AQA Qualifications

AQA CERTIFICATE

FURTHER MATHEMATICS

Paper 2 83602

Mark scheme

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Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.

Q	Answer	Mark	Comments
1	x^2 4 5	B3	All in appropriate boxes B1 for each correct box In the left hand boxes ignore inclusion of y = and/or f(x) =
2	**Alternative method 1**		
	A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
	$\frac{1}{2}$ × their 6 × y = 24	M1	
	y = 8	A1ft	Only ft B0 M1
	their 8 = 12 – 2x	M1	
	x = 2	A1ft	ft their y SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or y = 12 (for B)
	Alternative method 2		
	A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
	B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 × 12 or 36 and $\frac{1}{2}$ × 12 × x = their 36 – 24	M1	
	x = 2	A1ft	Only ft B0 M1
	y = 12 – 2 × their 2	M1	
	y = 8	A1ft	ft their y SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or y = 12 (for B)

2	**Alternative method 3**		
	A (6, 0) or $x = 6$ (for A)	B1	May be on diagram or be implied
	$\frac{1}{2} \times$ their $6 \times y = 24$	M1	
	$y = 8$	A1ft	Only ft B0 M1
	B (0, 12) or $y = 12$ (for B) and (area OAB =) $\frac{1}{2} \times$ their 6×12 or 36 and $\frac{1}{2} \times 12 \times x$ = their 36 – 24	M1	
	$x = 2$	A1ft	Only ft B0 with 2nd M1 gained SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or $y = 12$ (for B)
	Alternative method 4		
	A (6, 0) or $x = 6$ (for A)	B1	May be on diagram or be implied
	B (0, 12) or $y = 12$ (for B) and (area OAB =) $\frac{1}{2} \times$ their 6×12 or 36 and $\frac{1}{2} \times 12 \times x$ = their 36 – 24	M1	
	$x = 2$	A1ft	Only ft B0 M1
	$\frac{1}{2} \times$ their $6 \times y = 24$	M1	
	$y = 8$	A1ft	Only ft B0 with 2nd M1 gained SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or $y = 12$ (for B)

2	**Alternative method 5**		
	A (6, 0) or x = 6 (for A)	B1	May be on diagram or be implied
	B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 × 12 or 36 and $\frac{24}{\text{their } 36} \times 12$	M1	
	y = 8	A1ft	Only ft B0 M1
	B (0, 12) or y = 12 (for B) and (area OAB =) $\frac{1}{2}$ × their 6 × 12 or 36 and $\frac{\text{their } 36 - 24}{\text{their } 36} \times \text{their } 6$	M1	
	x = 2	A1ft	Only ft B0 with 2nd M1 gained SC2 Answer (8, 2) with no valid working SC1 B (0, 12) or y = 12 (for B)

3(a)	Valid reason e.g.1 Triangle OTS is isosceles e.g.2 OT = OS e.g.3 OT and OS are radii	B1	

3(b)	Correct equation e.g.1 $5x = 2(x + 30)$ e.g.2 $2.5x = x + 30$ e.g.3 $(180 – 2x) + 120 + 5x = 360$ e.g.4 $x + 30 + x + 30 + 360 – 5x = 360$	M1	oe Brackets not needed in e.g.3
	Collects terms for their initial equation e.g.1 $5x – 2x = 60$ e.g.2 $2.5x – x = 30$ e.g.3 $– 2x + 5x = 360 – 180 – 120$	M1	oe their initial equation must have ≥ 2 terms in x Any brackets must be expanded correctly
	20	A1	

4(a)	$x^3 – 2x^2$	B2	B1 for x^3 B1 for $–2x^2$

4(b)	$3x^2$ or $–4x$	M1	At least one term of their $x^3 – 2x^2$ differentiated correctly
	$3(3)^2 – 4(3)$ or $27 – 12$	M1dep	oe Substitutes $x = 3$ in their $\frac{dy}{dx}$ their $\frac{dy}{dx}$ must be an expression in x Allow even if their (a) has only one term
	15	A1ft	ft M2 and their (a) Only ft if their (a) has at least two terms of different order and all of their terms are differentiated correctly

4(c)	$y - 9$ = their 15$(x - 3)$ or y = their 15$x + c$ and substitutes (3, 9)	M1	oe e.g. $\frac{9-y}{3-x}$ = their 15 their 15 from (b) Allow $y - 9 = \frac{-1}{\text{their 15}}(x - 3)$ or $y = \frac{-1}{\text{their 15}}x + c$ and substitutes (3, 9) for M1 A0 only
	$y = 15x - 36$	A1ft	ft their 15 from (b) $15x - 36$ is M1 A0 unless $y = 15x - 36$ seen in working

5	$5(4c + 3)$ and $2(c - 8)$ or $20c + 15$ and $2c - 16$	M1	oe e.g. $10(4c + 3) + 4(c - 8)$ Allow one error in expansion if not showing brackets e.g. Allow $20c + 3$ and $2c - 16$ Equation or fractions not necessary
	Correct equation with no unexpanded brackets e.g.1 $20c + 15 + 2c - 16 = 10$ e.g.2 $22c - 1 = 10$ e.g.3 $\frac{(20c+15)}{10} + \frac{(2c-16)}{10} = 1$ e.g.4 $\frac{44c-2}{20} = 1$	A1	
	Eliminates denominators correctly and collects terms for their equation e.g.1 $20c + 2c = 10 - 15 + 16$ e.g.2 $22c = 11$	M1dep	dep on first M1 Do not award this mark if the denominator has been eliminated incorrectly at any time in the working Allow one sign error when collecting terms
	$\frac{1}{2}$ or $\frac{11}{22}$	A1ft	oe Only ft from M1 A0 M1 with a maximum of one error in expansions and collecting terms SC2 Answer $\frac{15}{22}$ oe

6	(radius =) $\sqrt{289}$ or 17 or (radius =) $\sqrt{121}$ or 11	B1	
	$(\frac{1}{4}\times)\ 2 \times \pi \times$ their 17 or 34π or $\frac{17\pi}{2}$ or [106.76, 107] or [26.69, 26.71] or $(\frac{1}{4}\times)\ 2 \times \pi \times$ their 11 or 22π or $\frac{11\pi}{2}$ or [69.08, 69.124] or [17.27, 17.3]	M1	oe their 17 can be 289 their 11 can be 121
	their 17 – their 11 or 6	M1	their 17 can be 289 their 11 can be 121 May be implied by 12 seen in next method mark
	$\frac{1}{4} \times 2 \times \pi \times$ their 17 + $\frac{1}{4} \times 2 \times \pi \times$ their 11 + 2 × their 6	M1	their 17 can be 289 their 11 can be 121
	$14\pi + 12$ or [55.96, 56(.0)]	A1	SC2 42π or [131.88, 132]

7(a)	x^7	B2	B1 $\sqrt{x^{14}}$ or $(x^{14})^{\frac{1}{2}}$ or $\sqrt{x^{5+9}}$ or $(x^{5+9})^{\frac{1}{2}}$ or $x^{\frac{14}{2}}$ or $x^{\frac{5+9}{2}}$ or $x^{\frac{5}{2}} \times x^{\frac{9}{2}}$ or $x^{2.5} \times x^{4.5}$

7(b)	0.2 or $\frac{1}{5}$ or 5^{-1}	B2	B1 $125^{-\frac{1}{3}}$ or $\sqrt[-3]{125}$ or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1}{125}}$ or $\frac{1}{125^{\frac{1}{3}}}$ or $\frac{1}{\sqrt[3]{125}}$ or $\left(\frac{1}{5^3}\right)^{\frac{1}{3}}$ or $\sqrt[3]{\frac{1}{5^3}}$ or $\frac{1^{\frac{1}{3}}}{5}$ or $\frac{\sqrt[3]{1}}{5}$ or $\frac{1}{y^3} = 125$ or $y^3 = \frac{1}{125}$ or $\frac{1}{y} = 5$ or $\frac{1}{y} = \sqrt[3]{125}$ or $\frac{1}{y} = 125^{\frac{1}{3}}$

8	$\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$	B2	B1 2 by 2 matrix with at least two elements correct
	their $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}(\times)\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$	M1	Multiplication can be in either order if their $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$ is a 2 by 2 matrix Do not award if their $\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$ is **M**
	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	A1	Must have B2 with M1 seen

9	**Alternative method 1**		
	angle ACD = 180 – 78 or 102	M1	
	angle ECD = 360 – 115 – their 102 or 143	M1	angle ECD = 143 implies M1 M1
	(143 + 32 =) 175 and No or 143 + 32 ≠ 180 (and No)	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)
	Alternative method 2		
	angle ACD = 180 – 78 or 102	M1	
	(Assumes CD is parallel to EF) angle DCE = 180 – 32 or 148	M1	
	(102 + 148 + 115 =) 365 and No or 102 + 148 + 115 ≠ 360 (and No)	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)
	Alternative method 3		
	Extends DC to X angle XCA = 78	M1	X may be a different letter or not labelled
	angle XCE = 115 – their 78 or 37	M1	angle XCE = 37 implies M1 M1
	37 and No	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)
	Alternative method 4		
	Extends DC to X angle XCA = 78	M1	X may be a different letter or not labelled
	(Assumes CD is parallel to EF) angle XCE = 32	M1	
	(32 + 78 =) 110 and No or 32 + 78 ≠ 115 (and No)	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)

9	**Alternative method 5**		
	Extends AC to meet EF at Y angle ECY = 180 – 115 or 65	M1	Y may be a different letter or not labelled
	angle EYC = 180 – their 65 – 32 or 83	M1	angle EYC = 83 implies M1 M1
	83 and No	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)
	Alternative method 6		
	Extends AC to meet EF at Y angle ECY = 180 – 115 or 65	M1	Y may be a different letter or not labelled
	(Assumes AB is parallel EF) angle EYC = 78	M1	
	(32 + 78 + 65 =) 175 and No or 32 + 78 + 65 ≠ 180 (and No)	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)
	Alternative method 7		
	Draws a line from X on AB to Y on EF passing through C with right angles marked at AXC and CYE (Assumes CD is parallel to EF) angle ACX = 180 – 90 – 78 or 12	M1	X and Y may be different letters or not labelled
	angle ECY = 180 – 90 – 32 or 58	M1	
	(12 + 115 + 58 =) 185 and No or 12 + 115 + 58 ≠ 180 (and No)	A1	oe SC3 32 + 78 = 110 and No or 32 + 78 ≠ 115 (and No)

10(a)	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	B2	B1 Rotation 180° (about/centre O) or indication that $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ or indication that $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}(\times)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(\times)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ or reflection in $y = -x$ and $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

10(b)	Correct square (vertices O, A'' (–3, 0) B'' (–3, –3) and C'' (0, –3)) with correct labelling	B3	B2 Correct square with incorrect or no labelling or correct points plotted with correct labelling B1 3 by 3 square in wrong position (ignore labelling) or correct points plotted with incorrect or no labelling or enlargement scale factor –3 (centre O) or $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

11(a)	$\frac{4c^5}{9d^3}$ or $\frac{4c^5d^{-3}}{9}$ or $\frac{0.\dot{4}c^5}{d^3}$ or $0.\dot{4}c^5d^{-3}$	B3	B2 Any two of these three components • numerator having c^5 (no c in denominator) • denominator having d^3 (no d in numerator) or numerator having d^{-3} (no d in denominator) • number $\frac{4}{9}$ or $0.\dot{4}$ B1 Any one of these three components • numerator having c^5 (no c in denominator) • denominator having d^3 (no d in numerator) or numerator having d^{-3} (no d in denominator) • number $\frac{4}{9}$ or $0.\dot{4}$ or $\frac{40c^7d^3}{90d^6c^2}$ or $\frac{20c^7d^3}{45d^6c^2}$ or $\frac{8c^7d^3}{18d^6c^2}$ or $\frac{1.\dot{3}c^7d^3}{3d^6c^2}$ or $\frac{\frac{4}{3}c^7d^3}{3d^6c^2}$ SC1 $\frac{9d^3}{4c^5}$ or $\frac{2.25d^3}{c^5}$ Always award SC1 if this is their final answer even if $\frac{4c^5}{9d^3}$ seen in working

11(b)	$(m + 1)(m - 4)$ or $m^2 - 3m - 4$ seen as a common denominator	B1	oe
	$5(m - 4) + 6(m + 1)$	M1	Allow one error in expansion if not showing brackets e.g. Allow $5m - 20 + m + 6$
	$\dfrac{5m - 20 + 6m + 6}{\text{their common denominator}}$ or $\dfrac{5m - 20}{\text{their common denominator}} + \dfrac{6m + 6}{\text{their common denominator}}$	M1	Allow one error in expansion of numerator(s) their common denominator must be a quadratic
	$\dfrac{11m - 14}{(m + 1)(m - 4)}$ or $\dfrac{11m - 14}{m^2 - 3m - 4}$	A1	

12	**Alternative method 1**		
	$x^2 + (2x)^2 = 20$ or $\sqrt{20 - x^2} = 2x$	M1	oe Condone absence of brackets
	$5x^2 = 20$ or $5x^2 - 20$ (= 0)	M1	oe e.g $x^2 = 4$ Collects terms for their quadratic to $ax^2 = b$ or $ax^2 - b$ (= 0) a and b both non-zero This mark implies the first M1
	$\sqrt{\dfrac{20}{\text{their } 5}}$ or $x = \sqrt{4}$ or $5(x + 2)(x - 2)$ (= 0)	M1	Correct attempt to solve their quadratic oe e.g. $(x + 2)(x - 2)$ (= 0) If using formula must substitute correctly If using completing the square must correctly obtain $(px + q)^2 = r$ or $(px + q)^2 - r$ (= 0) p, q and r non-zero
	$x = 2$ and $x = -2$ or $x = 2$ and $y = 4$ or $x = -2$ and $y = -4$	A1	Allow $x = \pm 2$
	D (2, 4) and E (–2 , –4)	A1	Correct letter must be linked to correct point SC2 Both points correct by T & I SC1 One point correct by T & I

12	**Alternative method 2**		
	$\left(\frac{y}{2}\right)^2 + y^2 = 20$ or $\sqrt{20 - y^2} = \frac{y}{2}$	M1	oe Condone absence of brackets
	$5y^2 = 80$ or $\frac{5}{4}y^2 = 20$ or $5y^2 - 80 = 0$	M1	oe e.g $y^2 = 16$ Collects terms for their quadratic to $ay^2 = b$ or $ay^2 - b$ (= 0) a and b both non-zero This mark implies the first M1
	$\sqrt{\frac{80}{\text{their } 5}}$ or $y = \sqrt{16}$ or $5(y + 4)(y - 4)$ (= 0)	M1	Correct attempt to solve their quadratic oe e.g. $(y + 4)(y - 4)$ (= 0) If using formula must substitute correctly If using completing the square must correctly obtain $(py + q)^2 = r$ or $(py + q)^2 - r$ $(= 0)$ p, q and r non-zero
	$y = 4$ and $y = -4$ or $y = 4$ and $x = 2$ or $y = -4$ and $x = -2$	A1	Allow $y = \pm 4$
	D (2, 4) and E (–2 , –4)	A1	Correct letter must be linked to correct point SC2 Both points correct by T & I SC1 One point correct by T & I

13(a)	C	B1	

13(b)	D	B1	

13(c)	A	B1	

14	$x(5 - 3w) = 2w + 1$	M1	
	$5x - 3xw = 2w + 1$ or $5 - 3w = \frac{2w}{x} + \frac{1}{x}$	M1dep	oe e.g. $5x - 3xw - 2w = 1$ Expands brackets correctly or divides each term by x
	$5x - 1 = 2w + 3xw$ or $5 - \frac{1}{x} = \frac{2w}{x} + 3w$	M1dep	oe e.g. $-3xw - 2w = 1 - 5x$ Collects terms in w (must have ≥ 2 terms containing w) Allow one sign error only dep on first M1 only
	$\frac{5x-1}{2+3x} = w$	A1	oe e.g. $w = \frac{1-5x}{-3x-2}$ Must have $= w$ or $w =$

15(a)	29 and 23 identified	B2	B1 $(n + 9)(n + 3)$ or 667 or 29 or 23

15(b)	**Alternative method 1**		
	$(n-3)^2$	M1	Allow $(n-3)(n-3)$ for $(n-3)^2$
	$(n-3)^2 - 9 + 14$ or $(n-3)^2 + 5$	A1	Allow $(n-3)(n-3)$ for $(n-3)^2$
	$(n-3)^2 \geq 0$ then adding 5 so always positive or States minimum value is 5 or States (3, 5) is minimum point	A1ft	oe Allow $(n-3)(n-3)$ for $(n-3)^2$ ft M1 A0 Must see M1 and attempt $(n-3)^2 + k$ ft $(n-3)^2 + k$ where $k > 0$ SC2 States minimum value is 5 or States (3, 5) is minimum point
	Alternative method 2		
	Quadratic curve sketched in first quadrant with minimum point above the x-axis	M1	Labelling on axes not required
	(discriminant =) –20	A1	
	States no (real) roots	A1ft	oe Allow roots → solutions ft M1 A0 Must see M1 and attempt a discriminant ft discriminant < 0 SC2 States minimum value is 5 or States (3, 5) is minimum point

15(b)	**Alternative method 3**		
	$2n - 6 = 0$	M1	oe equation e.g. $2n = 6$ or $n = 3$
	(second derivative =) 2	A1	
	States minimum value is 5 or States (3, 5) is minimum point	A1ft	oe ft M1 A0 Must see M1 and attempt a second derivative ft (second derivative) > 0 SC2 States minimum value is 5 or States (3, 5) is minimum point

16(a)	$a - 2$	B1	

16(b)	**Alternative method 1**		
	C (0, 4) or $y = 4$ (for C)	B1	May be on diagram
	$\frac{\text{their } 4 - 0}{0 - 2}$ or -2 or $\frac{(a-2)^2 - 0}{a - 2}$ or $a - 2$	M1	gradient BC or gradient AB
	$\frac{\text{their } 4 - 0}{0 - 2}$ or -2 and $\frac{(a-2)^2 - 0}{a - 2}$ or $a - 2$	M1dep	gradient BC or gradient AB
	their $a - 2 = \frac{-1}{\text{their } -2}$	M1dep	
	$2\frac{1}{2}$	A1	oe
	Alternative method 2		
	C (0, 4) or $y = 4$ (for C)	B1	May be on diagram
	$\frac{\text{their } 4 - 0}{0 - 2}$ or -2	M1	gradient BC
	$y = -\frac{1}{\text{their } -2}(x - 2)$ or $y = \frac{1}{2}(x - 2)$ or $y = \frac{1}{2}x - 1$	M1dep	Equation AB
	their $\frac{1}{2}(x - 2) = (x - 2)^2$	M1dep	oe e.g. $\frac{1}{2}x - 1 = x^2 - 4x + 4$
	$2\frac{1}{2}$	A1	oe

16b	**Alternative method 3**		
	C (0, 4) or $y = 4$ (for C)	B1	May be on diagram
	$a^2 + ((a-2)^2 -$ their $4)^2$ or $(a-2)^2 + ((a-2)^2)^2$	M1	AC^2 or AB^2 oe
	$a^2 + ((a-2)^2 -$ their $4)^2$ = $(a-2)^2 + ((a-2)^2)^2 + 2^2 +$ their 4^2	M1dep	$AC^2 = AB^2 + BC^2$ oe e.g. $AC^2 - AB^2 = BC^2$ Only ft their 4
	their $8a^2 - 36a + 40$ (= 0)	M1dep	oe their quadratic $pa^2 + qa + r$ (= 0) p, q and r all non-zero
	$2\frac{1}{2}$	A1	oe
	Alternative method 4		
	C (0, 4) or $y = 4$ (for C)	B1	May be on diagram
	tan $OBC = \frac{\text{their } 4}{2}$	M1	oe
	angle ABD = 180 – 90 – their $\tan^{-1}\frac{\text{their } 4}{2}$	M1dep	oe D on the x-axis such that angle $BDA = 90^\circ$
	tan their angle $ABD = \frac{(a-2)^2}{a-2}$	M1dep	oe
	$2\frac{1}{2}$	A1	oe

17(a)	$3d(4c^2 - 3d)$	B2	B1 $d(12c^2 - 9d)$ or $3(4c^2d - 3d^2)$

17(b)	**Alternative method 1**		
	$(w + 4)^2$ as a factor	M1	Allow $(w + 4)\,(w + 4)$
	$(w + 4)^2(w + 4 - (w + 1))$ or $(w + 4)^2(w + 4 - w + 1)$ or $(w + 4)^2(w + 4 - w - 1)$	M1dep	Allow $(w + 4)\,(w + 4)$ for $(w + 4)^2$
	$3(w + 4)^2$	A1	Allow $3(w + 4)\,(w + 4)$
	Alternative method 2		
	$(w + 4)[(w + 4)^2 - (w + 4)(w + 1)]$	M1	
	$(w + 4)(aw + b)$	M1dep	a and b both non-zero
	$3(w + 4)^2$	A1	Allow $3(w + 4)\,(w + 4)$
	Alternative method 3		
	$w^3 + 12w^2 + 48w + 64$ or $w^3 + 9w^2 + 24w + 16$ or $-w^3 - 9w^2 - 24w - 16$ or $-w^3 + 9w^2 + 24w + 16$ or $3w^2 + 24w + 48$ or $3(w^2 + 8w + 16)$	M1	Must collect terms
	$(3w + 12)(w + 4)$	M1dep	Correctly factorises their three term quadratic
	$3(w + 4)^2$	A1	Accept $3(w + 4)\,(w + 4)$

18	**Alternative method 1**		
	$\sqrt{14^2+8^2}$ or $\sqrt{260}$ or $2\sqrt{65}$ or [16.1, 16.125]	M1	AC
	tan (x) = $\frac{7}{\text{their } AC}$	M1dep	oe
	[23.4667, 23.5]	A1	
	Alternative method 2		
	$\sqrt{14^2+8^2+7^2}$ or $\sqrt{309}$ or [17.578, 17.6]	M1	EC May be seen in stages e.g. Work out AC with correct method then work out their AC^2 + 7^2 then square roots Condone use of $2\sqrt{65}\ ^2$ for AC^2
	sin (x) = $\frac{7}{\text{their } EC}$ (× sin 90) or cos (x) = $\frac{\sqrt{8^2+14^2}}{\text{their } EC}$	M1dep	cos (x) = $\frac{8^2+14^2+\text{their } EC^2 - 7^2}{2 \times \text{their } \sqrt{8^2+14^2} \times \text{their } EC}$ Condone use of $2\sqrt{65}\ ^2$ for AC^2
	[23.4667, 23.5]	A1	

19(a)	$2\pi r(r + 5)$ seen	M1	oe e.g. $2 \times \pi \times r(r + 5)$
	$\frac{9\pi r^2}{2}$	M1	oe e.g. $\pi \times r \times \frac{9r}{2}$
	$\pi r^2 + 2\pi r^2 + 10\pi r + \frac{9\pi r^2}{2}$ or $\frac{2\pi r^2 + 4\pi r^2 + 20\pi r + 9\pi r^2}{2}$ or $3\pi r^2 + 10\pi r + \frac{9\pi r^2}{2}$ or $\frac{6\pi r^2 + 20\pi r + 9\pi r^2}{2}$	A1	Correct unsimplified expression with brackets $2\pi r(r + 5)$ expanded May still contain multiplication signs
	$\frac{15\pi r^2}{2} + 10\pi r = \frac{5\pi r}{2}(3r + 4)$ or $\frac{15\pi r^2 + 20\pi r}{2} = \frac{5\pi r}{2}(3r + 4)$	A1	Must see M2 A1

19(b)	$\frac{5\pi r}{2}(3r + 4) = 1200\pi$	M1	oe Allow $1200\pi \rightarrow 1200$
	Correct equation or 3 term expression with no unexpanded brackets e.g.1 $3r^2 + 4r - 480$ (= 0) e.g.2 $15r^2 + 20r = 2400$ e.g.3 $\frac{15\pi}{2}r^2 + 10\pi r = 1200\pi$	A1	oe
	Attempt to factorise their 3 term quadratic e.g. for $3r^2 + 4r - 480$ $(3r + a)(r + b)$ where $ab = \pm480$ or $3b + a = \pm4$ or Attempt to substitute in the formula for their 3 term quadratic (allow one sign error) e.g. for $3r^2 + 4r - 480$ $\frac{-4 \pm \sqrt{4^2 - 4\times3\times-480}}{2\times3}$ or $\frac{4 \pm \sqrt{4^2 - 4\times3\times-480}}{2\times3}$ (1 sign error)	M1dep	oe Attempt to complete the square for their 3 term quadratic e.g. for $3r^2 + 4r - 480$ $(3)\ [(r + \frac{2}{3})^2 \ldots\ldots.]$
	Correctly factorises their 3 term quadratic e.g. for $3r^2 + 4r - 480$ (= 0) $(3r + 40)(r - 12)$ (= 0) or Correct substitution in formula for their 3 term quadratic e.g. for $3r^2 + 4r - 480$ (= 0) $\frac{-4 \pm \sqrt{4^2 - 4\times3\times-480}}{2\times3}$	A1ft	ft M1 A0 M1dep oe Correct completion of square for their 3 term quadratic e.g. for $3r^2 + 4r - 480$ $(3)\ [(r + \frac{2}{3})^2 - (\frac{2}{3})^2 - 160\ ]$ oe
	12	A1	Do not award if negative solution also included

20	$(8x - y)^2 = (6x)^2 + (x + y)^2$	M1	oe Allow $(8x - y)(8x - y)$ and $(x + y)(x + y)$ Condone $6x^2$
	Expands $(8x - y)^2$ to 4 terms with 3 correct from $64x^2 - 8xy - 8xy + y^2$	M1	oe If going straight to 3 terms must be $64x^2 - 16xy + ky^2 \quad (k \neq 0)$ or $ax^2 - 16xy + y^2 \quad (a \neq 0)$
	Expands $(x + y)^2$ to 4 terms with 3 correct from $x^2 + xy + xy + y^2$	M1	oe If going straight to 3 terms must be $x^2 + 2xy + ay^2 \quad (a \neq 0)$ or $bx^2 + 2xy + y^2 \quad (b \neq 0)$
	$27x^2 - 18xy$ (= 0) or $27x^2 = 18xy$ or better e.g.1 $9x^2 - 6xy$ (= 0) e.g.2 $3x^2 = 2xy$	A1	$64x - 16y = 36x + x + 2y$ or equivalent linear equation e.g.1 $64x - 16y - 36x = x + 2y$ e.g.2 $64x - 16y - x - 2y = 36x$
	Any correct factorisation of their $px^2 + qxy$ or correct division of their $px^2 = qxy$ by a multiple of x (p and q non zero) e.g.1 $9x(3x - 2y)$ (= 0) e.g.2 $3x(9x - 6y)$ (= 0) e.g.3 $27x = 18y$ e.g.4 $9x = 6y$	M1	Correct collection and correct simplification of terms for their linear equation in x and y e.g. $27x = 18y$ To gain this mark there must have been some expansion of brackets seen
	$3x = 2y$ or $\frac{x}{y} = \frac{2}{3}$ or $\frac{y}{x} = \frac{3}{2}$ or $x = \frac{2}{3}y$ or $y = \frac{3}{2}x$ or $\frac{x}{2} = \frac{y}{3}$ or $\frac{2}{x} = \frac{3}{y}$	A1	Must see M1 M1 M1 A1 Do not allow if a contradictory statement is also seen

21	**Alternative method 1**		
	$\sin(x) = \sqrt{\frac{1}{16}}$ or $\sin(x) = \frac{1}{4}$	M1	
	[14.4775, 14.5]	A1	Do not award if another solution in range $0 \le x < 90$ is given
	$\sin x = -\sqrt{\frac{1}{16}}$ or $\sin x = -\frac{1}{4}$ or –[14.4775, 14.5] or 180 + their [14.4775, 14.5]	M1	their [14.4775, 14.5] must be a positive acute angle
	[194.4775, 194.5]	A1	Do not award if another solution in range $180 \le x \le 270$ is given
	[165.5, 165.5225]	B1ft	ft 180 – their [14.4775, 14.5] their [14.4775, 14.5] must be a positive acute angle Do not award if another solution in range $90 \le x < 180$ is given

21	Alternative method 2		
	$\cos(x) = \sqrt{1-\frac{1}{16}}$ or $\cos(x) = \sqrt{\frac{15}{16}}$ or $\cos(x) = \frac{\sqrt{15}}{4}$ or $\cos(x)$ = [0.968, 0.97]	M1	
	[14.4775, 14.5]	A1	Do not award if another solution in range $0 \leq x < 90$ is given
	$\cos(x) = -\sqrt{1-\frac{1}{16}}$ or $\cos(x) = -\sqrt{\frac{15}{16}}$ or $\cos(x) = -\frac{\sqrt{15}}{4}$ or $\cos(x)$ = –[0.968, 0.97] or 180 + their [14.4775, 14.5]	M1	their [14.4775, 14.5] must be a positive acute angle
	[194.4775, 194.5]	A1	Do not award if another solution in range $180 \leq x \leq 270$ is given
	[165.5, 165.5225]	B1ft	ft 180 – their [14.4775, 14.5] their [14.4775, 14.5] must be a positive acute angle Do not award if another solution in range $90 \leq x < 180$ is given

22	**Alternative method 1**		
	Substitutes a value $0 < x < 3$ and obtains a correct expression in k e.g. $x = 2 \rightarrow 2k(2-3)^3$ or $2k(-1)^3$ and substitutes a value $x > 3$ and obtains a correct expression in k e.g. $x = 4 \rightarrow 4k(4-3)^3$ or $4k(1)^3$	M1	oe
	Obtains correct expressions for both and correctly indicates whether they are positive or negative e.g. $-2k$ positive and $4k$ negative	M1dep	
	Max(imum point)	A1	Must see the working for M1 M1
	Alternative method 2		
	Correct second derivative with $x = 3$ substituted in leading to 0 i.e. $4kx^3 - 27kx^2 + 54kx - 27k$ and $x = 3 \rightarrow 0$	M1	oe e.g. $3kx(x-3)^2 + k(x-3)^3$ and $x = 3 \rightarrow 0$
	Correct third derivative with $x = 3$ substituted in leading to 0 and correct fourth derivative with $x = 3$ substituted in leading to < 0 i.e. $12kx^2 - 54kx + 54k$ and $x = 3 \rightarrow 0$ and $24kx - 54k$ and $x = 3 \rightarrow 18k$ negative	M1dep	
	Max(imum point)	A1	Must see the working for M1 M1