Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16 – 17	
18 – 19	
20 – 21	
22 – 23	
24 – 25	
26 – 27	
28	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2014

Further Mathematics 8360/2

Level 2

Paper 2 Calculator

Friday 20 June 2014 9.00 am to 11.00 am

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

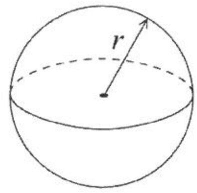
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

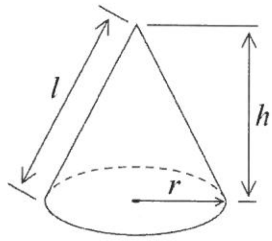
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

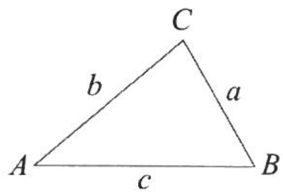


Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC



Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

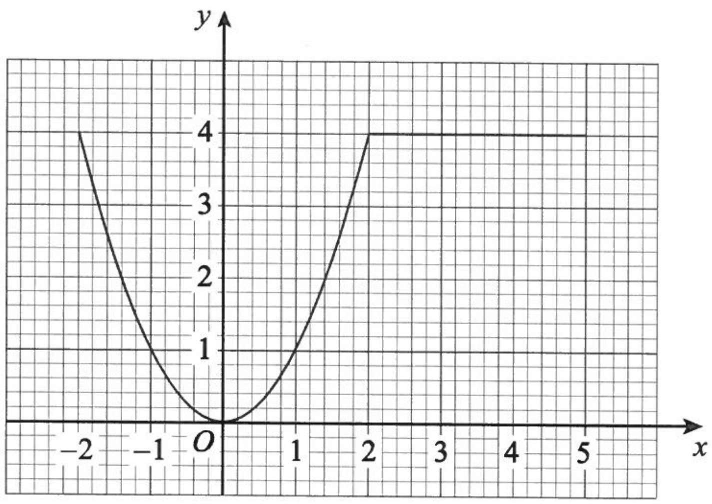
Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The graph of $y = f(x)$ for the full domain is shown.
The graph consists of a quadratic curve and a straight line.



Complete the boxes to describe $f(x)$.

[3 marks]

$f(x) = \boxed{x^2}$ $\quad -2 \leqslant x \leqslant 2$

$y = \boxed{4}$ $\quad 2 < x \leqslant \boxed{5}$

Turn over for the next question

3

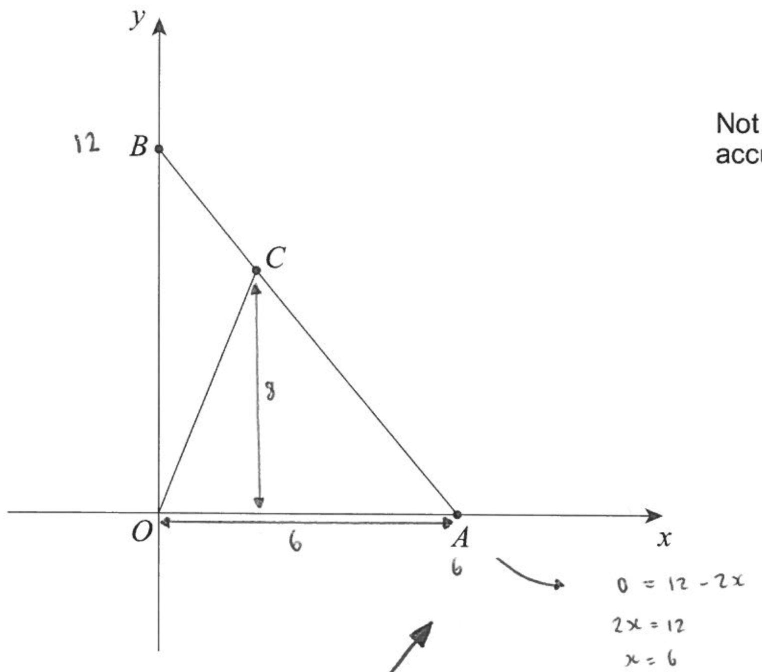
Turn over ▶

0 3

Do not write outside the box

2 The equation of line AB is $y = 12 - 2x$

The area of triangle OCA is 24 square units.



Work out the coordinates of C.

[5 marks]

$\frac{b \times h}{2} = 24 \rightarrow \frac{6 \times h}{2} = 24$

$6h = 48$

$h = 8$

y-coord is 8 at C so sub this into $y = 12 - 2x$ to find x-coord $8 = 12 - 2x$

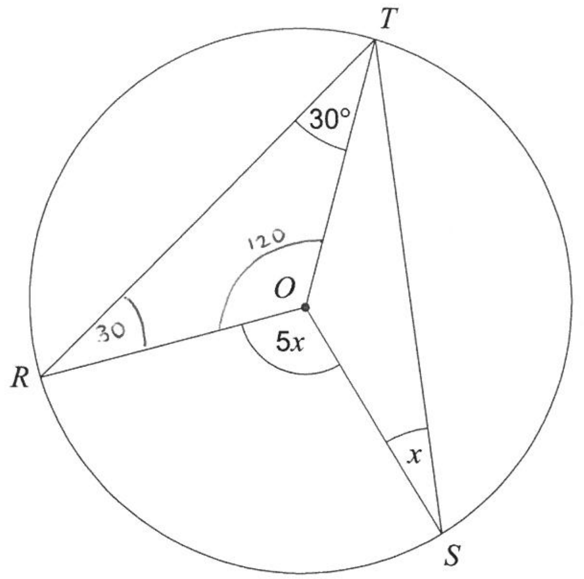
$8 + 2x = 12$

$2x = 4$

$x = 2$

Answer (2 , 8)

3 R, S and T are on the circumference of a circle, centre O.



Not drawn accurately

3 (a) Give a reason why angle $OTS = x$

[1 mark]

triangle TOS is isosceles because OT = OS (radius)
therefore base angles are equal OST = OTS
$x = x$ as req.

3 (b) Work out the value of x.

[3 marks]

TRO = 30° ∴ TOR = 180 − 30 − 30 = 120
TOS = 180 − x − x = 180 − $2x$
angle around the centre : 360 = 120 + 180 − $2x$ + $5x$
360 = 300 + $3x$
60 = $3x$
x = 20

Answer 20 degrees

Turn over for the next question

9

Do not write outside the box

4 (a) Expand $x^2(x-2)$

[2 marks]

Answer $x^3 - 2x^2$

4 (b) A curve has equation $y = x^2(x-2)$

Work out the gradient of the curve at the point (3, 9).

[3 marks]

$\frac{dy}{dx} = 3x^2 - 4x$

when $x = 3$: $3(3)^2 - 4(3) = 27 - 12 = 15$

Answer 15

4 (c) Line L is the tangent to the curve $y = x^2(x-2)$ at the point (3, 9).

Work out the equation of L.
Give your answer in the form $y = mx + c$

[2 marks]

$y - y_1 = m(x - x_1)$

$y - 9 = 15(x - 3)$

$y - 9 = 15x - 45$

$y = 15x - 36$

Answer $y = 15x - 36$

Do not write outside the box

5 Solve $\frac{4c+3}{2} + \frac{c-8}{5} = 1$

[4 marks]

$$\frac{5(4c+3)}{10} + \frac{2(c-8)}{10} = 1$$

$$\frac{20c+15+2c-16}{10} = 1$$

$$\frac{22c-1}{10} = 1$$

$$22c-1 = 10$$

$$22c = 11$$

$$c = \frac{11}{22}$$

$c = $ $\frac{1}{2}$

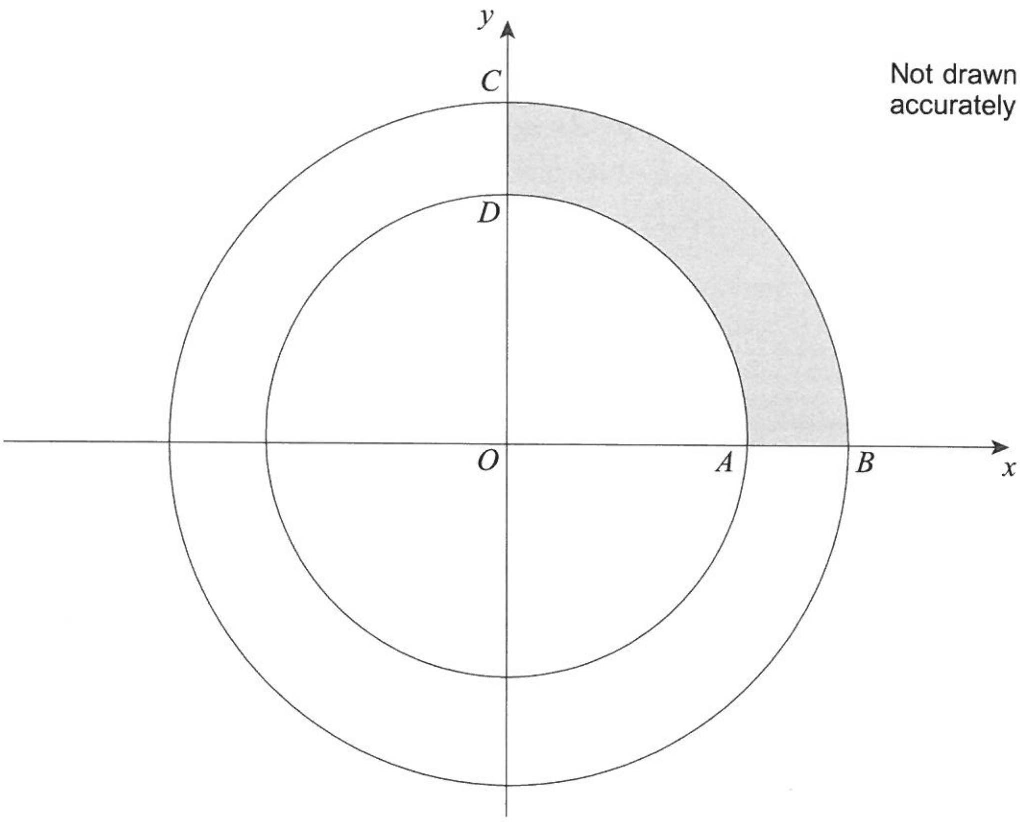
Turn over for the next question

11

Do not write outside the box

6 Two circles, each with centre O, are shown.
The equations of the circles are

$$x^2 + y^2 = 289 \quad \text{and} \quad x^2 + y^2 = 121$$



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Work out the **perimeter** of the shaded section $ABCD$.

[5 marks]

radius $OA = \sqrt{121} = 11$ radius $OB = \sqrt{289} = 17$

$\therefore$ CD and AB $= 17 - 11 = 6$

arc of CB $= \frac{\pi d}{4} = \frac{34\pi}{4} = 8.5\pi$

arc of DA $= \frac{\pi d}{4} = \frac{22\pi}{4} = 5.5\pi$

total perimeter $= 8.5\pi + 5.5\pi + 6 + 6$

Answer 55.98 (2dp)

7 (a) Simplify $\sqrt{x^5 \times x^9}$

Give your answer in the form x^p where p is an integer.

[2 marks]

$\sqrt{x^{14}} = (x^{14})^{\frac{1}{2}} = x^7$

Answer x^7

7 (b) Solve $y^{-3} = 125$

[2 marks]

$y^{-3} = \frac{1}{y^3}$

$\frac{1}{y^3} = 125$

$1 = 125y^3$

$\frac{1}{125} = y^3$ $y = \sqrt[3]{\frac{1}{125}}$

$y =$ $\frac{1}{5}$

Turn over for the next question

9

8 $\mathbf{M} = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$

Show that $\mathbf{M}^3 = \mathbf{I}$

[4 marks]

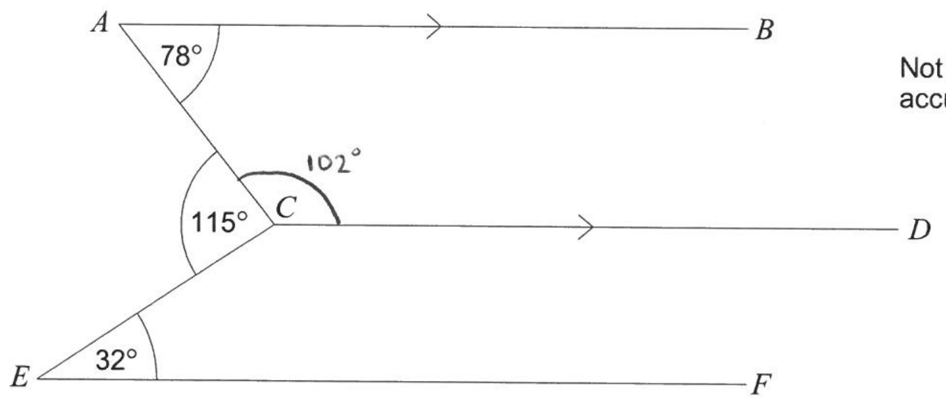
$$\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} (-2\times-2)+(-1\times3) & (-2\times-1)+(-1\times1) \\ (3\times-2)+(1\times3) & (3\times-1)+(1\times1) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} (1\times-2)+(1\times3) & (1\times-1)+(1\times1) \\ (-3\times-2)+(-2\times3) & (-3\times-1)+(-2\times1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \quad \text{as required.}$$

Do not write outside the box

9



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AB is parallel to CD.

Is EF parallel to CD?
You **must** show your working.

[3 marks]

co-interior angles add up to 180° ∴ ACD = 102°
(180 − 78 = 102)
angles around a point add up to 360° ∴ ECD = 143°
(360 − 115 − 102 = 143)
ECD + CEF = 143 + 32 = 175° which means
EF is not parallel to CD because they should
add up to 180° (co-interior angles)

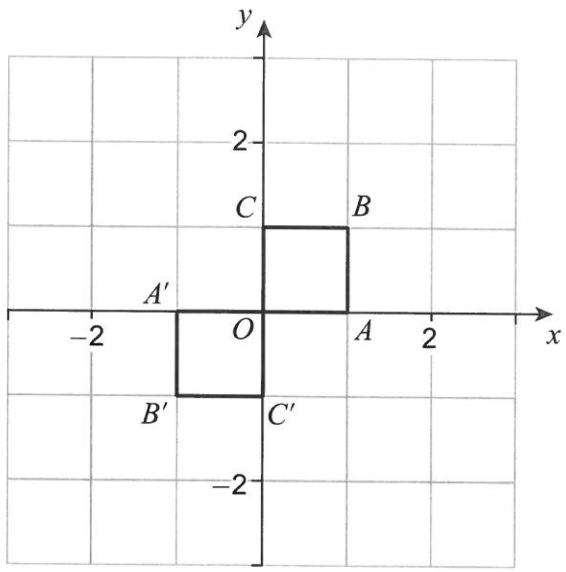
Turn over for the next question

7

10 The unit square $OABC$ has vertices

$$O(0,\ 0) \quad A(1,\ 0) \quad B(1,\ 1) \quad C(0,\ 1)$$

10 (a) $OABC$ is mapped to $OA'B'C'$ under transformation matrix **M**.



Work out matrix **M**.

[2 marks]

$A' = (-1, 0)$

$C' = (0, -1)$

$\longrightarrow \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Answer ....

Do not write outside the box

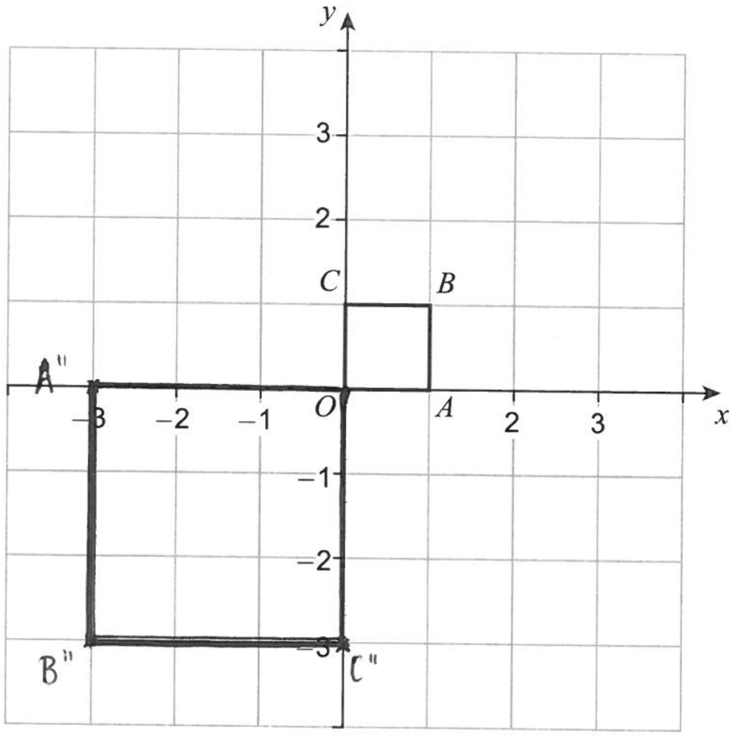
10 (b) $OABC$ is mapped to $OA''B''C''$ under transformation matrix $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

Draw **and** label $OA''B''C''$ on the diagram below.

$A'' = (-3, 0)$

$C'' = (0, -3)$

[3 marks]



Turn over for the next question

5

Do not write outside the box

11 (a) Simplify fully $\dfrac{8c^7}{15d^6} \div \dfrac{6c^2}{5d^3}$

[3 marks]

$$\frac{8c^{\cancel{7}\,5}}{15d^{\cancel{6}\,3}} \times \frac{5\cancel{d^3}}{6\cancel{c^2}} = \frac{\cancel{8}^{4}c^5}{{}_3\cancel{15}d^3} \times \frac{\cancel{5}^{1}}{\cancel{6}_3} = \frac{4c^5}{9d^3}$$

Answer $\dfrac{4c^5}{9d^3}$

11 (b) Write as a single fraction $\dfrac{5}{m+1} + \dfrac{6}{m-4}$

Give your answer in its simplest form.

[4 marks]

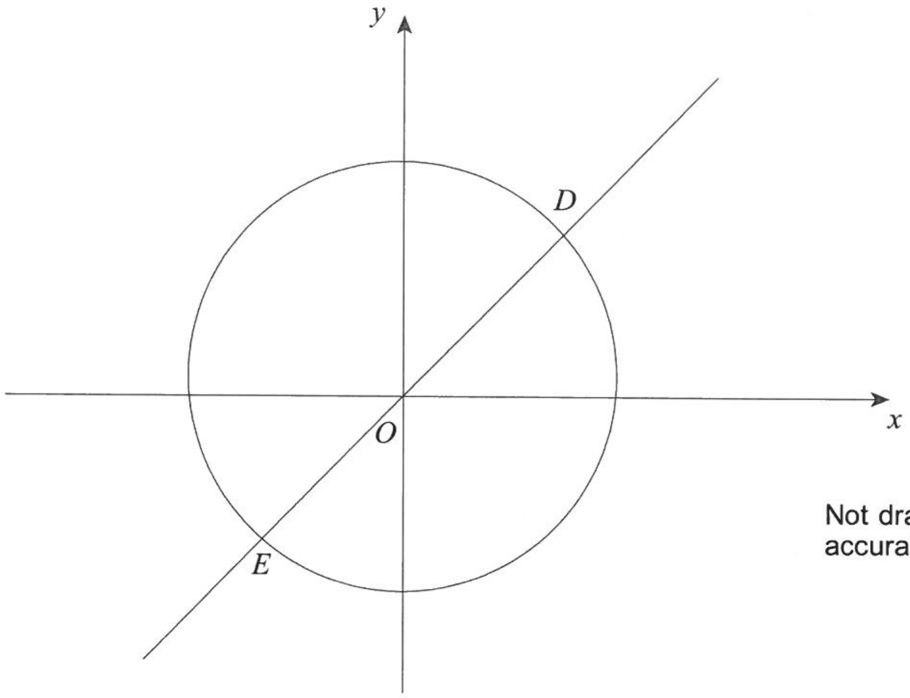
$$\frac{5(m-4) + 6(m+1)}{(m+1)(m-4)} = \frac{5m - 20 + 6m + 6}{(m+1)(m-4)}$$

$$= \frac{11m - 14}{(m+1)(m-4)}$$

Answer $\dfrac{11m-14}{(m+1)(m-4)}$

Do not write outside the box

12 The circle $x^2 + y^2 = 20$ and the line $y = 2x$ intersect at points D and E.



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Work out the coordinates of D and E.
Do **not** use trial and improvement.
You **must** show your working.

[5 marks]

$x^2 + (2x)^2 = 20$

$x^2 + 4x^2 = 20$

$5x^2 = 20$

$x^2 = 4$

$x = \pm 2$

when $x = 2$ when $x = -2$

$y = 2(2) = 4$ $y = 2(-2) = -4$

D (2 , 4) E (2 , -4)

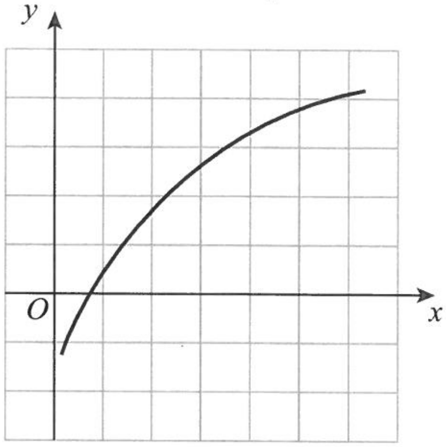
12

Turn over ▶

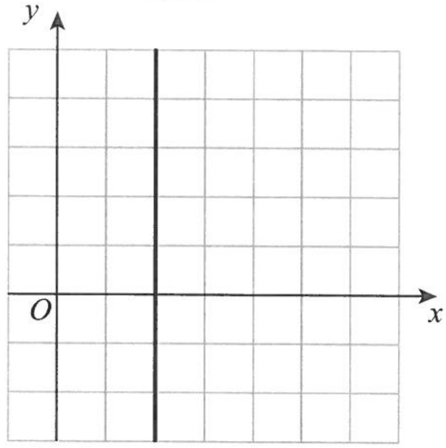
Do not write outside the box

13 Here are five graphs.

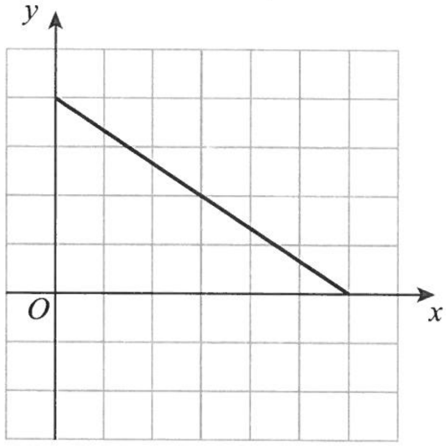
Graph A



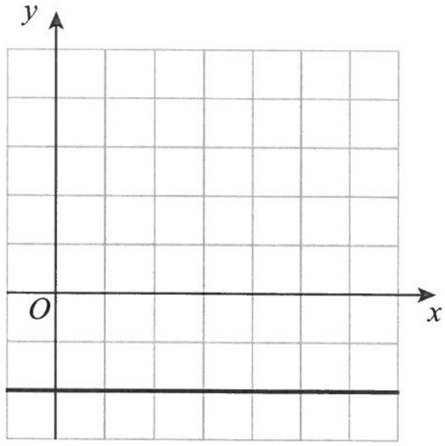
Graph B



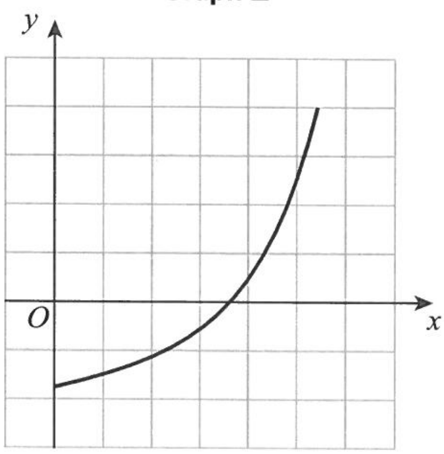
Graph C



Graph D



Graph E



Do not write outside the box

For each of the following statements, decide which graph is being described.
Circle your answer each time.

13 (a) The rate of change of y with respect to x is always negative.

[1 mark]

A B C D E

13 (b) The rate of change of y with respect to x is always zero.

[1 mark]

A B C D E

13 (c) As x increases, the rate of change of y with respect to x decreases.

[1 mark]

A B C D E

Turn over for the next question

3

Do not write outside the box

14 Rearrange $x = \frac{2w+1}{5-3w}$ to make w the subject.

[4 marks]

$x(5-3w) = 2w+1$

$5x - 3wx = 2w + 1$

$5x = 2w + 3wx + 1$

$5x - 1 = 2w + 3wx$

$5x - 1 = w(2+3x)$

$\frac{5x-1}{2+3x} = w$

$w = \frac{5x-1}{2+3x}$

Answer ....

Do not write outside the box

15 (a) The nth term of a sequence is $n^2 + 12n + 27$

By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.

[2 marks]

$(n+9)(n+3)$

20th: $(20+9)(20+3) = 29 \times 23$

both prime numbers as required

15 (b) The nth term of a different sequence is $n^2 - 6n + 14$

By completing the square, or otherwise, show that every term is positive.

[3 marks]

$(n-3)^2 - 9 + 14 = (n-3)^2 + 5$

minimum point is $(3,5)$

$\left((n-3)^2 = n^2 - 6n + 9\right)$ $\therefore$ always positive

Turn over for the next question

9

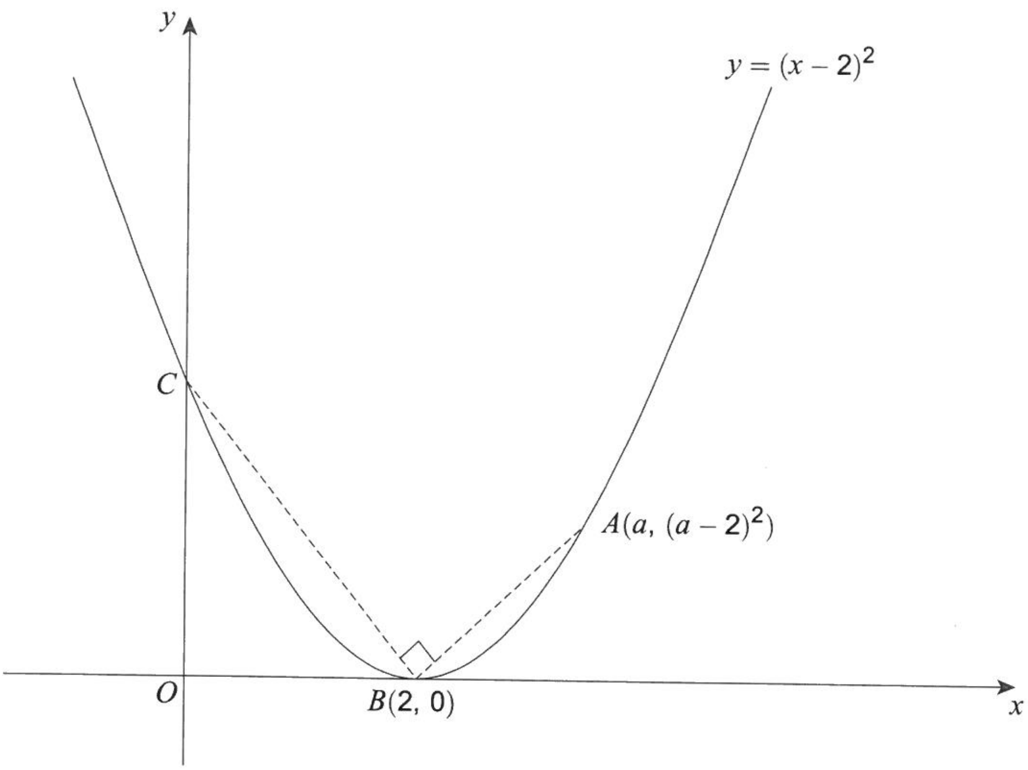
Do not write outside the box

16 (a) Simplify $\dfrac{(a-2)^{\cancel{2}}}{\cancel{a-2}}$

[1 mark]

Answer $a - 2$

16 (b) Here is a sketch of the curve $y = (x-2)^2$



- The curve touches the x-axis at B and intersects the y-axis at C.
- Angle ABC is 90°.
- The curve passes through $A(a, (a-2)^2)$

Do not write outside the box

Work out the value of a.

[5 marks]

$y = (x-2)^2 = x^2 - 4x + 4 \quad \therefore \quad C = (0,4)$

gradient of BC $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-0}{0-2} = \dfrac{4}{-2} = -2$

ABC $= 90^\circ \quad \therefore$ AB is perpendicular to BC.

$\Rightarrow m = \frac{1}{2}$

sub in $(2,0)$: $y - 0 = \frac{1}{2}(x-2)$

$y = \frac{1}{2}x - 1$

Sub in $(a, (a-2)^2)$: $(a-2)^2 = \frac{1}{2}(a) - 1$

$a^2 - 4a + 4 = \frac{1}{2}a - 1$

$a^2 - \frac{9}{2}a + 5 = 0$

$2a^2 - 9a + 10 = 0$

× to make 20
+ to make −9
−5 , −4

$(2a - 5)(2a - 4) = 0$

$(2a - 5)(a - 2) = 0$

$a = \frac{5}{2} \quad a = 2 \leftarrow$ already knew this

Answer $a = \frac{5}{2}$

$2a - 5 = 0$ OR $a - 2 = 0$

$2a = 5 \qquad a = 2$

$a = \frac{5}{2}$

Turn over for the next question

6

Do not write outside the box

17 (a) Factorise fully $12c^2d - 9d^2$

[2 marks]

$3d(4c^2 - 3d)$

Answer $3d(4c^2 - 3d)$

17 (b) Factorise fully $(w + 4)^3 - (w + 4)^2(w + 1)$

[3 marks]

$(w+4)^2[(w+4) - (w+1)]$

$(w+4)^2[w+4-w-1]$

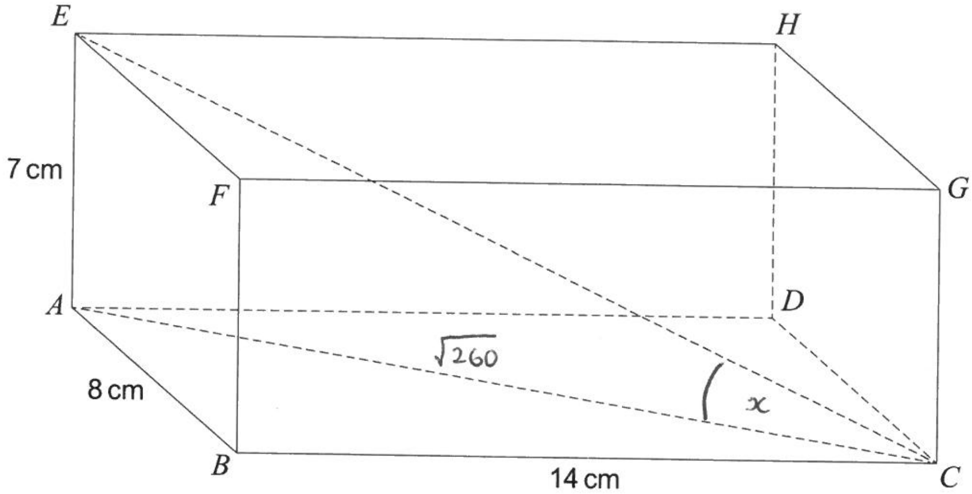
$(w+4)^2[3]$

$3(w+4)^2$

Answer $3(w+4)^2$

Do not write outside the box

18 *ABCDEFGH* is a cuboid.



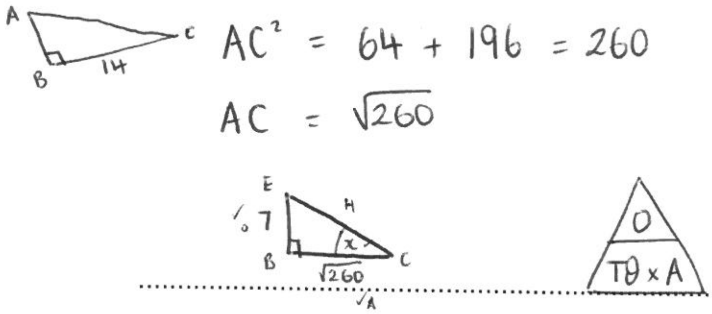
Work out the angle between *EC* and *ABCD*.

[3 marks]

$AC^2 = 8^2 + 14^2$

$AC^2 = 64 + 196 = 260$

$AC = \sqrt{260}$



$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{7}{\sqrt{260}}$

$\theta = \tan^{-1}\left(\frac{7}{\sqrt{260}}\right)$

$= 24.36670\ldots$

Answer 24.4 (1dp) degrees

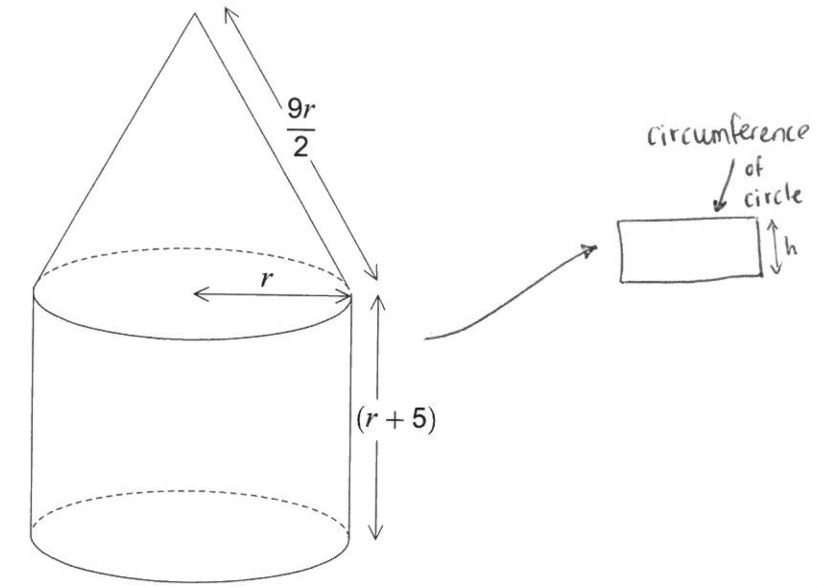
8

Turn over ▶

19 On this diagram all lengths are given in centimetres.
A cylinder and cone are joined together to make a solid.

The cylinder has radius r and height $(r + 5)$

The cone has radius r and slant height $\frac{9r}{2}$



19 (a) Show that the **total** surface area of the solid, in cm^2, is $\frac{5\pi r}{2}(3r + 4)$

[4 marks]

curved surface area of cone $= \pi r l$

$= \pi r \left(\frac{9r}{2}\right) = \frac{9\pi r^2}{2}$

SA of cylinder : rectangle $= \pi d \times h$

$= \pi (2r) \times (r+5)$

$= 2\pi r(r+5) = 2\pi r^2 + 10\pi r$

circle $= \pi r^2$

$= \pi r^2$

total : $\frac{9\pi r^2}{2} + 2\pi r^2 + 10\pi r + \pi r^2 = \frac{9\pi r^2}{2} + \frac{4\pi r^2}{2} + 10\pi r + \frac{2\pi r^2}{2}$

$= \frac{15\pi r^2}{2} + 10\pi r = \frac{15\pi r^2}{2} + \frac{20\pi r}{2}$

$= \frac{5\pi r}{2}(3r + 4)$ as required.

Do not write outside the box

19 (b) The total surface area of the solid is 1200π cm^2

Work out the value of r.

[5 marks]

$\frac{5\pi r}{2}(3r+4) = 1200\pi$

$\frac{15\pi r^2}{2} + \frac{20\pi r}{2} = 1200\pi$

$15\cancel{\pi}r^2 + 20\cancel{\pi}r = 2400\cancel{\pi}$

$15r^2 + 20r = 2400$

$3r^2 + 4r = 480$

$3r^2 + 4r - 480 = 0$

$a=3 \quad b=4 \quad c=-480$

$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$r = \frac{-4 \pm \sqrt{4^2-(4\times3\times-480)}}{2(3)}$

$r = \frac{-4 \pm \sqrt{16--5760}}{6} = \frac{-4 \pm \sqrt{5776}}{6} = \frac{-4 \pm 76}{6}$

$r = \frac{-4+76}{6} = 12 \qquad r = \frac{-4-76}{6} = \cancel{\frac{-80}{6}}$

radius can't be negative

Answer $r = 12$

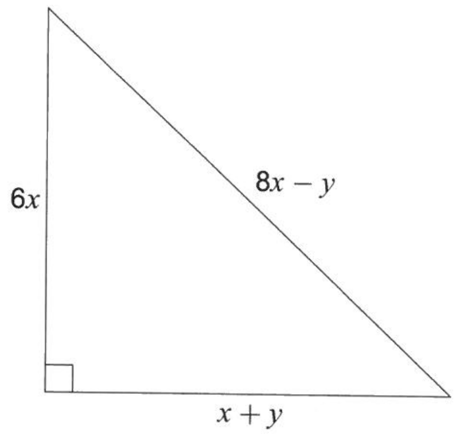
Turn over for the next question

9

Turn over ▶

Do not write outside the box

20 The diagram shows a right-angled triangle.



Not drawn accurately

Prove algebraically that $x : y = 2 : 3$

[6 marks]

$a^2 + b^2 = c^2$

$(6x)^2 + (x+y)^2 = (8x-y)^2$

$36x^2 + x^2 + 2xy + \cancel{y^2} = 64x^2 - 16xy + \cancel{y^2}$

$37x^2 + 2xy = 64x^2 - 16xy$

$2xy = 27x^2 - 16xy$

$18\cancel{x}y = 27x^{\cancel{2}}$

$18y = 27x$

$2y = 3x$

$\left(\begin{aligned} (x+y)(x+y) &= x^2 + xy + xy + y^2 \\ &= x^2 + 2xy + y^2 \\ (8x-y)(8x-y) &= 64x^2 - 8xy - 8xy + y^2 \\ &= 64x^2 - 16xy + y^2 \end{aligned} \right)$

$3x = 2y$

$\therefore \quad x : y = 2 : 3$ as required

Do not write outside the box

21 Solve $16\sin^2 x = 1$ for $0° \leqslant x \leqslant 270°$

[5 marks]

$$\sin^2 x = \frac{1}{16}$$

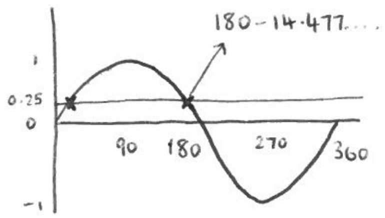
$$\sin x = \sqrt{\frac{1}{16}} = \pm\frac{1}{4}$$

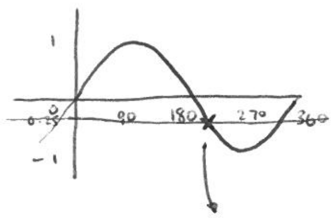
$\sin x = \frac{1}{4}$

$x = 14.477\ldots, 165.522\ldots$

$\sin x = -\frac{1}{4}$

$x = -\cancel{14.477}\ldots$





same distance from 180
as −14.477... is from 0

194.477....

Answer $14.48°$, $165.52°$, $194.48°$ (2 dp)

Turn over for the next question

11

22 The curve $y = \mathrm{f}(x)$ has $\dfrac{\mathrm{d}y}{\mathrm{d}x} = kx(x-3)^3$ where k is a **negative** constant.

There is a stationary point at $x = 3$

Determine the nature of this stationary point.
You **must** show your working.

[3 marks]

Try $x = 2.9$ in $\dfrac{dy}{dx}$

$K \times x \times (\cancel{x}\,2.9 - 3)^3$

$-ve \times +ve \times -ve = +ve$

Try $x = 3.1$ in $\dfrac{dy}{dx}$

$k \times x \times (3.1 - 3)^3$

$-ve \times +ve \times +ve = -ve$

Answer Maximum

END OF QUESTIONS

x	2.9	3	3.1
grad	/	—	\

$\therefore$

max

3