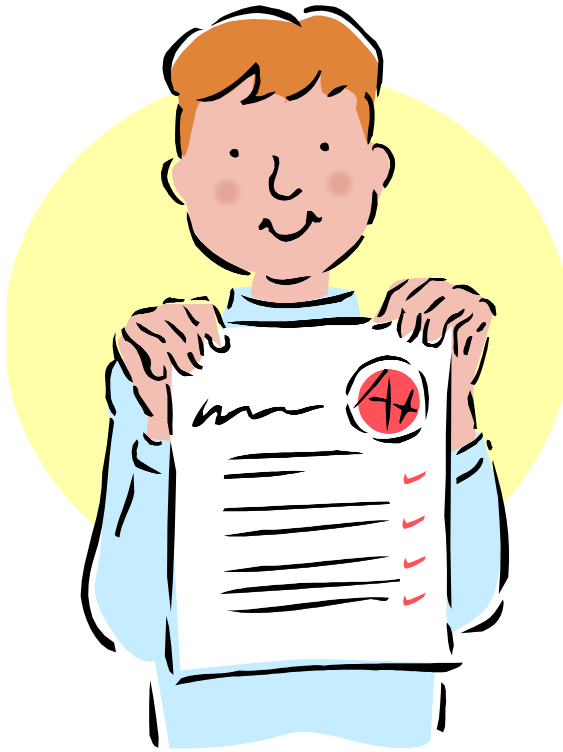
Mr A. Slack (2012)

Core 1



Practice Examination Questions

Mr A. Slack (2012)

Assessment Criteria by examination paper: Core 1

Algebra and Functions	I can use the laws of indices.	
	I can use and manipulate surds.	
	I can describe a quadratic function and its graph.	
	I can find the discriminant of a quadratic function and explain it.	
	I can complete the square of a quadratic function and explain it.	
	I can solve a quadratic by various means.	
	I can solve a pair of simultaneous equations.	
	I can expand brackets and collect like terms.	
	I can sketch curves defined by equations.	
	I can transform graphs.	
Coordinate Geometry	I can find the equation of a straight line given information.	
	I can find the equation of a perpendicular line.	
	I know the conditions for a line to be perpendicular to another.	
	I know the conditions for a line to be parallel to another.	
	I can find the length of a line segment.	
	I can write the equation of a straight line in different forms	
Arithmetic Series	I can find the n^{th} term of an arithmetic sequence.	
	I can prove the formula to find the sum of the first n terms of a series.	
	I can find the sum of the first n numbers in an arithmetic series.	
	I can generate sequences from a recurrence relation.	
	I can use the $\sum U_n$ notation.	

Differentiation	I can differentiate a function.	
	I know the differentiation is the gradient of the tangents of the function.	
	I can find and understand the second order derivative of a function.	
	I can manipulate functions in order to differentiate them.	
	I know the link between the order of a function and the order of the differential.	
	I can find the equation of a tangent at a point to a given curve.	
	I can find the equation of the normal at a point to a given curve.	
Integration	I know that indefinite integration is the reverse of differentiation.	
	I can integrate functions.	
	I can use integration to find the equation of a curve, given f'(x).	
	I understand the term 'constant of integration' and can find it.	

This appendix lists formulae that candidates are expected to <u>remember</u> and that will not be included in formulae booklets.

Quadratic equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation

function	derivative
x^n	nx^{n-1}

Integration

function	integral
x^n	$\dfrac{1}{n+1}x^{n+1} + c,\ n \neq -1$

Surds and Indices

1. (*a*) Write $\sqrt{45}$ in the form $a\sqrt{5}$, where a is an integer.

(1)

(*b*) Express $\dfrac{2(3+\sqrt{5})}{(3-\sqrt{5})}$ in the form $b + c\sqrt{5}$, where b and c are integers.

(5)

2. Write

$$\sqrt{(75)} - \sqrt{(27)}$$

in the form $k\sqrt{x}$, where k and x are integers.

(2)

3. Simplify

$$\frac{5-\sqrt{3}}{2+\sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(4)

4. (*a*) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(*b*) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

5. Simplify $(3 + \sqrt{5})(3 - \sqrt{5})$.

(2)

6. Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(2)

7. (*a*) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

(3)

(*b*) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

8. Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1},$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

9. Simplify

(a) $(3\sqrt{7})^2$

(1)

(b) $(8 + \sqrt{5})(2 - \sqrt{5})$

(3)

10. (a) Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$.

(2)

(b) Express $\dfrac{26}{4+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

(2)

11. (a) Find the value of $8^{\frac{4}{3}}$.

(2)

(b) Simplify $\dfrac{15x^{\frac{4}{3}}}{3x}$.

(2)

12. (a) Write down the value of $16^{\frac{1}{2}}$.

(1)

(b) Find the value of $16^{-\frac{3}{2}}$.

(2)

13. (a) Write down the value of $16^{\frac{1}{4}}$.

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$.

(2)

14. (*a*) Write down the value of $125^{\frac{1}{3}}$.

(1)

(*b*) Find the value of $125^{-\frac{2}{3}}$.

(2)

15. (*a*) Find the value of $16^{-\frac{1}{4}}$.

(2)

(*b*) Simplify $x\left(2x^{-\frac{1}{4}}\right)^4$.

(2)

16. (*a*) Write down the value of $8^{\frac{1}{3}}$.

(1)

(*b*) Find the value of $8^{-\frac{2}{3}}$.

(2)

17. Find the value of

(*a*) $25^{\frac{1}{2}}$,

(1)

(*b*) $25^{-\frac{3}{2}}$.

(2)

18. Given that $32\sqrt{2} = 2^a$, find the value of a.

(3)

Differentiation and Integration

1. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^p + 3x^q$, where p and q are constants.

(2)

Given that $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(*b*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, simplifying the coefficient of each term.

(4)

2. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2+2}{x}, \qquad x > 0,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(6)

3. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(*a*) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(*b*) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, simplifying the coefficient of each term.

(4)

4. Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(*a*) write down the value of p and the value of q.

(2)

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ and that $y = 90$ when $x = 4$,

(*b*) find y in terms of x, simplifying the coefficient of each term.

(5)

5. $$\mathrm{f}(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0.$$

(*a*) Show that $\mathrm{f}(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(*b*) Find $\mathrm{f}'(x)$.

(3)

(*c*) Evaluate $\mathrm{f}'(9)$.

(2)

6. (*a*) Show that $\dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$ can be written as $9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$.

(2)

Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(3-\sqrt{x})^2}{\sqrt{x}}$, $x > 0$, and that $y = \frac{2}{3}$ at $x = 1$,

(*b*) find y in terms of x.

(6)

7. $$\mathrm{f}(x) = 3x + x^3, \qquad x > 0.$$

(*a*) Differentiate to find $\mathrm{f}'(x)$.

(2)

Given that $\mathrm{f}'(x) = 15$,

(*b*) find the value of x.

8. Given that $y = 3x^2 + 4\sqrt{x}$, $x > 0$, find

(*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$,

(2)

(*b*) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$,

(2)

(*c*) $\displaystyle\int y \;\mathrm{d}x$.

(3)

9. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x}$,

(3)

(*b*) $\int y \;\mathrm{d}x$.

(4)

10. (i) Given that $y = 5x^3 + 7x + 3$, find

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x}$,

(3)

(*b*) $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$.

(1)

(ii) Find $\int \left(1 + 3\sqrt{x} - \frac{1}{x^2}\right) \mathrm{d}x$.

(4)

11. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$, find

(*a*) $\frac{\mathrm{d}y}{\mathrm{d}x}$,

(3)

(*b*) $\int y \;\mathrm{d}x$, simplifying each term.

(3)

12. Given that $y = 6x - \frac{4}{x^2}$, $x \neq 0$,

(*a*) find $\frac{\mathrm{d}y}{\mathrm{d}x}$,

(2)

(*b*) find $\int y \;\mathrm{d}x$.

(3)

13. Given that $y = 2x^2 - \dfrac{6}{x^3}$, $x \neq 0$,

(a) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$,

(2)

(b) find $\displaystyle\int y \ \mathrm{d}x$.

(3)

14. (a) Show that $(4 + 3\sqrt{x})^2$ can be written as $16 + k\sqrt{x} + 9x$, where k is a constant to be found.

(2)

(b) Find $\displaystyle\int (4 + 3\sqrt{x})^2 \ \mathrm{d}x$.

(3)

15. Find $\displaystyle\int (3x^2 + 4x^5 - 7) \ \mathrm{d}x$.

(4)

16. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5) \ \mathrm{d}x,$$

giving each term in its simplest form.

(4)

17. Find $\displaystyle\int (12x^5 - 8x^3 + 3) \ \mathrm{d}x$, giving each term in its simplest form.

(4)

18. Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}}) \ \mathrm{d}x,$$

giving each term in its simplest form.

(5)

19. Find $\int (2+5x^2)\ \mathrm{d}x$.

(3)

20. Find $\int (6x^2+2+x^{-\frac{1}{2}})\ \mathrm{d}x$, giving each term in its simplest form.

(4)

21. A curve has equation $y = \mathrm{f}(x)$ and passes through the point (4, 22).

Given that

$$\mathrm{f}'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $\mathrm{f}(x)$, giving each term in its simplest form.

(5)

22. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

23. Differentiate with respect to x

(*a*) $x^4 + 6\sqrt{x}$,

(3)

(*b*) $\dfrac{(x+4)^2}{x}$.

(4)

24. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

25.

$$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \qquad x > 0.$$

Given that $y = 35$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(7)

26. The curve C has equation $y = f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2.$$

Given that the point P (4, 5) lies on C, find

(*a*) $f(x)$,

(5)

(*b*) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Quadratics

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1. The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k.

(4)

2. The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has different real roots.

(*a*) Show that $k^2 - 4k - 12 > 0$.

(2)

(*b*) Find the set of possible values of k.

(4)

3.

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

(*a*) Find the value of a and the value of b.

(3)

(*b*) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(3)

4.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(*a*) Find the values of the constants a and b.

(2)

(*b*) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(*c*) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (*b*).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(*d*) Find the set of possible values of k, giving your answer in surd form.

(4)

5. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

6. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

7. The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0.$$

(3)

(b) Find the set of possible values of k.

(4)

8. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)

9. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

10. Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

11. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(*a*) Find the value of p.

(4)

(*b*) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

12. (*a*) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q,$$

where p and q are integers to be found.

(2)

(*b*) Sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(*c*) Find the value of the discriminant of $x^2 + 6x + 11$.

(2)

13.

$$\mathrm{f}(x) = x^2 + (k + 3)x + k,$$

where k is a real constant.

(*a*) Find the discriminant of $\mathrm{f}(x)$ in terms of k.

(2)

(*b*) Show that the discriminant of $\mathrm{f}(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(*c*) Show that, for all values of k, the equation $\mathrm{f}(x) = 0$ has real roots.

(2)

3. On separate diagrams, sketch the graphs of

(a) $y = (x + 3)^2$,

(3)

(b) $y = (x + 3)^2 + k$, where k is a positive constant.

(2)

Show on each sketch the coordinates of each point at which the graph meets the axes.

10. Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

(4)

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

10. $$f(x) = x^2 + 4kx + (3 + 11k), \quad \text{where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation $f(x) = 0$ has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

Simultaneous Equations

1. Solve the simultaneous equations

$$x + y = 2$$

$$x^2 + 2y = 12.$$

(6)

2. Solve the simultaneous equations

$$y = x - 2,$$

$$y^2 + x^2 = 10.$$

(7)

3. Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

4. Solve the simultaneous equations

$$x - 2y = 1,$$

$$x^2 + y^2 = 29.$$

(6)

5. Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

(7)

6. (*a*) By eliminating y from the equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0.$$

(2)

(*b*) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

Arithmetic Sequences and Series

1. The rth term of an arithmetic series is $(2r - 5)$.

(a) Write down the first three terms of this series.

(2)

(b) State the value of the common difference.

(1)

(c) Show that $\sum_{r=1}^{n}(2r - 5) = n(n - 4)$.

(3)

2. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

(a) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200.

(1)

(b) Find the amount of Alice's annual allowance on her 18th birthday.

(2)

(c) Find the total of the allowances that Alice had received up to and including her 18th birthday.

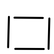
(3)

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(d) Find how old Alice was when she received her last allowance.

(7)

3. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

Row 1 

Row 2 

Row 3 

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row.

(3)

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(*b*) Find the total number of sticks Ann uses in making these 10 rows.

(3)

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k + 1)$th row,

(*c*) show that k satisfies $(3k - 100)(k + 35) < 0$.

(4)

(*d*) Find the value of k.

(2)

4. The first term of an arithmetic sequence is 30 and the common difference is -1.5.

(*a*) Find the value of the 25th term.

(2)

The rth term of the sequence is 0.

(*b*) Find the value of r.

(2)

The sum of the first n terms of the sequence is S_n.

(*c*) Find the largest positive value of S_n.

(3)

5. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(*a*) Use this information to write down two equations for a and d.

(2)

(*b*) Show that $a = -17.5$ and find the value of d.

(2)

The sum of the first n terms of the series is 2750.

(*c*) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(*d*) Hence find the value of n.

(3)

6. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and son on, so that the amounts of money she gave each year formed an arithmetic sequence.

(*a*) Find the amount of money she gave in Year 10.

(2)

(*b*) Calculate the total amount of money she gave over the 20-year period.

(3)

Kevin also gave money to charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(*c*) Calculate the value of A.

(4)

7. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(*a*) Show that $10a + 45d = 162$.

(2)

Given also that the sixth term of the sequence is 17,

(*b*) write down a second equation in a and d,

(1)

(*c*) find the value of a and the value of d.

(4)

8. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(*a*) Show that on the 4th Saturday of training she runs 11 km.

(1)

(*b*) Find an expression, in terms of n, for the length of her training run on the nth Saturday.

(2)

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.

(3)

On the nth Saturday Sue runs 43 km.

(d) Find the value of n.

(2)

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

(2)

9. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d,

(3)

(b) the value of a,

(2)

(c) the total number of houses built in Oldtown over the 40-year period.

(3)

10. An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is

$$\tfrac{1}{2}n[2a + (n - 1)d].$$

(4)

Sean repays a loan over a period of n months. His monthly repayments form an arithmetic sequence.

He repays £149 in the first month, £147 in the second month, £145 in the third month, and so on. He makes his final repayment in the nth month, where $n > 21$.

(b) Find the amount Sean repays in the 21st month.

(2)

Over the n months, he repays a total of £5000.

(c) Form an equation in n, and show that your equation may be written as

$$n^2 - 150n + 5000 = 0.$$

(3)

(d) Solve the equation in part (c).

(3)

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible solution to the repayment problem.

(1)

11. An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of a and the value of d.

(7)

12. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.

(3)

(b) Calculate her total savings over the complete 200 week period.

(3)

13. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a for their first day, £($a + d$) for their second day, £($a + 2d$) for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d.

(2)

A picker who works for all 30 days will earn a total of £1005.

(b) Show that $15(a + 40.75) = 1005$.

(2)

(c) Hence find the value of a and the value of d.

(4)

14. (*a*) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \dots\dots + 100.$$

(3)

(*b*) In the arithmetic series

$$k + 2k + 3k + \dots\dots + 100,$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}.$$

(4)

(*c*) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), \quad (4k + 4), \quad (6k + 7), \ \dots ,$$

giving your answer in its simplest form.

(2)

Transformations of Functions

1.

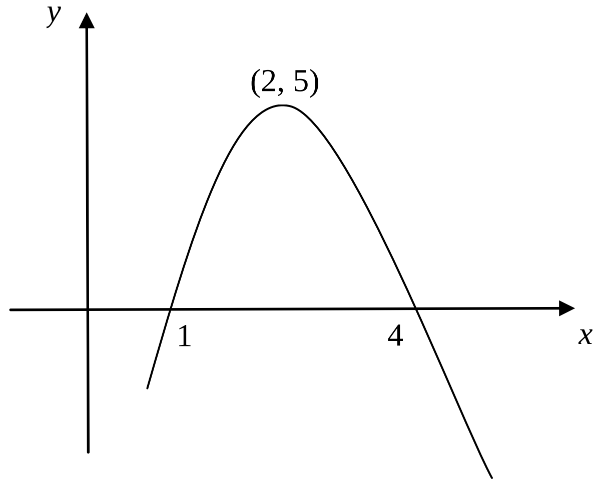


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the points (1, 0) and (4, 0). The maximum point on the curve is (2, 5).

In separate diagrams, sketch the curves with the following equations. On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(*a*) $y = 2\mathrm{f}(x)$,

(3)

(*b*) $y = \mathrm{f}(-x)$.

(3)

The maximum point on the curve with equation $y = \mathrm{f}(x + a)$ is on the y-axis.

(*c*) Write down the value of the constant a.

(1)

2.

Figure 1

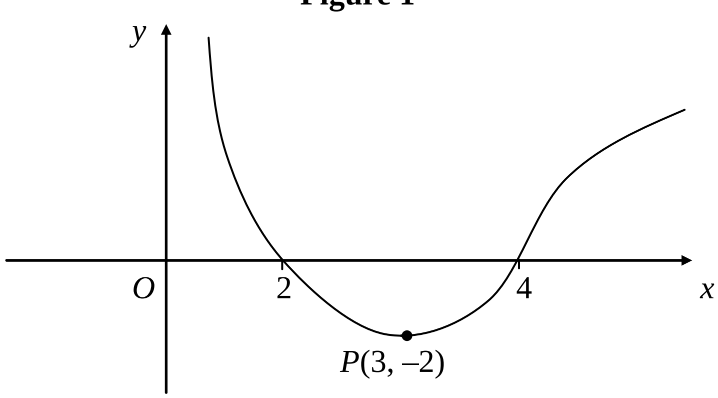


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Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve crosses the x-axis at the points (2, 0) and (4, 0). The minimum point on the curve is $P(3, -2)$.

In separate diagrams sketch the curve with equation

(*a*) $y = -f(x)$,

(3)

(*b*) $y = f(2x)$.

(3)

On each diagram, give the coordinates of the points at which the curve crosses the x-axis, and the coordinates of the image of P under the given transformation.

3.

Figure 1

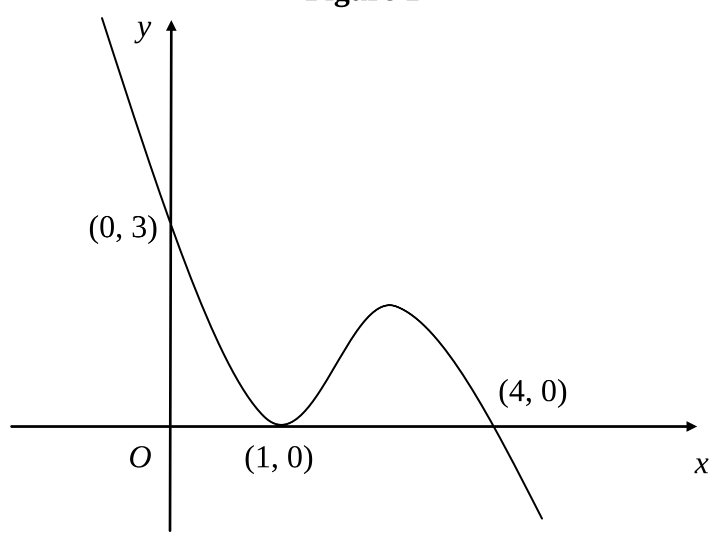

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curve with equation

(*a*) $y = f(x + 1)$,

(3)

(*b*) $y = 2f(x)$,

(3)

(*c*) $y = f\left(\frac{1}{2}x\right)$.

(3)

On each diagram show clearly the coordinates of all the points at which the curve meets the axes.

4.

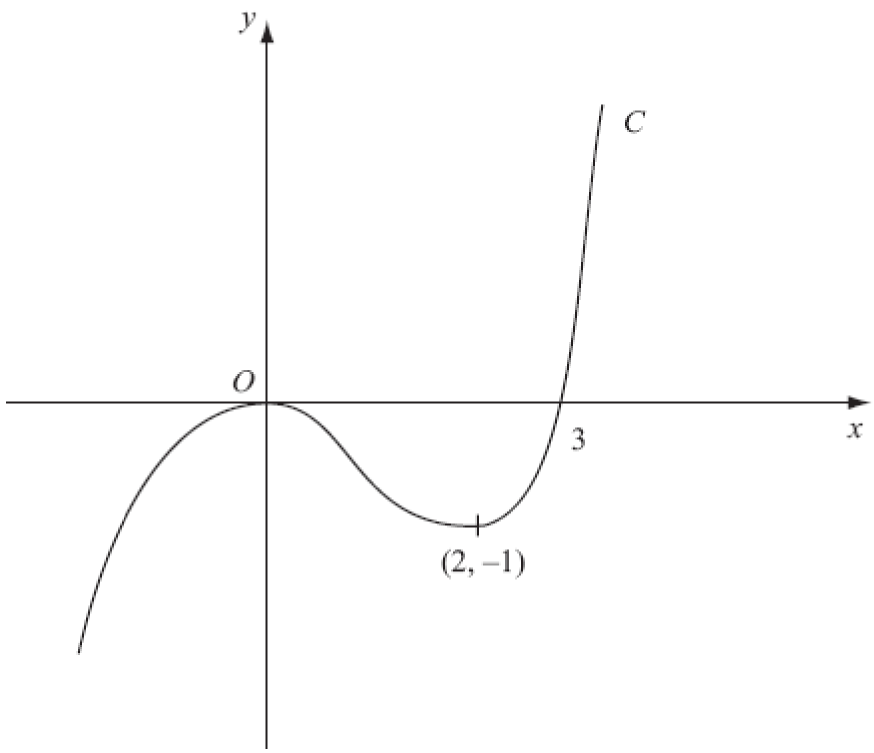

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$. There is a maximum at (0, 0), a minimum at (2, –1) and C passes through (3, 0).

On separate diagrams, sketch the curve with equation

(*a*) $y = \mathrm{f}(x + 3)$,

(3)

(*b*) $y = \mathrm{f}(-x)$.

(3)

5.

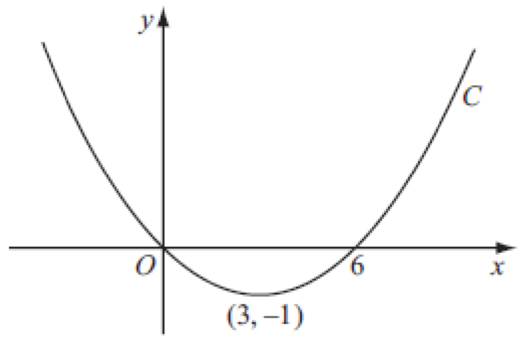

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.
The curve C passes through the origin and through (6, 0).
The curve C has a minimum at the point (3, –1).

On separate diagrams, sketch the curve with equation

(*a*) $y = \mathrm{f}(2x)$,

(3)

(*b*) $y = -\mathrm{f}(x)$,

(3)

(*c*) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$.

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.

6.

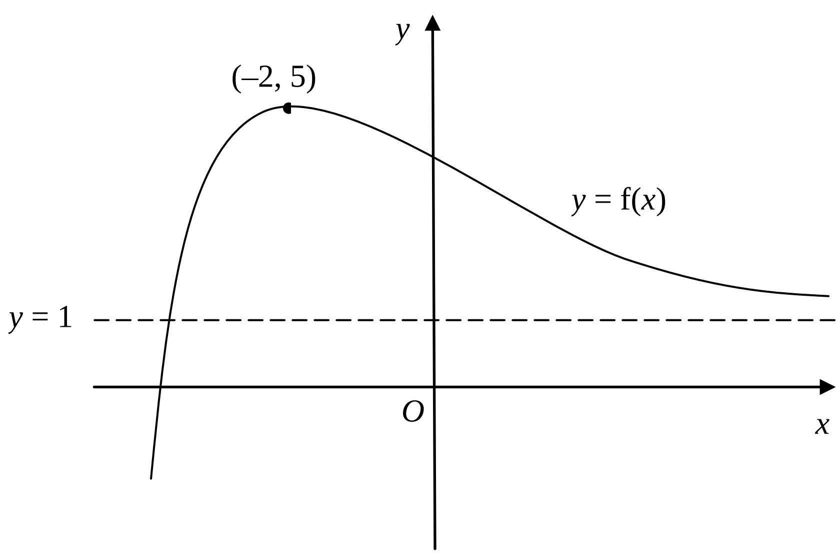

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$.

The curve has a maximum point $(–2, 5)$ and an asymptote $y = 1$, as shown in Figure 1.

On separate diagrams, sketch the curve with equation

(*a*) $y = f(x) + 2$,

(2)

(*b*) $y = 4f(x)$,

(2)

(c) $y = f(x + 1)$.

(3)

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

7.

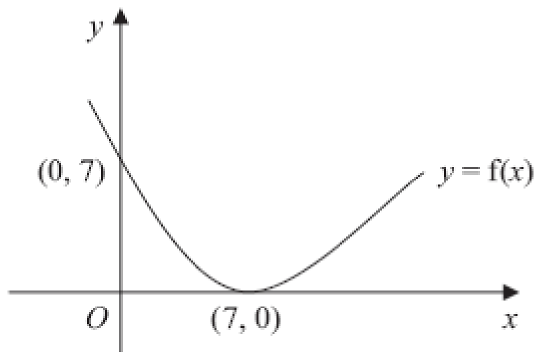

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the point $(0, 7)$ and has a minimum point at $(7, 0)$.

On separate diagrams, sketch the curve with equation

(*a*) $y = f(x) + 3$,

(3)

(*b*) $y = f(2x)$.

(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

8. **Figure 1**

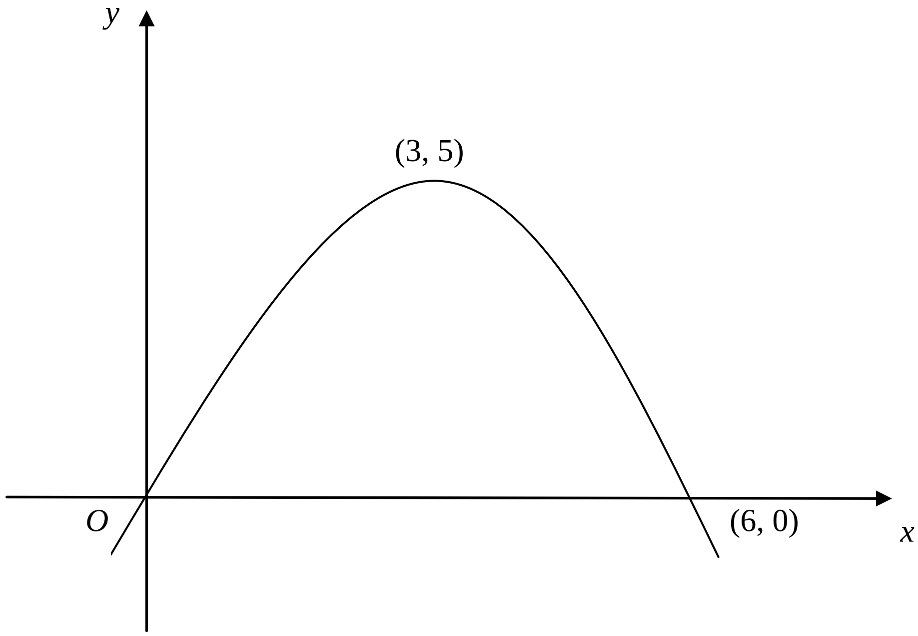

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the origin O and through the point (6, 0). The maximum point on the curve is (3, 5).

On separate diagrams, sketch the curve with equation

(*a*) $y = 3\mathrm{f}(x)$,

(2)

(*b*) $y = \mathrm{f}(x + 2)$.

(3)

On each diagram, show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

9.

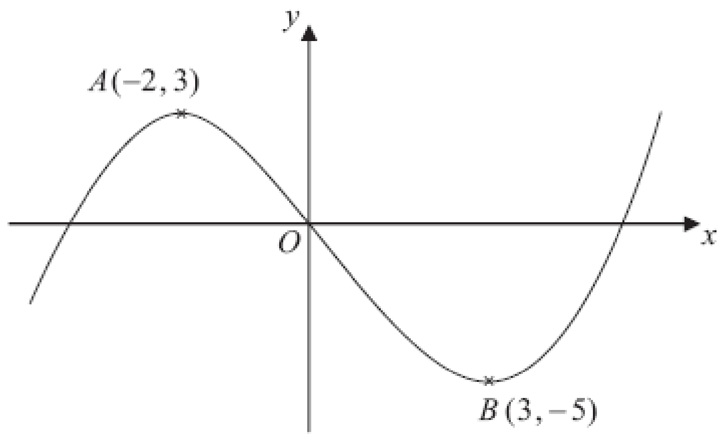

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at (–2, 3) and a minimum point B at (3, – 5).

On separate diagrams sketch the curve with equation

(*a*) $y = \mathrm{f}(x + 3)$,

(3)

(*b*) $y = 2\mathrm{f}(x)$.

(3)

On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = \mathrm{f}(x) + a$ has a minimum at (3, 0), where a is a constant.

(*c*) Write down the value of a.

(1)

10.

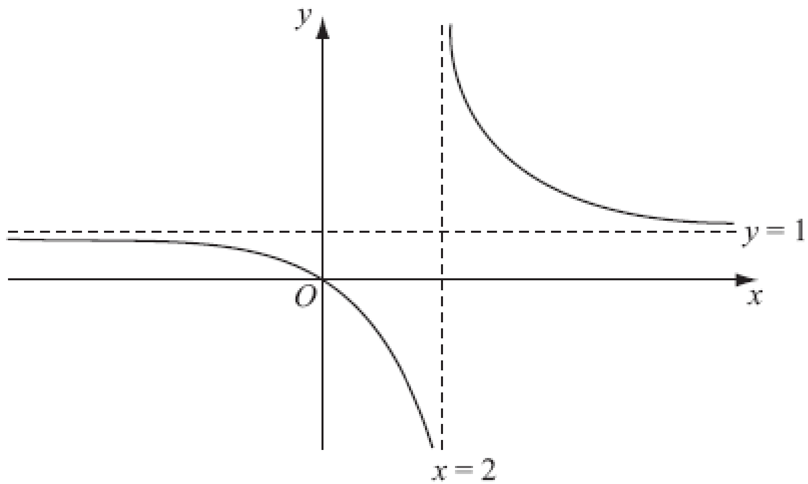

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2.$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

(*a*) In the space below, sketch the curve with equation $y = \mathrm{f}(x - 1)$ and state the equations of the asymptotes of this curve.

(3)

(*b*) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x - 1)$ crosses the coordinate axes.

(4)

11. Given that $$\mathrm{f}(x) = \frac{1}{x}, \quad x \neq 0,$$

(*a*) sketch the graph of $y = \mathrm{f}(x) + 3$ and state the equations of the asymptotes.

(4)

(*b*) Find the coordinates of the point where $y = \mathrm{f}(x) + 3$ crosses a coordinate axis.

(2)

Coordinate Geometry

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1. The line L has equation $y = 5 - 2x$.

(*a*) Show that the point P (3, –1) lies on L.

(1)

(*b*) Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

2. The points P and Q have coordinates (–1, 6) and (9, 0) respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ.

Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

3. The line l_1 has equation $3x + 5y - 2 = 0$.

(*a*) Find the gradient of l_1.

(2)

The line l_2 is perpendicular to l_1 and passes through the point (3, 1).

(*b*) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

4. The point $A(-6, 4)$ and the point $B(8, -3)$ lie on the line L.

(*a*) Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

(*b*) Find the distance AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

5. The line l_1 passes through the point $A(2, 5)$ and has gradient $-\frac{1}{2}$.

(*a*) Find an equation of l_1, giving your answer in the form $y = mx + c$.

(3)

The point B has coordinates $(–2, 7)$.

(b) Show that B lies on l_1.

(1)

(c) Find the length of AB, giving your answer in the form $k\sqrt{5}$, where k is an integer.

(3)

The point C lies on l_1 and has x-coordinate equal to p.

The length of AC is 5 units.

(d) Show that p satisfies

$$p^2 - 4p - 16 = 0.$$

(4)

6. The line L_1 has equation $2y - 3x - k = 0$, where k is a constant.

Given that the point $A(1, 4)$ lies on L_1, find

(a) the value of k,

(1)

(b) the gradient of L_1.

(2)

The line L_2 passes through A and is perpendicular to L_1.

(c) Find an equation of L_2 giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

The line L_2 crosses the x-axis at the point B.

(d) Find the coordinates of B.

(2)

(e) Find the exact length of AB.

(2)

7.

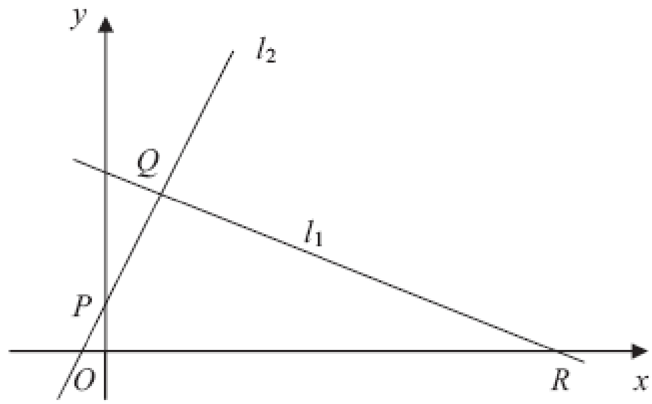


Figure 2

The points Q (1, 3) and R (7, 0) lie on the line l_1, as shown in Figure 2.

The length of QR is $a\sqrt{5}$.

(*a*) Find the value of a.

(3)

The line l_2 is perpendicular to l_1, passes through Q and crosses the y-axis at the point P, as shown in Figure 2. Find

(*b*) an equation for l_2,

(5)

(*c*) the coordinates of P,

(1)

(*d*) the area of ΔPQR.

(4)

8. The line l_1 passes through the point (9, –4) and has gradient $\frac{1}{3}$.

(*a*) Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line l_2 passes through the origin O and has gradient –2. The lines l_1 and l_2 intersect at the point P.

(*b*) Calculate the coordinates of P.

(4)

Given that l_1 crosses the y-axis at the point C,

(*c*) calculate the exact area of $\triangle OCP$.

(3)

9. (*a*) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

(*b*) Find the length of AB, leaving your answer in surd form.

(2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(*c*) Find the value of t.

(1)

(*d*) Find the area of triangle ABC.

(2)

10. The curve C has equation $y = f(x)$, $x \neq 0$, and the point $P(2, 1)$ lies on C. Given that

$$f'(x) = 3x^2 - 6 - \frac{8}{x^2},$$

(*a*) find $f(x)$.

(5)

(*b*) Find an equation for the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are integers.

(4)

11. The curve with equation $y = f(x)$ passes through the point $(-1, 0)$.

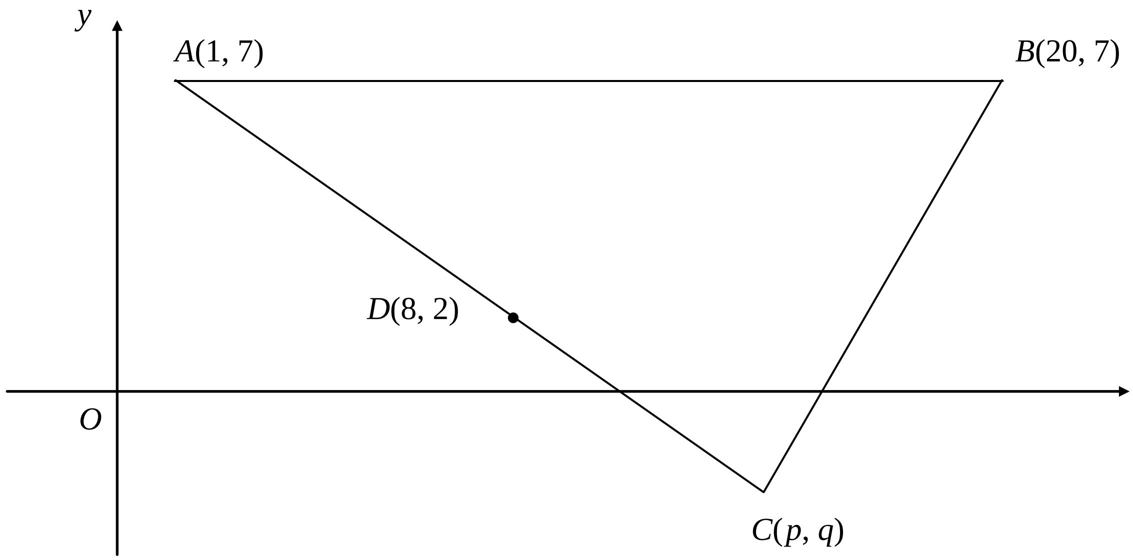
Given that

$$f'(x) = 12x^2 - 8x + 1,$$

find $f(x)$.

(5)

12. **Figure 2**



The points $A(1, 7)$, $B(20, 7)$ and $C(p, q)$ form the vertices of a triangle ABC, as shown in Figure 2. The point $D(8, 2)$ is the mid-point of AC.

(*a*) Find the value of p and the value of q.

(2)

The line l, which passes through D and is perpendicular to AC, intersects AB at E.

(*b*) Find an equation for l, in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(*c*) Find the exact x-coordinate of E.

(2)

13.

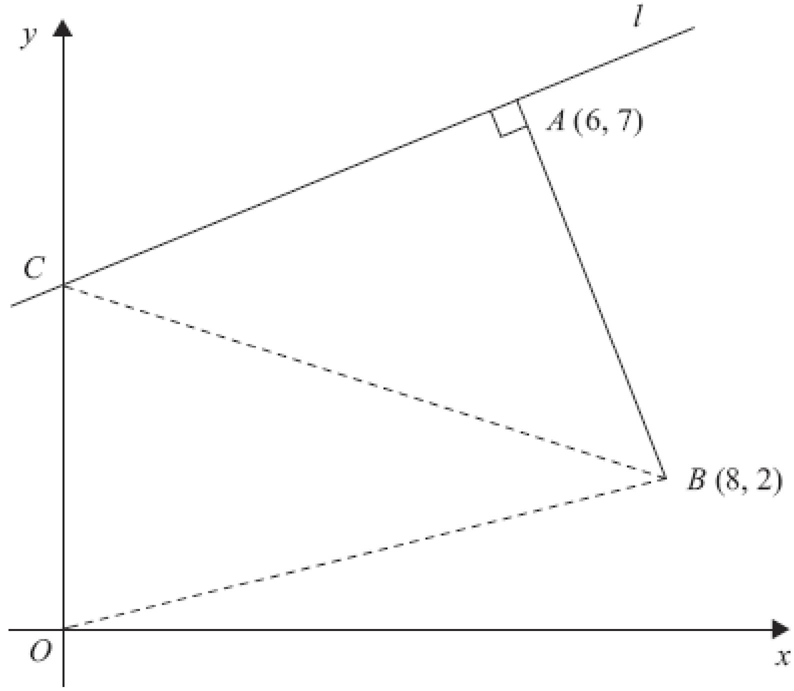


Figure 1

The points A and B have coordinates (6, 7) and (8, 2) respectively.

The line l passes through the point A and is perpendicular to the line AB, as shown in Figure 1.

(a) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Given that l intersects the y-axis at the point C, find

(b) the coordinates of C,

(2)

(c) the area of ΔOCB, where O is the origin.

(2)

14. The line l_1 passes through the points $P(–1, 2)$ and $Q(11, 8)$.

(a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants.

(4)

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

(b) Calculate the coordinates of S.

(5)

(c) Show that the length of RS is $3\sqrt{5}$.

(2)

(d) Hence, or otherwise, find the exact area of triangle PQR.

(4)

15. The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$.

(a) Find the gradient of the line l_2.

(2)

The point of intersection of l_1 and l_2 is P.

(b) Find the coordinates of P.

(3)

The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively.

(c) Find the area of triangle ABP.

(4)

Coordinate Geometry with differentiation

1. The curve C has equation $y = 4x^2 + \frac{5-x}{x}$, $x \neq 0$. The point P on C has x-coordinate 1.

(*a*) Show that the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P is 3.

(5)

(*b*) Find an equation of the tangent to C at P.

(3)

This tangent meets the x-axis at the point $(k, 0)$.

(*c*) Find the value of k.

(2)

2. The curve C has equation

$$y = \frac{(x+3)(x-8)}{x}, \quad x > 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form.

(4)

(*b*) Find an equation of the tangent to C at the point where $x = 2$.

(4)

3. The curve C has equation

$$y = \frac{1}{2}x^3 - 9x^{\frac{3}{2}} + \frac{8}{x} + 30, \qquad x > 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

(*b*) Show that the point $P(4, -8)$ lies on C.

(2)

(*c*) Find an equation of the normal to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

4. The curve C has equation $y = x^2(x - 6) + \dfrac{4}{x}$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 2 respectively.

(a) Show that the length of PQ is $\sqrt{170}$.

(4)

(b) Show that the tangents to C at P and Q are parallel.

(5)

(c) Find an equation for the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

5. The gradient of the curve C is given by

$$\frac{dy}{dx} = (3x - 1)^2.$$

The point $P(1, 4)$ lies on C.

(a) Find an equation of the normal to C at P.

(4)

(b) Find an equation for the curve C in the form $y = f(x)$.

(5)

(c) Using $\dfrac{dy}{dx} = (3x - 1)^2$, show that there is no point on C at which the tangent is parallel to the line $y = 1 - 2x$.

(2)

6. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \dfrac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C,

(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

7. The curve C has equation $y = 4x + 3x^{\frac{3}{2}} - 2x^2, \quad x > 0$.

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Show that the point $P(4, 8)$ lies on C.

(1)

(c) Show that an equation of the normal to C at the point P is

$$3y = x + 20.$$

(4)

The normal to C at P cuts the x-axis at the point Q.

(d) Find the length PQ, giving your answer in a simplified surd form.

(3)

8. The curve C has equation

$$y = 9 - 4x - \frac{8}{x}, \quad x > 0.$$

The point P on C has x-coordinate equal to 2.

(a) Show that the equation of the tangent to C at the point P is $y = 1 - 2x$.

(6)

(b) Find an equation of the normal to C at the point P.

(3)

The tangent at P meets the x-axis at A and the normal at P meets the x-axis at B.

(c) Find the area of the triangle APB.

(4)

9. **Figure 2**

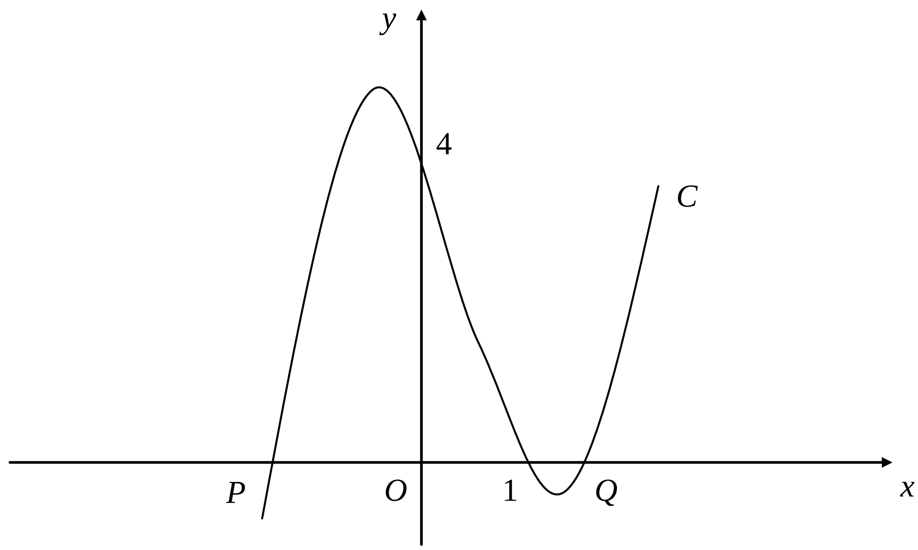


Figure 2 shows part of the curve C with equation

$$y = (x - 1)(x^2 - 4).$$

The curve cuts the x-axis at the points P, (1, 0) and Q, as shown in Figure 2.

(a) Write down the x-coordinate of P and the x-coordinate of Q.

(2)

(b) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 2x - 4$.

(3)

(c) Show that $y = x + 7$ is an equation of the tangent to C at the point (–1, 6).

(2)

The tangent to C at the point R is parallel to the tangent at the point (–1, 6).

(d) Find the exact coordinates of R.

(5)

10. The curve with equation $y = \mathrm{f}(x)$ passes through the point (1, 6). Given that

$$\mathrm{f}'(x) = 3 + \frac{5x^2 + 2}{x^{\frac{1}{2}}}, \quad x > 0,$$

find $\mathrm{f}(x)$ and simplify your answer.

(7)

11. The gradient of a curve C is given by $\dfrac{dy}{dx} = \dfrac{(x^2+3)^2}{x^2}$, $x \neq 0$.

(a) Show that $\dfrac{dy}{dx} = x^2 + 6 + 9x^{-2}$.

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = f(x)$.

(6)

12. The curve C with equation $y = f(x)$, $x \neq 0$, passes through the point $(3, 7\tfrac{1}{2})$.

Given that $f'(x) = 2x + \dfrac{3}{x^2}$,

(a) find $f(x)$.

(5)

(b) Verify that $f(-2) = 5$.

(1)

(c) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

13. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0.$$

The point P has coordinates (2, 7).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

The point Q also lies on C.

Given that the tangent to C at Q is perpendicular to the tangent to C at P,

(c) show that the x-coordinate of Q is $\dfrac{1}{3}(2 + \sqrt{6})$.

(5)

14. The curve C has equation $y = \frac{1}{3}x^3 - 4x^2 + 8x + 3$.

The point P has coordinates (3, 0).

(a) Show that P lies on C.

(1)

(b) Find the equation of the tangent to C at P, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

Another point Q also lies on C. The tangent to C at Q is parallel to the tangent to C at P.

(c) Find the coordinates of Q.

(5)

Recurrence Formula

1. The sequence of positive numbers $u_1, u_2, u_3, \ldots$, is given by

$$u_{n+1} = (u_n - 3)^2, \qquad u_1 = 1.$$

(*a*) Find u_2, u_3 and u_4.

(3)

(*b*) Write down the value of u_{20}.

(1)

2. A sequence is given by

$$x_1 = 1,$$

$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(*a*) Find x_2 in terms of p.

(1)

(*b*) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(*c*) find the value of p,

(3)

(*d*) write down the value of x_{2008}.

(2)

3. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = 2,$$

$$a_{n+1} = 3a_n - c$$

where c is a constant.

(*a*) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$,

(*b*) find the value of c.

(4)

4. A sequence x_1, x_2, x_3, … is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geq 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a.

(3)

5. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \quad n \geq 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k.

(4)

6. A sequence a_1, a_2, a_3, . . . is defined by

$$a_1 = 3,$$

$$a_{n+1} = 3a_n - 5, \quad n \geq 1.$$

(*a*) Find the value a_2 and the value of a_3.

(2)

(*b*) Calculate the value of $\sum_{r=1}^{5} a_r$.

(3)

7. A sequence $a_1,\ a_2,\ a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5,\quad n \geq 1,$$

where *k* is a positive integer.

(*a*) Write down an expression for a_2 in terms of *k*.

(1)

(*b*) Show that $a_3 = 9k + 20$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of *k*.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

(4)

8. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)},\qquad n \geq 1,$$

$$a_1 = 2.$$

(*a*) Find a_2 and a_3, leaving your answers in surd form.

(2)

(*b*) Show that $a_5 = 4$.

(2)

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9. A sequence a_1, a_2, a_3, …, is defined by

$$a_1 = k,$$

$$a_{n+1} = 5a_n + 3, \quad n \geq 1,$$

where k is a positive integer.

(*a*) Write down an expression for a_2 in terms of k.

(1)

(*b*) Show that $a_3 = 25k + 18$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

(4)

Curve Sketching and functions

Mr A. Slack (2012)

1. (*a*) On the same axes sketch the graphs of the curves with equations

(i) $y = x^2(x - 2)$,

(3)

(ii) $y = x(6 - x)$,

(3)

and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

(*b*) Use algebra to find the coordinates of the points where the graphs intersect.

(7)

2. (*a*) On the axes below sketch the graphs of

(i) $y = x\,(4 - x)$,

(ii) $y = x^2\,(7 - x)$,

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(*b*) Show that the x-coordinates of the points of intersection of

$$y = x\,(4 - x) \quad \text{and} \quad y = x^2\,(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$.

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(*c*) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r and s are integers.

(7)

3. The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

(*a*) Sketch C, showing clearly the coordinates of the points where the curve meets the coordinate axes.

(4)

(*b*) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k.

(2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x-coordinates of these two points.

(6)

4. The point $P(1, a)$ lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a.

(1)

(b) Sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$,

(ii) $y = \dfrac{2}{x}$.

On your diagram show clearly the coordinates of any points at which the curves meet the axes.

(5)

(c) With reference to your diagram in part (b), state the number of real solutions to the equation

$$(x + 1)^2(2 - x) = \frac{2}{x}.$$

(1)

5. (a) Factorise completely $x^3 - 4x$.

(3)

(b) Sketch the curve C with equation

$$y = x^3 - 4x,$$

showing the coordinates of the points at which the curve meets the axis.

(3)

The point A with x-coordinate -1 and the point B with x-coordinate 3 lie on the curve C.

(*c*) Find an equation of the line which passes through A and B, giving your answer in the form $y = mx + c$, where m and c are constants.

(5)

(*d*) Show that the length of AB is $k\sqrt{10}$, where k is a constant to be found.

(2)

6. (*a*) Sketch the graphs of

(i) $y = x(x + 2)(3 - x)$,

(ii) $y = -\frac{2}{x}$.

showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(*b*) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x + 2)(3 - x) + \frac{2}{x} = 0.$$

(2)

7. The curve C with equation $y = \mathrm{f}(x)$ passes through the point (5, 65).

Given that $\mathrm{f}'(x) = 6x^2 - 10x - 12$,

(*a*) use integration to find $\mathrm{f}(x)$.

(4)

(*b*) Hence show that $\mathrm{f}(x) = x(2x + 3)(x - 4)$.

(2)

(*c*) Sketch C, showing the coordinates of the points where C crosses the x-axis.

(3)

8. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

(*a*) Sketch the graphs of C and l, indicating clearly the coordinates of any intersections with the axes.

(3)

(*b*) Find the coordinates of the points of intersection of C and l.

(6)

9. (*a*) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(*b*) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (*b*), or otherwise,

(*c*) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis.

(2)

10. The curve C has equation

$$y = (x + 1)(x + 3)^2.$$

(*a*) Sketch C, showing the coordinates of the points at which C meets the axes.

(4)

(*b*) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 14x + 15$.

(3)

The point A, with x-coordinate -5, lies on C.

(*c*) Find the equation of the tangent to C at A, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Another point B also lies on C. The tangents to C at A and B are parallel.

(*d*) Find the x-coordinate of B.

(3)

11. Factorise completely $x^3 - 9x$.

(3)

12. Factorise completely

$$x^3 - 4x^2 + 3x.$$

(3)

13. Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

(*a*) express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants.

(3)

(*b*) Hence factorise $f(x)$ completely.

(2)

(*c*) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes.

(3)

14.

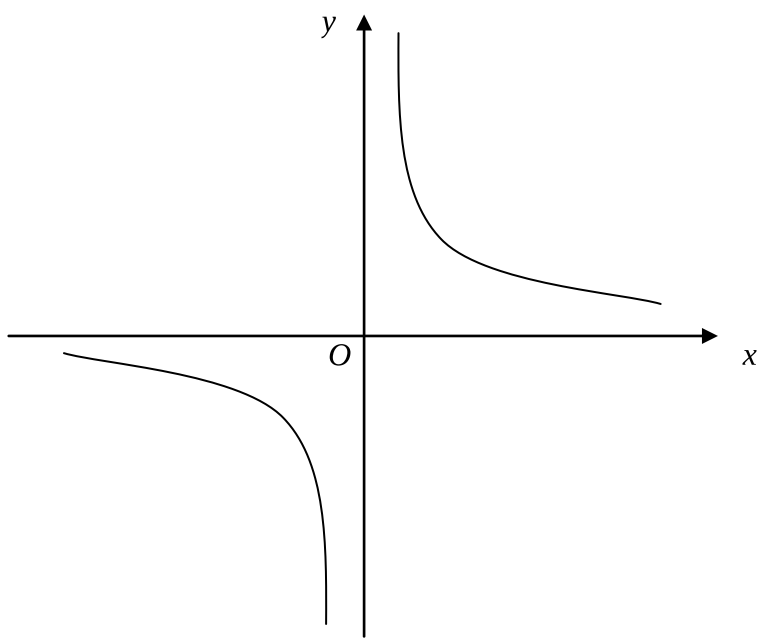

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$.

(*a*) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \neq -2$, showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(*b*) Write down the equations of the asymptotes of the curve in part (*a*).

(2)

15. Find the set of values of x for which

(*a*) $3(2x + 1) > 5 - 2x$,

(2)

(*b*) $2x^2 - 7x + 3 > 0$,

(4)

(*c*) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)

16. Find the set of values of x for which

$$x^2 - 7x - 18 > 0.$$

(4)

17. Find the set of values of x for which

(*a*) $4x - 3 > 7 - x$

(2)

(*b*) $2x^2 - 5x - 12 < 0$

(4)

(*c*) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

18. Find the set of values of x for which

(*a*) $3(x - 2) < 8 - 2x$,

(2)

(*b*) $(2x - 7)(1 + x) < 0$,

(3)

(*c*) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$.

(1)