AQA

Level 2 Certificate in Further Mathematics
June 2015

Further Mathematics   8360/2

Level 2

Paper 2   Calculator

Friday 19 June 2015   9.00 am to 11.00 am

For this paper you must have:
• a calculator
• mathematical instruments.

Time allowed
• 2 hours

Instructions
• Use black ink or black ball-point pen. Draw diagrams in pencil.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
• Do all rough work in this book. Cross through any work that you do not want to be marked.
• In all calculations, show clearly how you work out your answer.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 105.
• You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
• The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
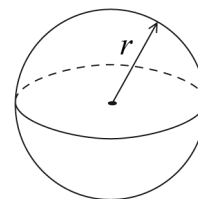
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
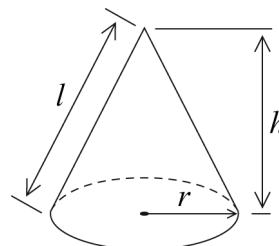
In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
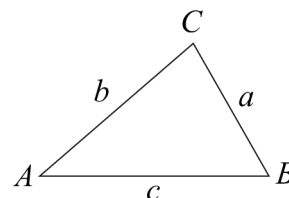
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$$

Answer **all** questions in the spaces provided.

1 A circle, centre $(0, 0)$, has circumference 12π

Work out the equation of the circle.

[2 marks]

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Answer ....

2 $a : b : c = 5 : 3 : 2$

Work out $\quad 4a - c : 3b$

Give your answer in its simplest form.

[2 marks]

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Answer .... : ....

3 The distance between the points $(2, 5p)$ and $(2, -10)$ is 30 units.

Work out the **two** possible values of p.

[3 marks]

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Answer....................................and ....

4   The first term of a sequence is $1 - a$

The term-to-term rule of a sequence is

         add $2a$ then   multiply by 3

4 (a)   Show that the second term is $3 + 3a$

[1 mark]

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4 (b)   The third term is 16

Work out the value of a.

[3 marks]

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Answer ....

5 A straight line L

is parallel to the straight line $y = 1 - 2x$
passes through $(3, -1)$

On the grid below, draw the straight line L for values of x from -3 to 3.

[4 marks]

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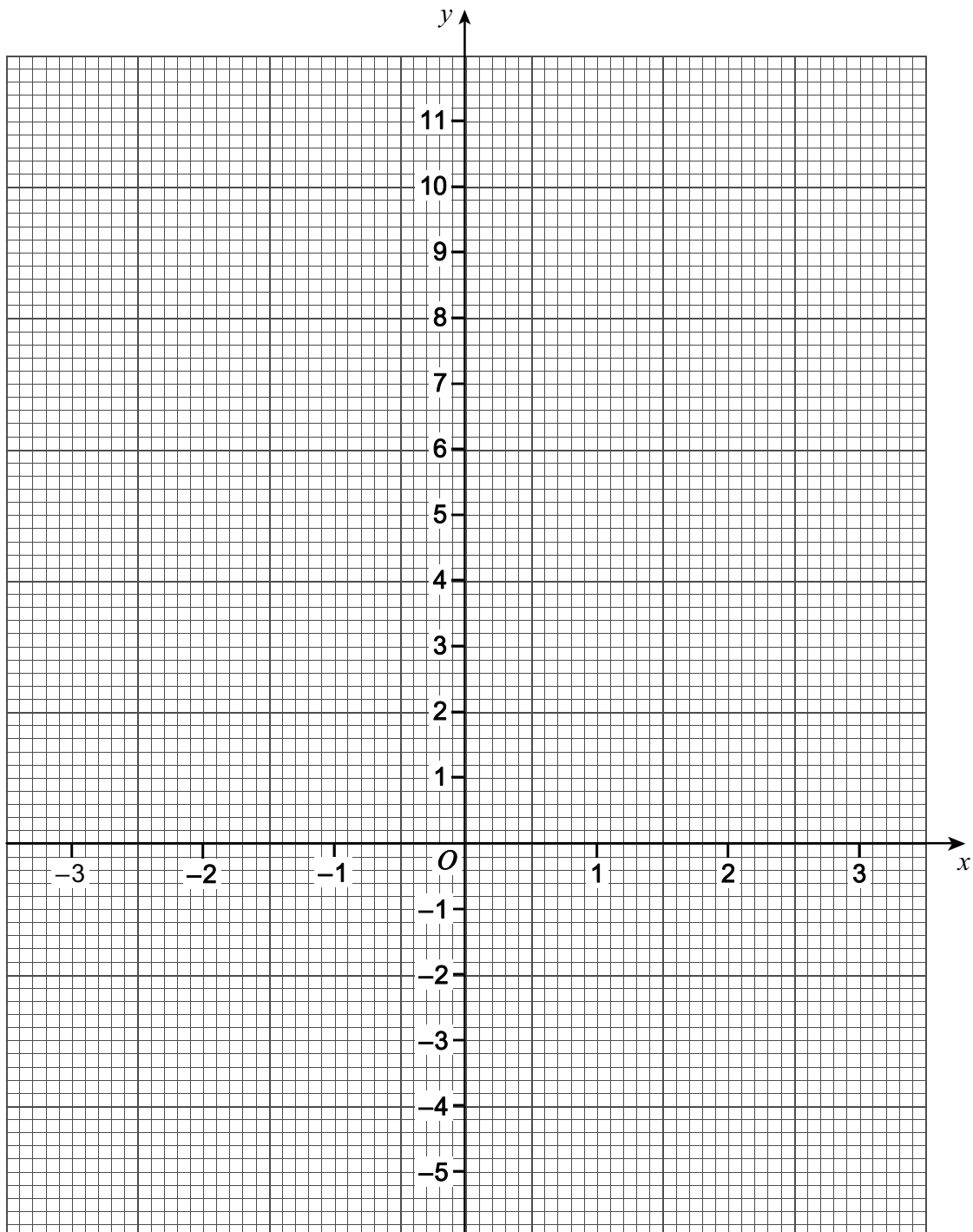
6 Write $\dfrac{15x^8 - 18x^7}{3x^2}$ in the form $ax^n - nx^a$ where a and n are integers.

[2 marks]

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7 $y = \dfrac{2}{3}x^6 - 8x^3$

Work out the rate of change of y with respect to x when $x = -1$

[3 marks]

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Answer ....

8 (a) $f(x) = x^4$
The domain of $f(x)$ is $x \geqslant 2$

Work out the range of $f(x)$.

[1 mark]

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Answer ....

8 (b) $g(x) = x^2 - 1$
The domain of $g(x)$ is $-2 \leqslant x \leqslant 3$

Work out the range of $g(x)$.

[2 marks]

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Answer ....

8 (c) $h(x) = 5x - 3$
The **range** of $h(x)$ is $-2 < h(x) < 1$

Work out the domain of $h(x)$.

[2 marks]

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Answer ....

9 (a)   Solve $6(2y - 3) - 10 = 2y$

[3 marks]

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$y = $ ....

9 (b)   Solve $\dfrac{\sqrt{w + 4}}{2} = 6$

[3 marks]

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$w = $ ....

9 (c)   Solve $3m^{\frac{1}{5}} + 9 = 0$

[2 marks]

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$m = $ ....

13

Turn over ▶

10 The diagram shows a circle, centre *C*.
TP is a tangent to the circle at *P*.

Not drawn
accurately

P(2, 8)

C(3, 6)

T(−4, *t*)

O

y

x
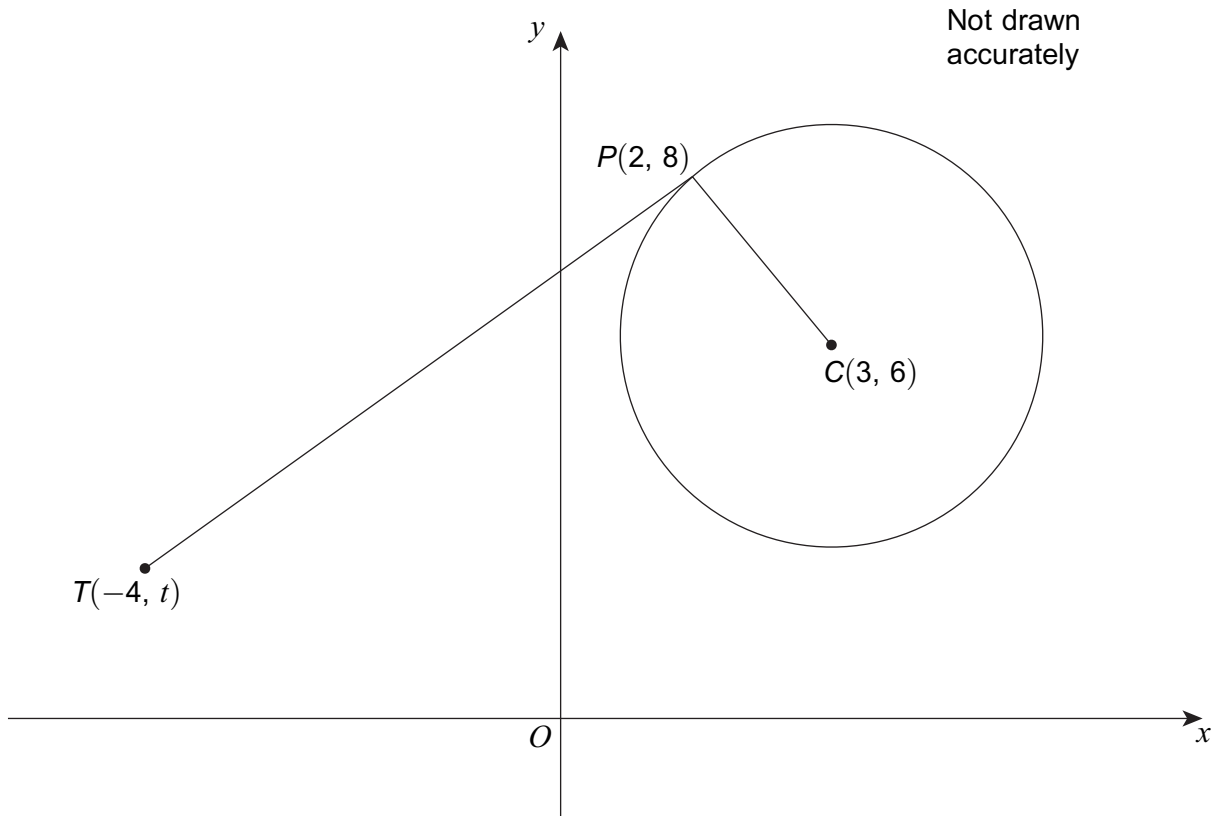
Work out the value of *t*.

[4 marks]

Answer ....

11 (a) Expand and simplify $(3w + 2y)(w - 4y)$

[3 marks]

Answer ....

11 (b) Expand and simplify $\dfrac{3}{x^2}\left(\dfrac{x}{3} + 3x^2 - 1\right)$

[3 marks]

Answer ....

10

Turn over ▶

12   The area of the triangle is equal to the area of the square.
All dimensions are in centimetres.

Not drawn accurately
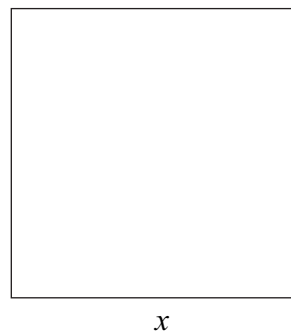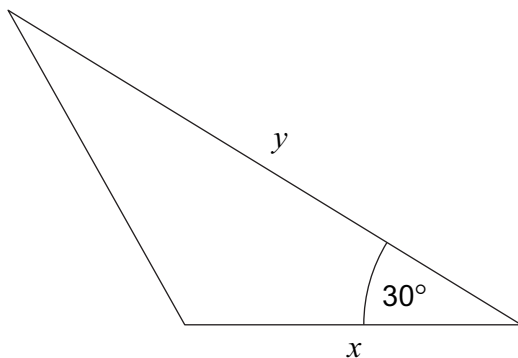
Write y in terms of x.

[2 marks]

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Answer ....

13 The diagram shows a circle, centre *P*, and a straight line passing through points *P* and *Q*.
Q lies on the *y*-axis and on the circumference of the circle.

The equation of the circle is $(x-3)^2 + y^2 = 25$

Not drawn accurately
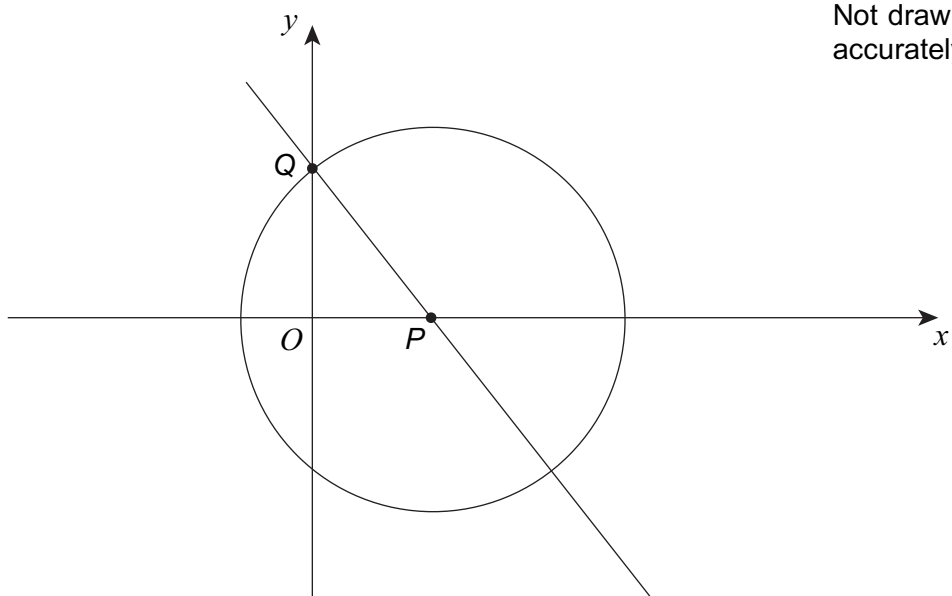
Work out the equation of the straight line through *P* and *Q*.
Give your answer in the form $ax + by + c = 0$ where *a*, *b* and *c* are integers.

[4 marks]

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Answer ....

14   *PQR* is a straight line.
          PQ : *QR* is 2 : 3

Not drawn
accurately

R(*a*, 7)

Q(6, 9*b*)

P(3, *b*)

y

O

x

14 (a)   Show that $a = 10.5$

[2 marks]

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14 (b) Work out the value of b.

[3 marks]

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Answer ....

15 Use algebra to prove that the value of $\dfrac{8c^2+16}{3c^2+6}+\dfrac{1}{3}$ is an integer for all values of c.

[3 marks]

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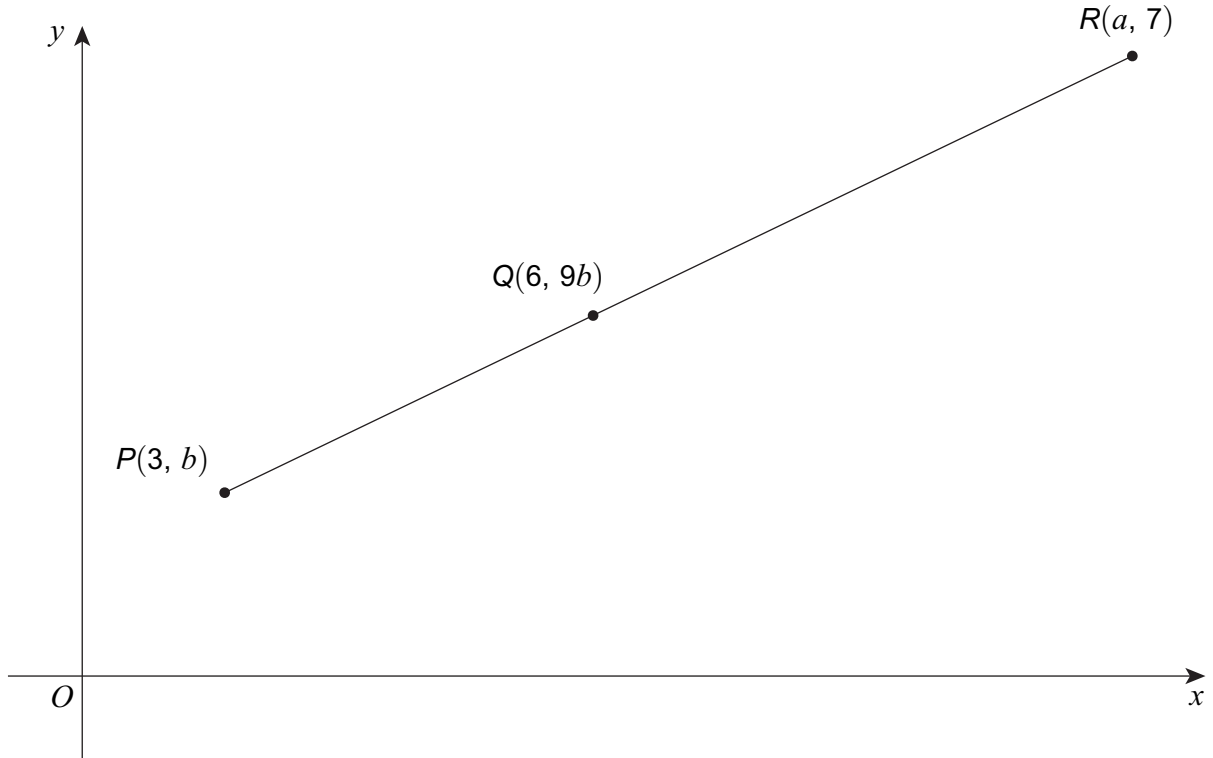...

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Turn over ▶

16 The diagram shows a rectangle with area 9 cm^2

x cm

$(2x - 1)$ cm
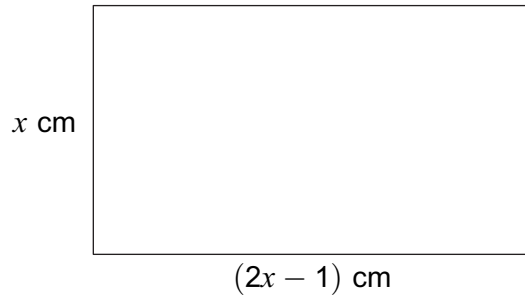
Not drawn accurately

Set up and solve an equation to work out the value of x.
Give your answer to 3 significant figures.

[5 marks]

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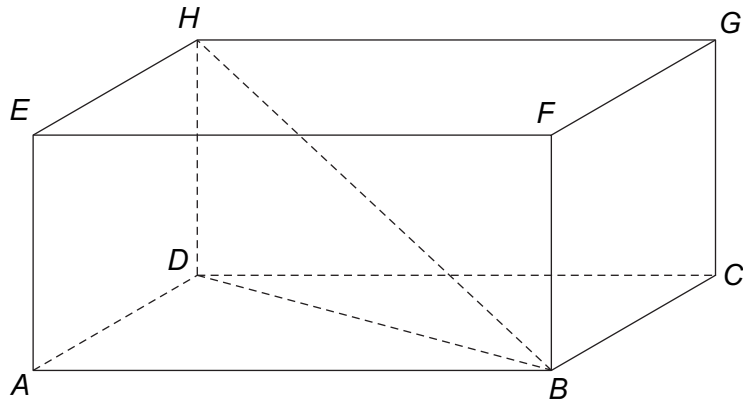$x =$ ....

17   *ABCDEFGH* is a cuboid.

$HB = 34$ cm
$HD = 16$ cm
$AD = 18$ cm

17 (a)   Work out the length of *AB*.

[3 marks]

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Answer.. cm

17 (b)   Work out the angle between *HB* and *ABCD*.

[2 marks]

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Answer... degrees

Turn over ▶

10

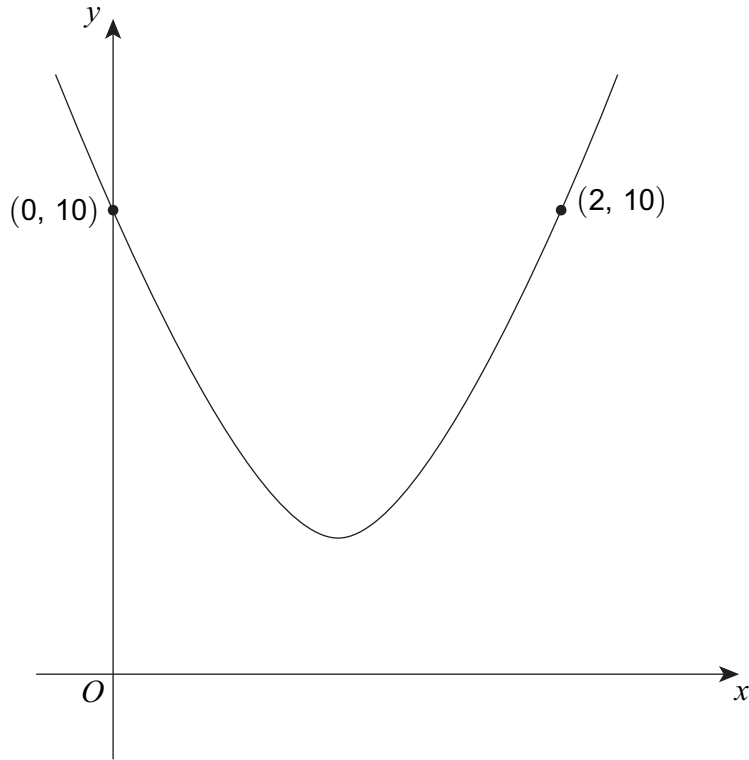**18** The sketch shows the quadratic curve $y = 4(x - a)^2 + b$
The curve passes through $(0, 10)$ and $(2, 10)$

Not drawn
accurately

18 (a) Give reasons why the value of a is 1.

[2 marks]

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18 (b)   Work out the value of b.

[2 marks]

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Answer ....

18 (c)   Write the equation of the curve in the form $y = px^2 + qx + r$

[2 marks]

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Answer ....

19   Use the factor theorem to show that $(x - 3)$ is **not** a factor of $x^3 - 10x - 3$

[2 marks]

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Turn over ▶

20 (a) The transformation matrix **P** represents a 90° anti-clockwise rotation about the origin.

Describe fully the **single** transformation represented by the matrix P^3

[2 marks]

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20 (b) The transformation matrix **Q** is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The transformation matrix **R** is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Describe fully the **single** transformation represented by the matrix **QR**.

[2 marks]

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21 A cubic curve has
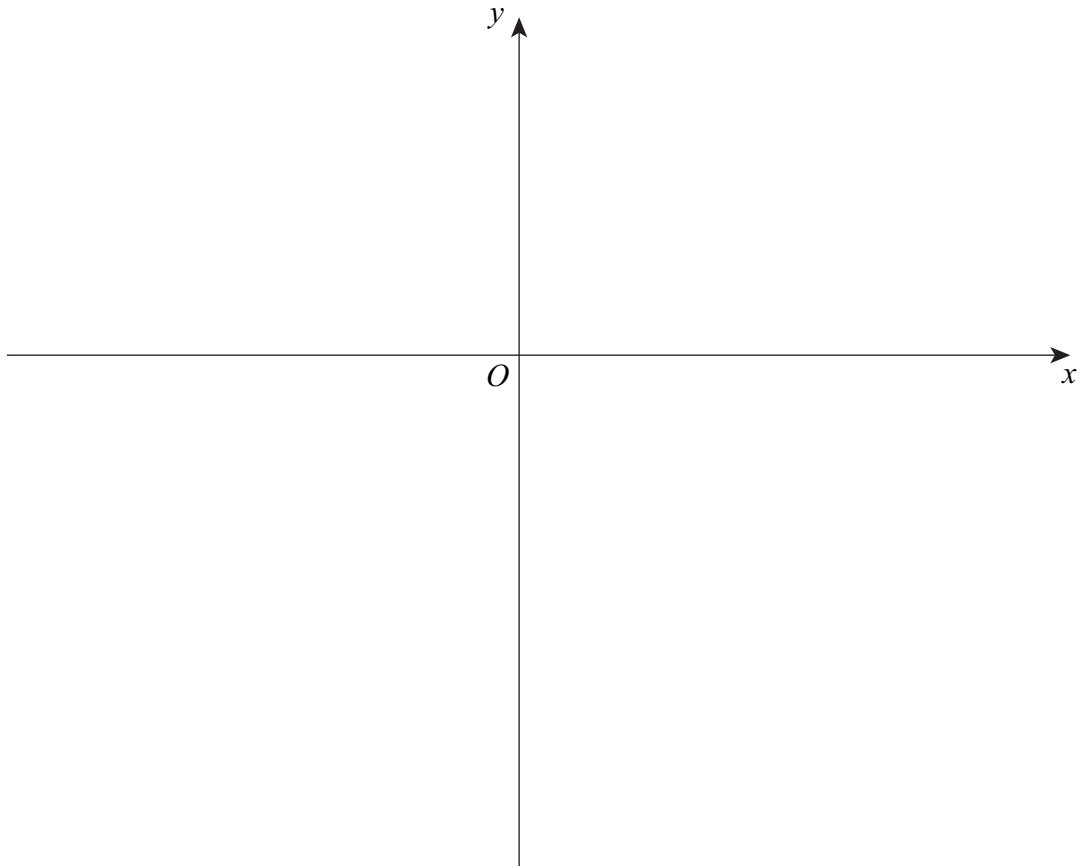
a maximum point at A $(-4, 10)$

a minimum point at B $(2, -26)$

The tangent to the curve at A and the normal to the curve at B intersect at point C.

Work out the area of triangle ABC.
You may sketch a diagram to help you.

[3 marks]

Answer..square units

Turn over ▶

22 A quadratic sequence starts

302   600   894   1184   ....

22 (a) Work out an expression for the nth term.

[3 marks]

Answer ....

22 (b) A term in the sequence has value 0

Find the position of this term.

[2 marks]

Answer ....

23 The continuous curve $y = f(x)$ has exactly **two** stationary points.

P is a maximum point when $x = a$

Q is a stationary point of inflection when $x = b$

$a < b$

Which of these is correct?

Tick **one** box only.

[1 mark]

When $a < x < b$, $\dfrac{dy}{dx}$ is positive

and

when $x > b$, $\dfrac{dy}{dx}$ is positive

☐

When $a < x < b$, $\dfrac{dy}{dx}$ is positive

and

when $x > b$, $\dfrac{dy}{dx}$ is negative

☐

When $a < x < b$, $\dfrac{dy}{dx}$ is negative

and

when $x > b$, $\dfrac{dy}{dx}$ is positive

☐

When $a < x < b$, $\dfrac{dy}{dx}$ is negative

and

when $x > b$, $\dfrac{dy}{dx}$ is negative

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6

Turn over ▶

24 $a^2 < 4$ and $a + 2b = 8$

Work out the range of possible values of b.
Give your answer as an inequality.

[4 marks]

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Answer ....

25   Work out the values of x between 0° and 360° for which

$$25\cos^2 x = 9$$

Give your answers to 1 decimal place.

[4 marks]

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Answer ....
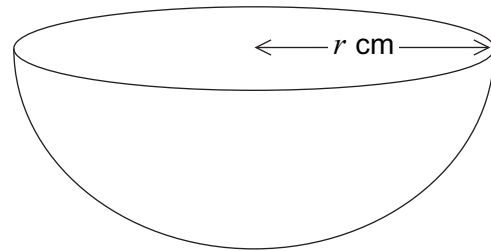
8

26 A cone has base radius r cm and slant height l cm

A hemisphere has radius r cm
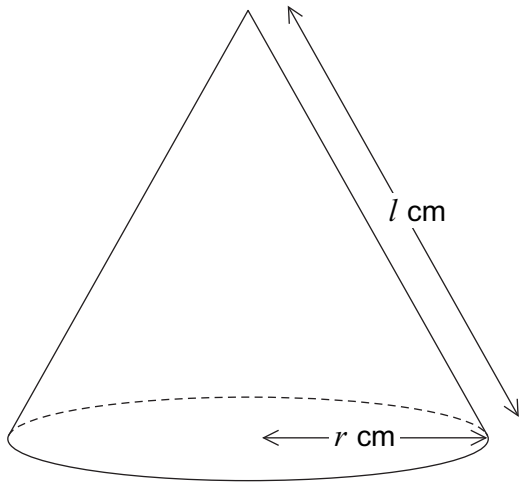
26 (a) The curved surface area of the cone equals the curved surface area of the hemisphere.

Show that $l = 2r$

[1 mark]

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26 (b)   The cone has vertical height h cm

The cone and hemisphere are joined to make the shape shown below.
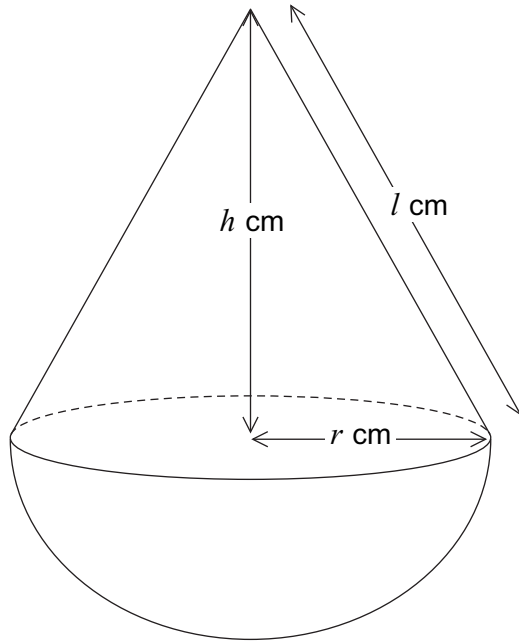

Show that the volume of the shape can be written as

$$\frac{1}{3}\pi r^3 (a + \sqrt{b}) \text{ cm}^3 \qquad \text{where } a \text{ and } b \text{ are integers.}$$

[4 marks]

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Turn over ▶

27 Work out the values of a when

$$2^{a^2} = 8^a \times 16$$

Do **not** use trial and improvement.
You **must** show your working.

[4 marks]

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Answer ....

END OF QUESTIONS

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