Centre Number						Candidate Number				
Surname	MR BARTON'S									
Other Names	SOLUTIONS									
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2015

Further Mathematics 8360/1

Level 2

Paper 1 Non-Calculator

Monday 15 June 2015 9.00 am to 10.30 am

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

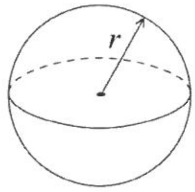
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

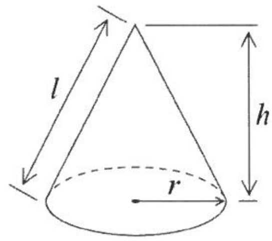
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

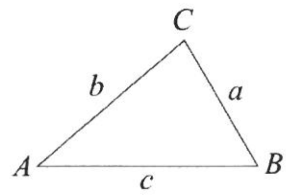


Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC



Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

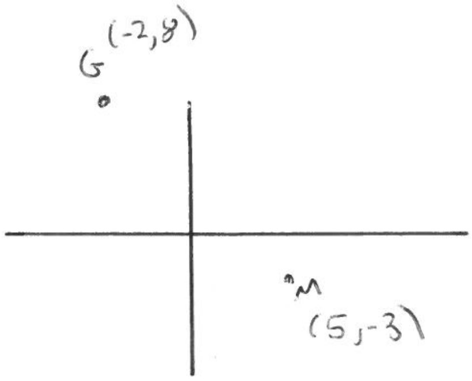
Answer **all** questions in the spaces provided.

1 *GH* is a straight line.

The coordinates of *G* are $(-2, 8)$
The midpoint of *GH* is $(5, -3)$

Work out the coordinates of *H*.

[2 marks]



$\boxed{x}$ $\quad \dfrac{-2 + x}{2} = 5 \rightarrow x = 12$

$\boxed{y}$ $\quad \dfrac{8 + y}{2} = -3 \rightarrow y = -14$

Answer (12 , -14)

Turn over for the next question

2

2 A straight line with equation $y = mx + c$ has gradient m and y-intercept c.

Here are the equations of four straight lines, P, Q, R and S.

P $2y - 4x = 5$ Q $5y = 2x - 4$

R $2y - 4 = 5x$ S $4y = 5 - 2x$

2 (a) Circle the line that passes through (7, 2) **[1 mark]**

P Q R S

$5(2) = 2(7) - 4$ ✓

2 (b) Circle the line with gradient $2\frac{1}{2}$ **[1 mark]**

P Q R S

$\rightarrow 2y = 5x + 4$

$\rightarrow y = 2.5x + 2$

2 (c) Circle the line with y-intercept $2\frac{1}{2}$ **[1 mark]**

P Q R S

$\rightarrow 2y = 4x + 5$

$\rightarrow y = 2x + 2.5$

2 (d) Circle the line with a negative gradient. **[1 mark]**

P Q R S

$\rightarrow y = \frac{5}{4} - 0.5x$

$\rightarrow$ grad $= -0.5$

2 (e) Circle a pair of perpendicular lines. **[1 mark]**

P Q R S

$\rightarrow y = 2x + 2.5$

$\rightarrow$ grad $= 2$

grad $= -0.5$

Do not write outside the box

3 Solve $2(3x + 1) > 3 - 4x$

[2 marks]

$6x + 2 > 3 - 4x$

$+4x$ $10x + 2 > 3$

-2 $10x > 1$

$\div 10$ $x > \frac{1}{10}$ or 0.1

Answer ....

Turn over for the next question

7

Do not write outside the box

4 The equation of a curve is $y = x^2 - 5x$

4 (a) Work out $\frac{dy}{dx}$

[2 marks]

Answer $\frac{dy}{dx} = 2x - 5$

4 (b) P is a point on the curve.
The tangent to the curve at P has gradient 1

Work out the coordinates of P.

[2 marks]

Gradient $= 1 \rightarrow \frac{dy}{dx} = 1$

$\rightarrow 2x - 5 = 1$

$+5$ $2x = 6$

$\div 2$ $x = 3$

$y = x^2 - 5x \rightarrow (3)^2 - 5(3) = 9 - 15 = -6$

Answer (3 , -6)

Do not write outside the box

5 In the expansion of $(x+2)(x^2+kx-3)$ the coefficient of x^2 is zero.

5 (a) Work out the value of k.

Group Terms Together

[1 mark]

$x^3 + kx^2 - 3x + 2x^2 + 2kx - 6$

$\Rightarrow x^3 + (k+2)x^2 + (2k-3)x - 6$

$\uparrow$

$k = -2$

Answer $k = -2$

5 (b) Work out the coefficient of x.

[2 marks]

From above $\Rightarrow (2k-3)x$

$\Rightarrow [2(-2)-3]x$

$\Rightarrow -7x$

Answer -7

Turn over for the next question

7

Do not write outside the box

6 A bag contains $5x$ red balls and $2x$ blue balls.

The number of red balls is **decreased** by 20%

The number of blue balls is **increased** by 30%

There are now 35 **more** red balls than blue balls in the bag.

Work out the value of x.

[4 marks]

$\boxed{\text{RED}}$ $5x \times 0.8 = 4x$

$\boxed{\text{BLUE}}$ $2x \times 1.3 = 2.6x$

New Amounts

$4x - 2.6x = 35$

$\rightarrow 1.4x = 35$

$x = \frac{35}{1.4} = \frac{350}{14} = \frac{50}{2}$

$\rightarrow x = 25$

Answer 25

7 $3x^3 - 2x^2 - 147x + 98 \equiv (ax - c)(bx + d)(bx - d)$

where a, b, c and d are positive integers.

Work out the values of a, b, c and d.

[3 marks]

$\boxed{x^3}$ $\boxed{3}x^3 = \boxed{ab^2}x^3$

Only positive integers that work are $a = 3$, $b = 1$

$\boxed{\text{NUMBER}}$ $98 = -c \times d \times -d$

$\Rightarrow 98 = cd^2$

Only positive integers that work are $c = 2$, $d = 7$

as $2 \times 7^2 = 98$

$a =$ 3 $b =$ 1 $c =$ 2 $d =$ 7

Turn over for the next question

7

Do not write outside the box

8 Simplify fully $\dfrac{5x}{(x+4)(x-6)} - \dfrac{3}{(x-6)}$

[4 marks]

Need common denominator:

$$\frac{5x}{(x+4)(x-6)} - \frac{3(x+4)}{(x+4)(x-6)}$$

$$= \frac{5x - 3x - 12}{(x+4)(x-6)}$$

$$= \frac{2x - 12}{(x+4)(x-6)}$$

$$= \frac{2(x-6)}{(x+4)(x-6)}$$

$$= \frac{2}{(x+4)}$$

Answer ....

Do not write outside the box

9 Given that $\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ a+1 \end{pmatrix}$

work out the values of a and b.

[5 marks]

$$\begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \; ① \\ a+1 \; ② \end{bmatrix}$$

① $\rightarrow 3a - b = b$

$\rightarrow 3a = 2b$

$\rightarrow b = 1.5a$

② $2a + b = a + 1$

Sub in ①

$\rightarrow 2a + 1.5a = a + 1$

$\rightarrow 3.5a = a + 1$

$\rightarrow 2.5a = 1$

$\rightarrow 5a = 2$

$\rightarrow a = \frac{2}{5}$

use ①

$b = 1.5a$

$\rightarrow b = 1.5 \times \frac{2}{5} = \frac{3}{2} \times \frac{2}{5} = \frac{6}{10}$ or $\frac{3}{5}$

$a = \frac{2}{5}$, $b = \frac{3}{5}$

9

Turn over ▶

Do not write outside the box

10 (b) Work out the equation of the normal to the curve at the point $(1, -2)$
Give your answer in the form $y = mx + c$

[5 marks]

when $x = 1$, $\frac{dy}{dx} = 3(1)^2 - 4(1) - 4 = -5$

Grad of tangent $= -5$, so grad of normal $= \frac{1}{5}$

$m = \frac{1}{5}$ $\quad y - y_1 = m(x - x_1)$

$x_1 = 1$ $\quad y - (-2) = \frac{1}{5}(x - 1)$

$y_1 = -2$ $\quad y + 2 = \frac{1}{5}x - \frac{1}{5}$

$y = \frac{1}{5}x - 2\frac{1}{5}$

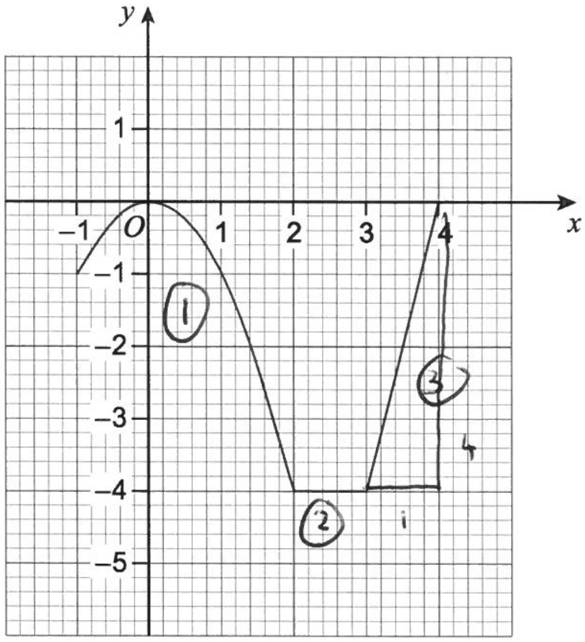
Answer ....

Turn over for the next question

9

11 Here is the graph of $y = f(x)$

It consists of a quadratic curve and two straight lines.



③ grad $= \frac{4}{1} = 4$

$m = 4$ $\quad y - y_1 = m(x - x_1)$

$x_1 = 4$ $\quad y - 0 = 4(x - 4)$

$y_1 = 0$ $\quad \boxed{y = 4x - 16}$

Define $f(x)$, stating clearly the domain for each part.

[4 marks]

① $y = -x^2$

② $y = -4$

③ $y = 4x - 16$

$\Rightarrow$ $f(x) = -x^2 \quad -1 \le x \le 2$

$-4 \quad 2 \le x \le 3$

$4x - 16 \quad 3 \le x \le 4$

$f(x) =$ ....

$=$ ....

$=$ ....

Do not write outside the box

12 Make y the subject of $\sqrt{\dfrac{3xy}{x+y}} = 4$

[4 marks]

square ^{2} $\dfrac{3xy}{x+y} = 16$

$\times(x+y)$ $3xy = 16(x+y)$

$3xy = 16x + 16y$

$-16y$ $3xy - 16y = 16x$

FACT $y(3x - 16) = 16x$

$\div(3x-16)$ $y = \dfrac{16x}{3x-16}$

Answer ....

Turn over for the next question

8

Do not write outside the box

13 $x^2 + 2ax + b \equiv (x-5)^2 - a$

Work out the values of a and b.

[3 marks]

$(x-5)^2 - a$

$= x^2 - 10x + 25 - a$

$= x^2 + 2ax + b$

~~so~~ $\boxed{x}$ $-10 = 2a \rightarrow a = -5$

$\boxed{\text{NUMBER}}$ $25 - a = b$

$25 - (-5) = b$

$\rightarrow b = 30$

$a =$ -5, $b =$ 30

Do not write outside the box

14 Write $\frac{5\sqrt{2}}{3\sqrt{6}-7}$ in the form $\sqrt{w}+\sqrt{k}$ where w and k are integers.

[5 marks]

RATIONALISE DENOMINATOR

$$\frac{5\sqrt{2}}{3\sqrt{6}-7} \times \frac{3\sqrt{6}+7}{3\sqrt{6}+7}$$

$$= \frac{15\sqrt{12} + 35\sqrt{2}}{54 + 21\sqrt{6} - 21\sqrt{6} - 49}$$

$$= \frac{15\sqrt{12} + 35\sqrt{2}}{5}$$

$$= 3\sqrt{12} + 7\sqrt{2}$$

$\cancel{= 3\sqrt{4} \times \sqrt{3} + 7\sqrt{2}}$

$\cancel{= 12\sqrt{3}}$

$$= \sqrt{9} \times \sqrt{12} + \sqrt{49} \times \sqrt{2}$$

$$= \sqrt{108} + \sqrt{98}$$

Answer $\sqrt{108} + \sqrt{98}$

8

Turn over ▶

15 (a) Give reasons why angle $RZT = b$

[2 marks]

Tangents from an external point are equal in length

→ RTZ is an Isosceles Triangle

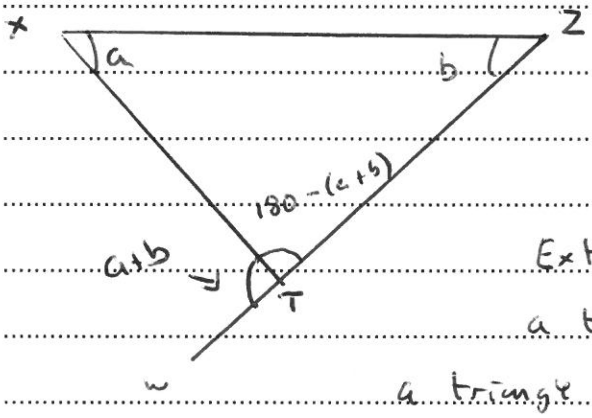
So base angles are equal

15 (b) Angle $RZT = b$

Prove that angle XTW = angle YTZ

[3 marks]

YXT = a (angles in alternate segment are equal)



XTW = a + b

Exterior angles in a triangle, or angles in a triangle + straight line = 180°

YTZ = a + b

XTW = a + b

Proved!

5

Turn over ▶

Do not write outside the box

16 By factorising fully, simplify $\dfrac{x^4 - x^3 - 2x^2}{x^4 - 5x^2 + 4}$ ← Think of $y^2 - 5y + 4$ where $y = x^2$ **[5 marks]**

$$\frac{x^2\left[x^2 - x - 2\right]}{(x^2-1)(x^2-4)}$$

$$= \frac{x^2\left[(x-2)(x+1)\right]}{(x+1)(x-1)(x+2)(x-2)}$$ ← Diff of 2 squares

$$= \frac{x^2}{(x-1)(x+2)}$$

Answer ....

Do not write outside the box

17 Prove that $2\tan^2\theta + 1 \equiv \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$ where $\sin^2\theta \neq 1$

[3 marks]

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \rightarrow \tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

$\boxed{\text{LHS}}$ $\dfrac{2\sin^2\theta}{\cos^2\theta} + 1$

$\dfrac{2\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta}$

$= \dfrac{2\sin^2\theta + \cos^2\theta}{\cos^2\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\rightarrow \cos^2\theta = 1 - \sin^2\theta$

$= \dfrac{2\sin^2\theta + 1 - \sin^2\theta}{1 - \sin^2\theta}$

$= \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

END OF QUESTIONS

8