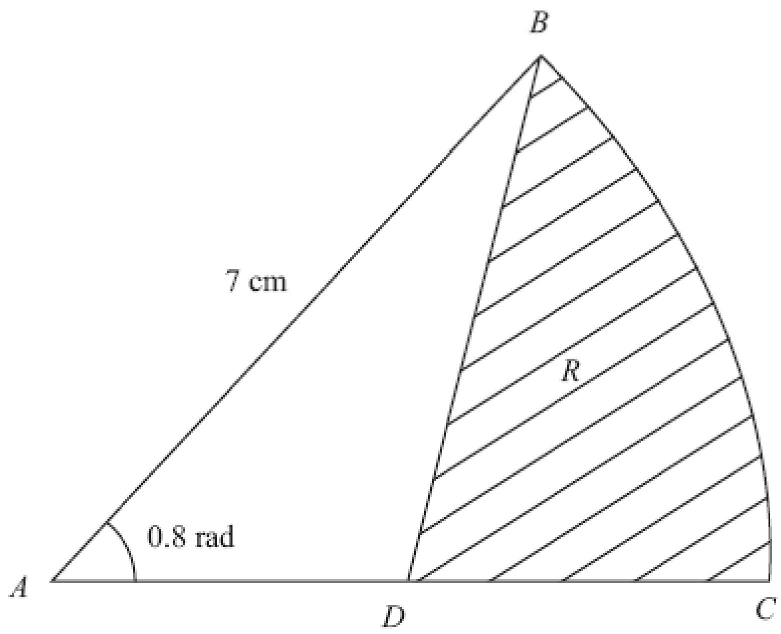


Figure 1

Figure 1 shows *ABC*, a sector of a circle with centre *A* and radius 7 cm.

Given that the size of $\angle BAC$ is exactly 0.8 radians, find

(*a*) the length of the arc *BC*,

(2)

(*b*) the area of the sector *ABC*.

(2)

The point *D* is the mid-point of *AC*. The region *R*, shown shaded in Figure 1, is bounded by *CD*, *DB* and the arc *BC*.

Find

(*c*) the perimeter of *R*, giving your answer to 3 significant figures,

(4)

(*d*) the area of *R*, giving your answer to 3 significant figures.

(4)

8.

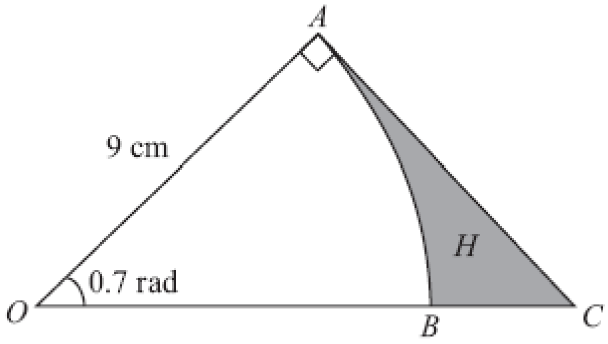


Figure 1

Figure 1 shows the sector *OAB* of a circle with centre *O*, radius 9 cm and angle 0.7 radians.

(*a*) Find the length of the arc *AB*.

(2)

(*b*) Find the area of the sector *OAB*.

(2)

The line *AC* shown in Figure 1 is perpendicular to *OA*, and *OBC* is a straight line.

(*c*) Find the length of *AC*, giving your answer to 2 decimal places.

(2)

The region *H* is bounded by the arc *AB* and the lines *AC* and *CB*.

(*d*) Find the area of *H*, giving your answer to 2 decimal places.

(3)

9.

Figure 2

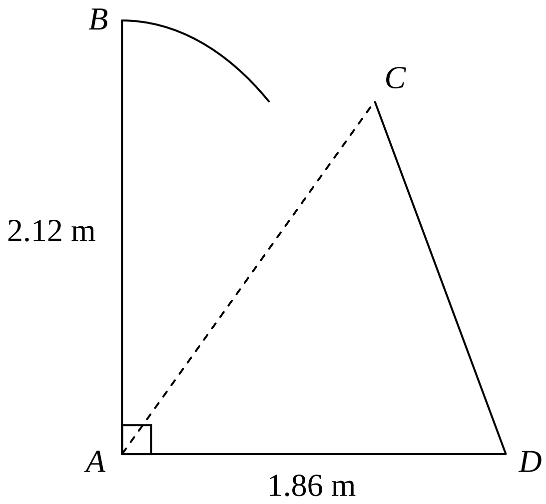


Figure 2 shows the cross-section $ABCD$ of a small shed.

The straight line AB is vertical and has length 2.12 m.

The straight line AD is horizontal and has length 1.86 m.

The curve BC is an arc of a circle with centre A, and CD is a straight line.

Given that the size of $\angle BAC$ is 0.65 radians, find

(a) the length of the arc BC, in m, to 2 decimal places,

(2)

(b) the area of the sector BAC, in m^2, to 2 decimal places,

(2)

(c) the size of $\angle CAD$, in radians, to 2 decimal places,

(2)

(d) the area of the cross-section $ABCD$ of the shed, in m^2, to 2 decimal places.

(3)

10.

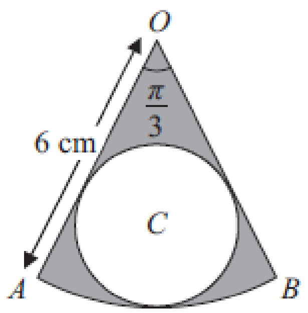


Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector OAB of a circle centre O, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle C, inside the sector, touches the two straight edges, OA and OB, and the arc AB as shown.

Find

(a) the area of the sector OAB,

(2)

(b) the radius of the circle C.

(3)

The region outside the circle C and inside the sector OAB is shown shaded in Figure 1.

(c) Find the area of the shaded region.

(2)

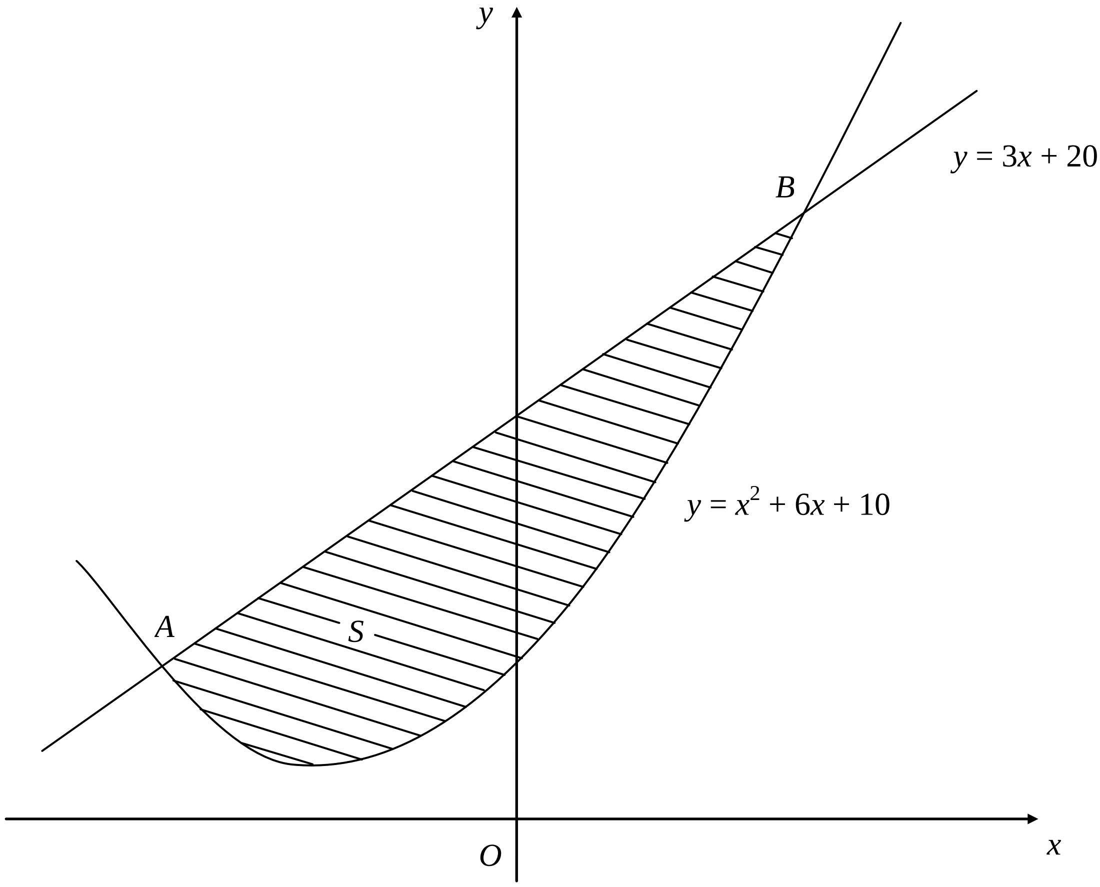
Area under curves

1. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x}) \;\; \mathrm{d}x \, .$$

(5)

2. **Figure 2**



The line with equation $y = 3x + 20$ cuts the curve with equation $y = x^2 + 6x + 10$ at the points A and B, as shown in Figure 2.

(a) Use algebra to find the coordinates of A and the coordinates of B.

(5)

The shaded region S is bounded by the line and the curve, as shown in Figure 2.

(b) Use calculus to find the exact area of S.

(7)

3.

Figure 3

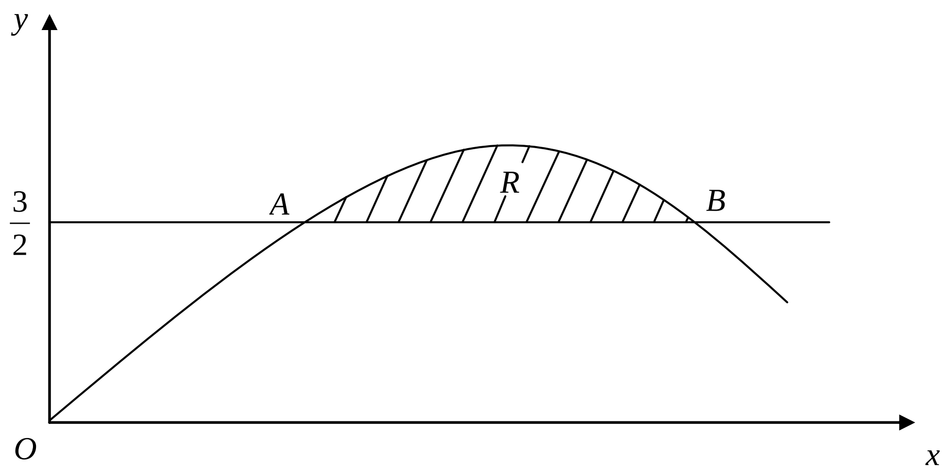


Figure 3 shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve.

Find

(a) the x-coordinates of the points A and B,

(4)

(b) the exact area of R.

(6)

4.

Figure 1

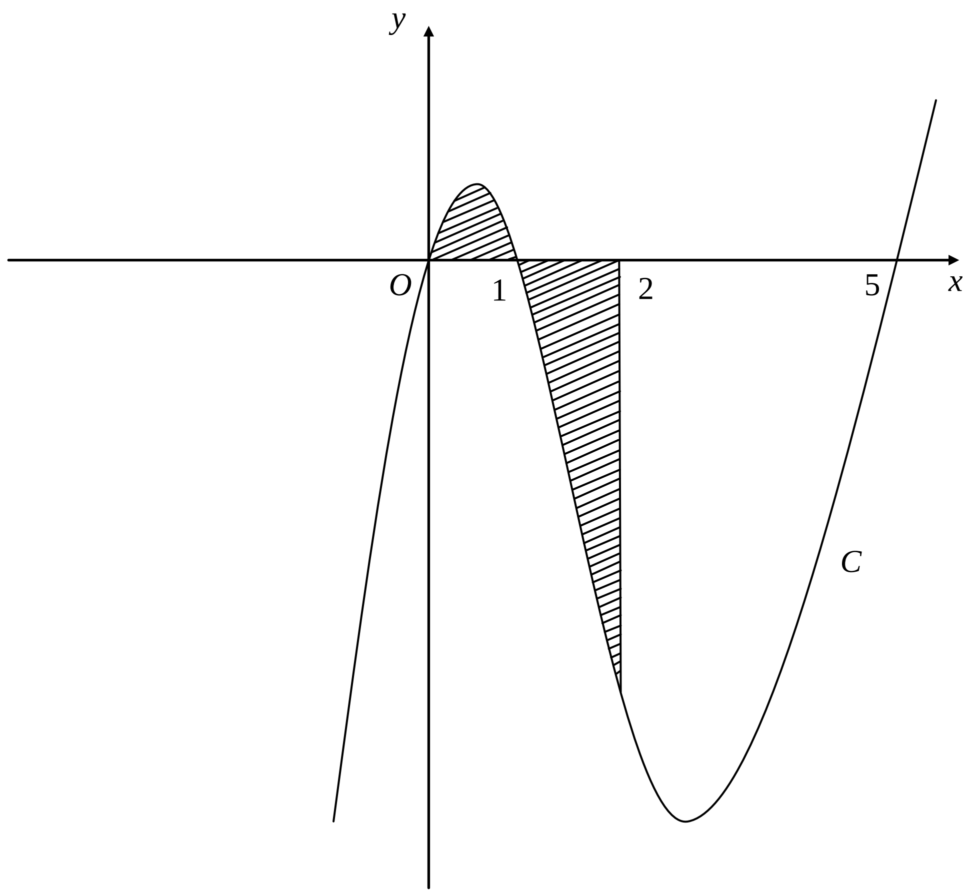


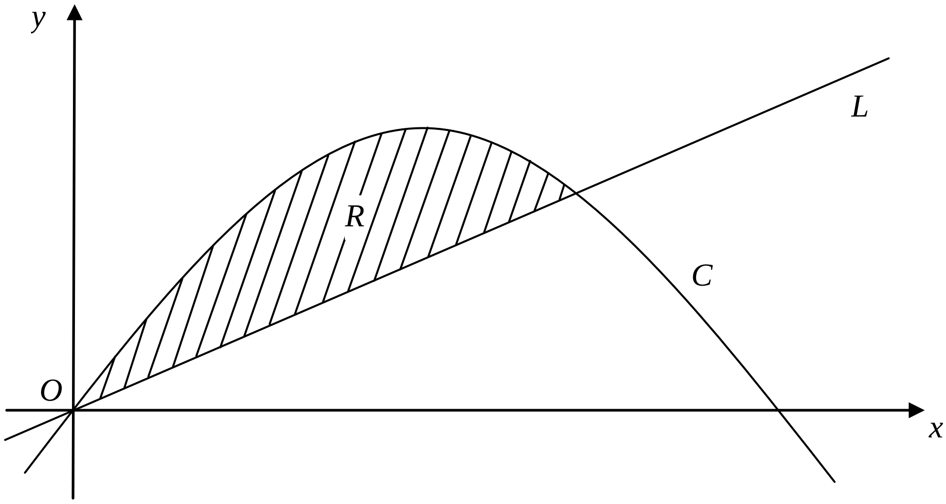
Figure 1 shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in Figure 1, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(9)

5. **Figure 2**



In Figure 2 the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects with the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points (0, 0) and (4, 8).

(3)

The region R, bounded by the curve C and the line L, is shown shaded in Figure 2.

(c) Use calculus to find the area of R.

(6)

6.

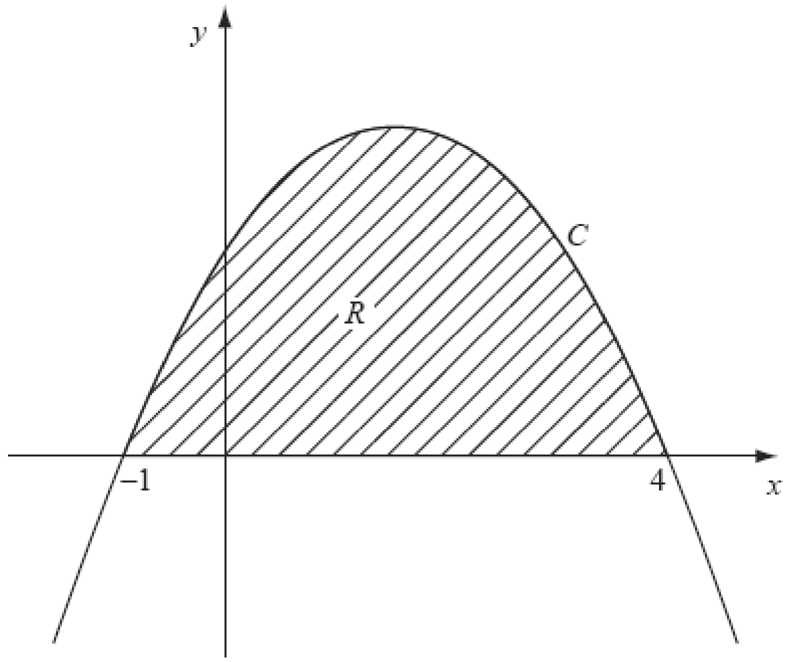


Figure 1

Figure 1 shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = -1$ and $x = 4$. The region R, shown shaded in Figure 1, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(5)

7.

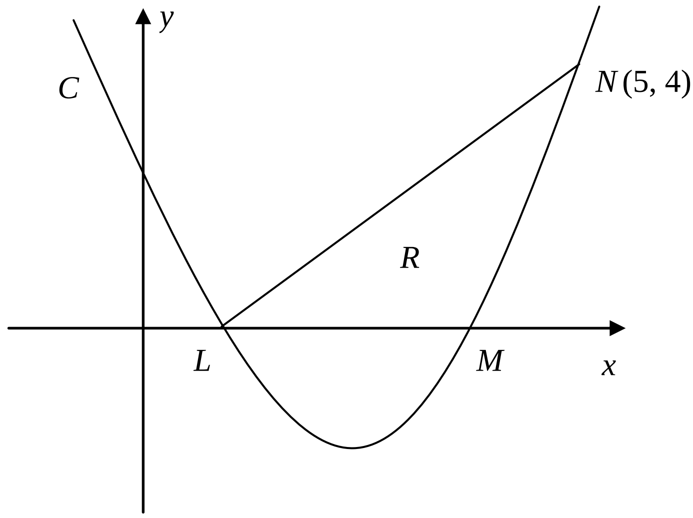


Figure 2

The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in Figure 2.

(a) Find the coordinates of the point L and the point M.

(2)

(b) Show that the point $N(5, 4)$ lies on C.

(1)

(c) Find $\int (x^2 - 5x + 4) \ \mathrm{d}x$.

(2)

The finite region R is bounded by LN, LM and the curve C as shown in Figure 2.

(d) Use your answer to part (c) to find the exact value of the area of R.

(5)

8.

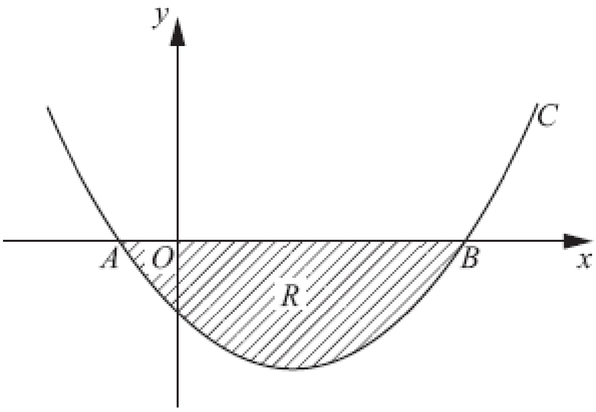


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5).$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

9. **Figure 1**

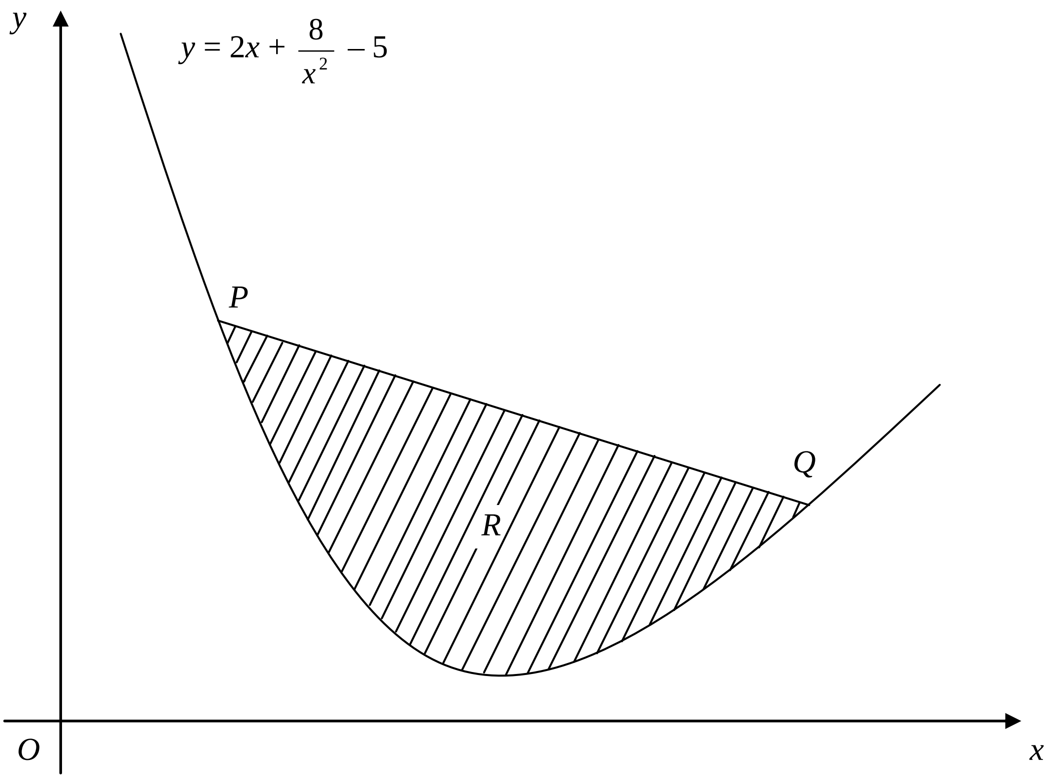


Figure 1 shows part of a curve C with equation $y = 2x + \frac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in Figure 1, is bounded by C and the straight line joining P and Q.

(*a*) Find the exact area of R.

(8)

(*b*) Use calculus to show that y is increasing for $x > 2$.

(4)

10.

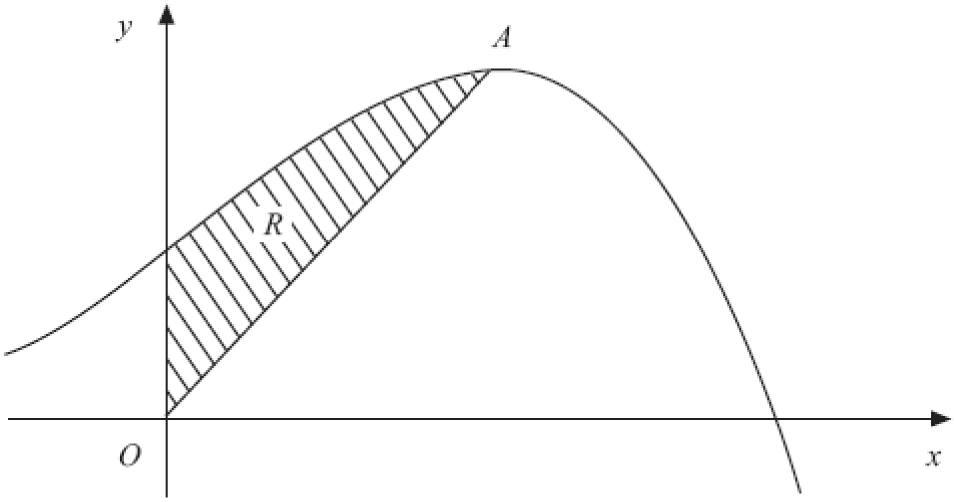


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

The region R, shown shaded in Figure 2, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

11. The speed, v m s^{-1}, of a train at time t seconds is given by

$$v = \sqrt{(1.2^t - 1)}, \qquad 0 \leq t \leq 30.$$

The following table shows the speed of the train at 5 second intervals.

t	0	5	10	15	20	25	30
v	0	1.22	2.28		6.11		

(a) Complete the table, giving the values of v to 2 decimal places.

(3)

The distance, s metres, travelled by the train in 30 seconds is given by

$$s = \int_0^{30} \sqrt{(1.2^t - 1)} \; \mathrm{d}t .$$

(*b*) Use the trapezium rule, with all the values from your table, to estimate the value of *s*.

(3)

12. $$y = \sqrt{(10x - x^2)}.$$

(*a*) Copy and complete the table below, giving the values of *y* to 2 decimal places.

x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

(2)

(*b*) Use the trapezium rule, with all the values of *y* from your table, to find an approximation for the value of $\int_1^3 \sqrt{(10x - x^2)}\ \mathrm{d}x$.

(4)

13. (*a*) Sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the *y*-axis.

(2)

(*b*) Copy and complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(*c*) Use the trapezium rule, with all the values from your tables, to find an approximation for the value of $\int_0^1 3^x\ \mathrm{d}x$.

(4)

14. $$y = \frac{5}{3x^2 - 2}$$

(*a*) Copy and complete the table below, giving the values of *y* to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2-2}\,dx$.

(4)

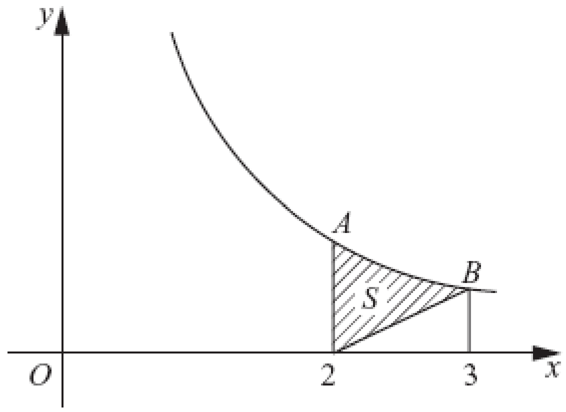


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2-2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(*c*) Use your answer to part (*b*) to find an approximate value for the area of S.

15.

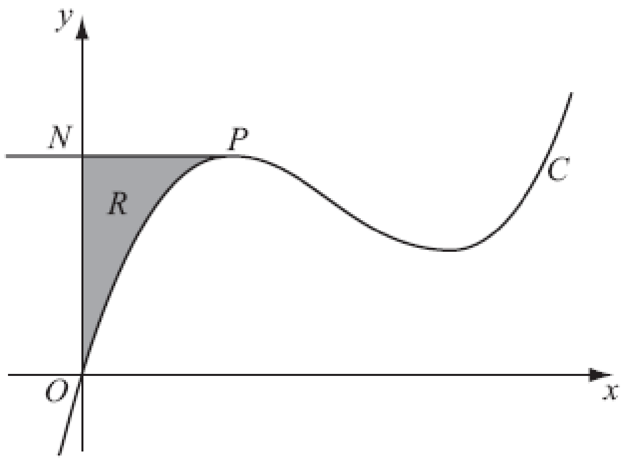


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

16. **Figure 3**

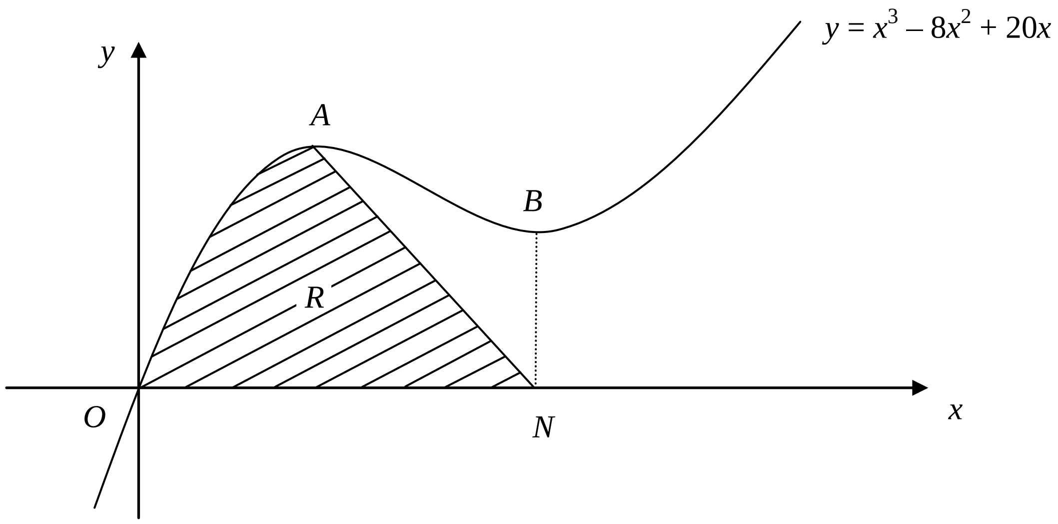


Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$. The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\dfrac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N. The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x) \ \mathrm{d}x$.

(3)

(d) Hence calculate the exact area of R.

(5)

17. A river, running between parallel banks, is 20 m wide. The depth, y metres, of the river measured at a point x metres from one bank is given by the formula

$$y = \frac{1}{10}x\sqrt{(20 - x)}, \qquad 0 \le x \le 20.$$

(a) Complete the table below, giving values of y to 3 decimal places.

x	0	4	8	12	16	20
y	0		2.771			0

(2)

(b) Use the trapezium rule with all the values in the table to estimate the cross-sectional area of the river.

(4)

Given that the cross-sectional area is constant and that the river is flowing uniformly at 2 m s^{-1},

(c) estimate, in m^3, the volume of water flowing per minute, giving your answer to 3 significant figures.

(2)

18. $$y = \sqrt{(5^x + 2)}$$

(*a*) Copy and complete the table below, giving the values of y to 3 decimal places.

x	0	0.5	1	1.5	2
y			2.646	3.630	

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of $\int_0^2 \sqrt{(5^x + 2)}\ \mathrm{d}x$.

(4)

19. (*a*) Complete the table below, giving values of $\sqrt{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

(2)

20. $$y = 3^x + 2x.$$

(*a*) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\ \mathrm{d}x$.

(4)

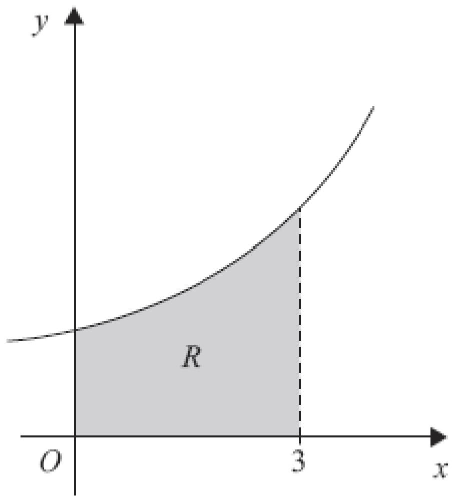


Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{(2^x + 1)}$, the x-axis and the lines $x = 0$ and $x = 3$

(b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R.

(4)

(c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R.

(2)

21. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \, dx$.

(5)

22. Evaluate $\int_1^8 \frac{1}{\sqrt{x}} \, dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

23. The curve C has equation

$$y = x\sqrt{(x^3 + 1)}, \qquad 0 \le x \le 2.$$

(a) Copy and complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

(*b*) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\ \mathrm{d}x$, giving your answer to 3 significant figures.

(4)

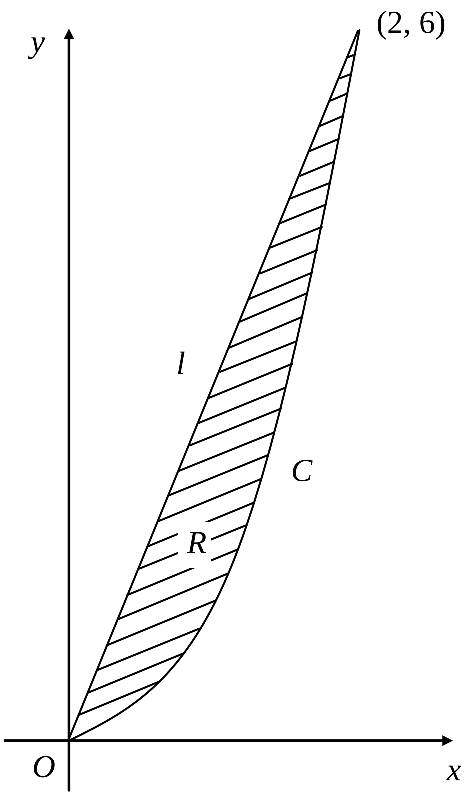


Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt{(x^3 + 1)}$, $0 \le x \le 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(*c*) Use your answer to part (*b*) to find an approximation for the area of R, giving your answer to 3 significant figures.

(3)

24.

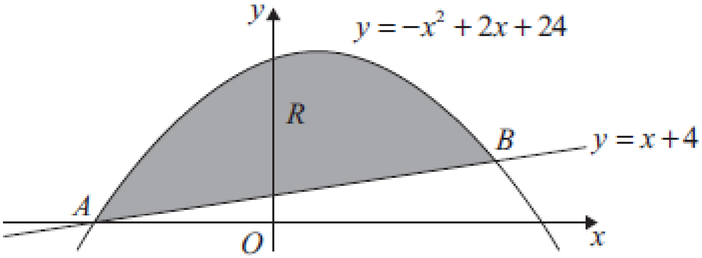


Figure 3

The straight line with equation $y = x + 4$ cuts the curve with equation $y = -x^2 + 2x + 24$ at the points A and B, as shown in Figure 3.

(a) Use algebra to find the coordinates of the points A and B.

(4)

The finite region R is bounded by the straight line and the curve and is shown shaded in Figure 3.

(b) Use calculus to find the exact area of R.

(7)

Applied Differentiation

1. **Figure 3**

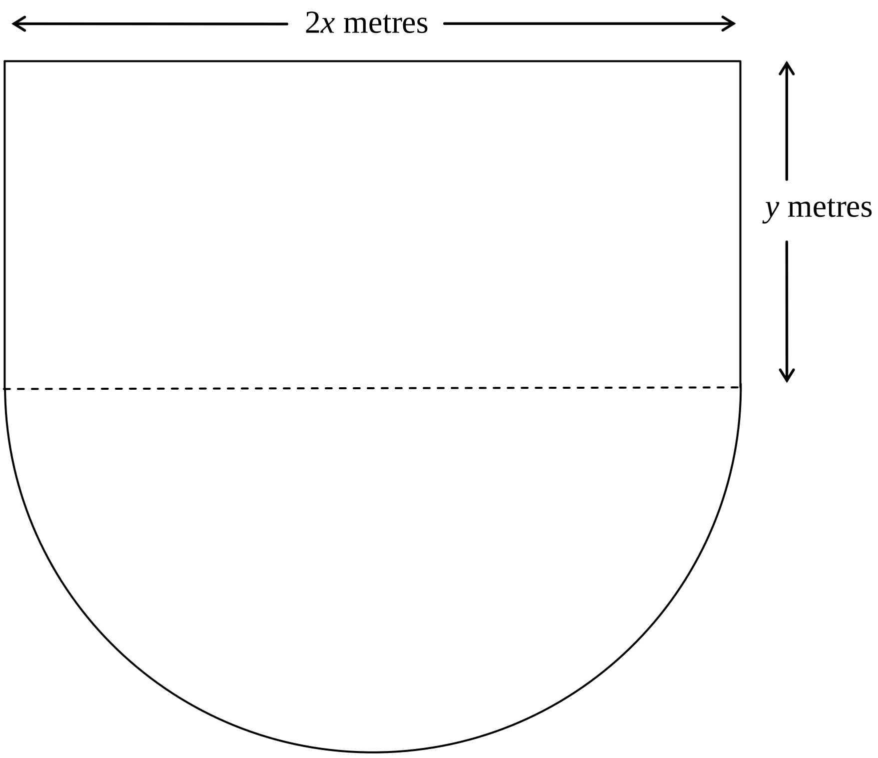


Figure 3 shows the plan of a stage in the shape of a rectangle joined to a semicircle. The length of the rectangular part is $2x$ metres and the width is y metres. The diameter of the semicircular part is $2x$ metres. The perimeter of the stage is 80 m.

(*a*) Show that the area, A m^2, of the stage is given by

$$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2.$$

(4)

(*b*) Use calculus to find the value of x at which A has a stationary value.

(4)

(*c*) Prove that the value of x you found in part (*b*) gives the maximum value of A.

(2)

(*d*) Calculate, to the nearest m^2, the maximum area of the stage.

(2)

2. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(*a*) $f''(x)$,

(3)

(*b*) $\int_1^2 f(x) \, dx$.

(4)

3. The curve *C* has equation

$$y = 2x^3 - 5x^2 - 4x + 2.$$

(*a*) Find $\frac{dy}{dx}$.

(2)

(*b*) Using the result from part (*a*), find the coordinates of the turning points of *C*.

(4)

(*c*) Find $\frac{d^2y}{dx^2}$.

(2)

(*d*) Hence, or otherwise, determine the nature of the turning points of *C*.

(2)

4. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(*a*) Find the value of v for which C is a minimum.

(5)

(*b*) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v.

(2)

(*c*) Calculate the minimum total cost of the journey.

(2)

5. **Figure 4**

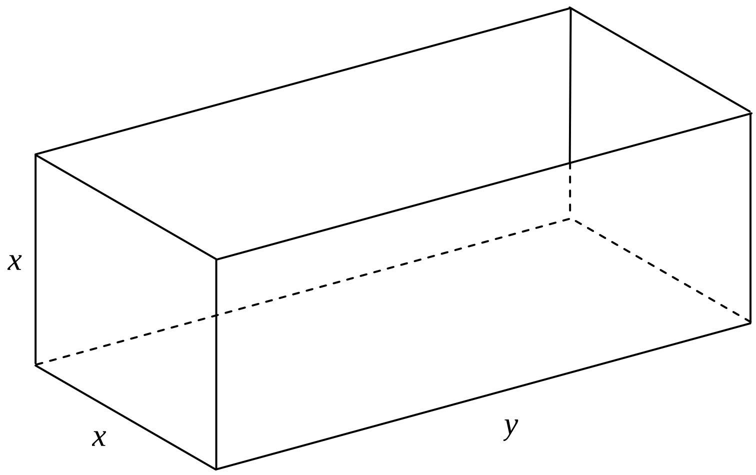


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(*a*) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(*b*) Use calculus to find the value of x for which A is stationary.

(4)

(*c*) Prove that this value of x gives a minimum value of A.

(2)

(*d*) Calculate the minimum area of sheet metal needed to make the tank.

(2)

6. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(*a*) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(*b*) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(*c*) Justify that the value of V you have found is a maximum.

(2)

7. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10$, $x > 0$.

(a) Use calculus to find the coordinates of the turning point on C.

(7)

(b) Find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

(2)

(c) State the nature of the turning point.

(1)

8. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5 - x)^2, \quad 0 < x < 5.$$

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

9. Find the coordinates of the stationary point on the curve with equation $y = 2x^2 - 12x$.

(4)

10.

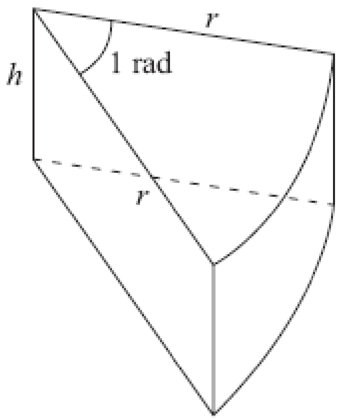


Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(a) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}.$$

(5)

(b) Use calculus to find the value of r for which S is stationary.

(4)

(c) Prove that this value of r gives a minimum value of S.

(2)

(d) Find, to the nearest cm^2, this minimum value of S.

(2)

11. $y = x^2 - k\sqrt{x}$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

12.

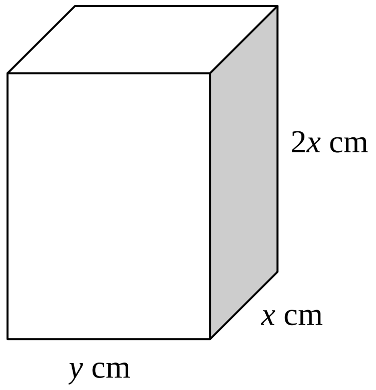


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(*a*) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(*b*) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(*c*) Justify that the value of V you have found is a maximum.

(2)

13.

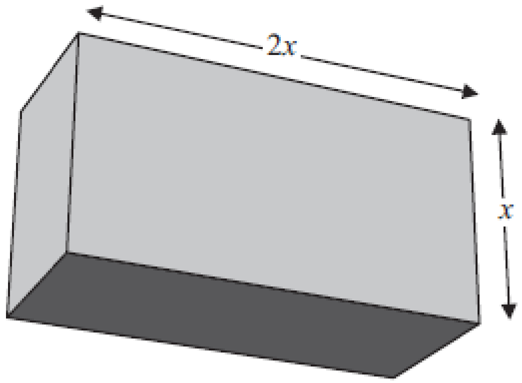


Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.

The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}.$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)