MR LYNCH'S ANSWERS

Centre Number		Candidate Number	
Surname			
Other Names			
Candidate Signature			

For Examiner's Use	
Examiner's Initials	
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16	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2014

Further Mathematics 8360/1

Level 2

Paper 1 Non-Calculator

Monday 16 June 2014 9.00 am to 10.30 am

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

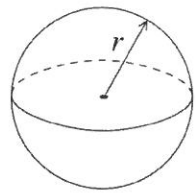
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

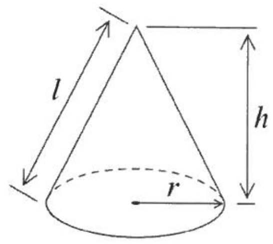
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



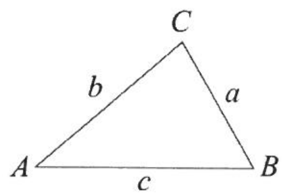
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$



Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\quad\quad$ $\sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A straight line has gradient -2 and passes through the point $(-3, 10)$.

Work out the equation of the line.
Give your answer in the form $y = mx + c$

[2 marks]

$y - y_1 = m(x - x_1)$

$y - 10 = -2(x - -3)$

$y - 10 = -2x - 6$

$y = -2x + 4$

Answer $y = -2x + 4$

2 $y = 4x^3 - 7x$

Work out $\dfrac{dy}{dx}$

[2 marks]

Answer $\dfrac{dy}{dx} = 12x^2 - 7$

Turn over for the next question

4

3 A transformation is given by the matrix **M**, where $\mathbf{M} = \begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix}$

The image of the point $(b, 5)$ under **M** is $(5, b)$.

Work out the values of a and b.

[3 marks]

$$\begin{pmatrix} 1 & a \\ 0 & 2 \end{pmatrix}\begin{pmatrix} b \\ 5 \end{pmatrix} = \begin{pmatrix} (1 \times b) + (a \times 5) \\ (0 \times b) + (2 \times 5) \end{pmatrix} = \begin{pmatrix} b + 5a \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} b + 5a \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ b \end{pmatrix}$$

$b + 5a = 5 \longrightarrow 10 + 5a = 5$

$10 = b$

$5a = -5$

$a = -1$

$a =$ .... -1 ...., $b =$ .... 10 ....

4 Solve $20 + w < 3(w + 2)$

[3 marks]

$20 + w < 3w + 6$

$20 < 2w + 6$

$14 < 2w$

$7 < w$

Answer .... $w > 7$ ....

Do not write outside the box

5 $f(x) = 10 - x^2$ for all values of x.

$g(x) = (x + 2a)(x + 3)$ for all values of x.

5 (a) Circle the correct value of $f(-4)$

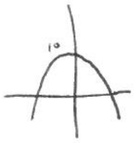
[1 mark]

26 −6 36 16 196

5 (b) Write down the range of $f(x)$.

[1 mark]

$10 - x^2$:



Answer $f(x) \leq 10$

5 (c) $g(0) = 24$

Show that $a = 4$

[1 mark]

$24 = (0 + 2a)(0 + 3)$

$24 = (2a)(3)$

$24 = 6a$

$4 = a$

5 (d) Hence solve $f(x) = g(x)$

[4 marks]

$10 - x^2 = (x + 2a)(x + 3)$

$10 - x^2 = (x + 8)(x + 3)$

$10 - x^2 = x^2 + 11x + 24$

$0 = 2x^2 + 11x + 14$

$0 = (2x + 7)(2x + 4)$

$0 = (2x + 7)(x + 2)$

$2x + 7 = 0$ or $x + 2 = 0$

$2x = -7$ $x = -2$

$x = -\frac{7}{2}$

Answer $x = -\frac{7}{2}$ or -2

13

6 The nth term of a sequence is $\dfrac{2n^2+7}{3n^2-2}$

6 (a) Work out the 7th term.
Give your answer as a fraction in its simplest form.

[2 marks]

$$\frac{2(7^2)+7}{3(7^2)-2} = \frac{2(49)+7}{3(49)-2} = \frac{98+7}{147-2} = \frac{105}{145} = \frac{21}{29}$$

Answer $\dfrac{21}{29}$

6 (b) Show that the limiting value of $\dfrac{2n^2+7}{3n^2-2}$ as $n \to \infty$ is $\dfrac{2}{3}$

[2 marks]

$$\frac{2+\frac{7}{n^2}}{3-\frac{2}{n^2}}$$

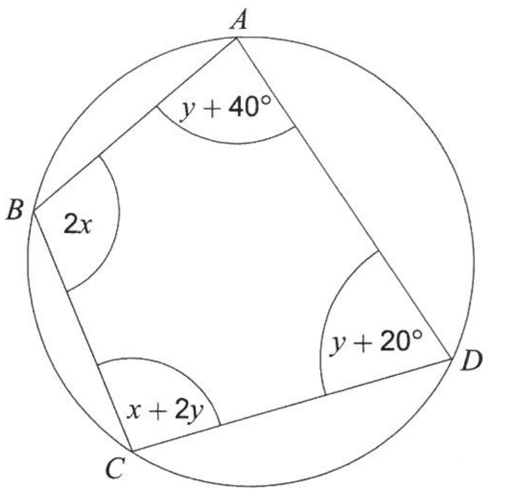
as $n^2 \to \infty$ $\dfrac{7}{n^2} \to 0$

and $\dfrac{2}{n^2} \to 0$

therefore $\dfrac{2+0}{3-0} = \dfrac{2}{3}$ as $n \to \infty$

Do not write outside the box

7 $ABCD$ is a cyclic quadrilateral.



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Work out the values of x and y.

[5 marks]

opposite angles in a cyclic quadrilateral add up to 180°

$A + C = 180$

$y + 40 + x + 2y = 180$

$40 + x + 3y = 180$

$x + 3y = 140$

(OR $B + D = 180$

$2x + y + 20 = 180$

$2x + y = 160$)

$2x + y = 160 \quad \times 3$

$x + 3y = 140$

$6x + 3y = 480$

$-\ x + 3y = 140$

$5x = 340$

$x = 68°$

sub $x = 68$ into $x + 3y = 140$

$68 + 3y = 140$

$3y = 72$

$y = 24$

$x =$ 68°, $y =$ 24°

9

Turn over ▶

Do not write outside the box

8 (a) Factorise fully $3x^2 - 12$

[2 marks]

$3(x^2 - 4)$

$3(x+2)(x-2)$

Answer $3(x+2)(x-2)$

8 (b) Factorise $5x^2 + 4xy - 12y^2$

[3 marks]

$(\square x \pm \square y)(\square x \pm \square y)$

2 numbers that × to make $5 \times -12 = -60$ & add to make 4

10, −6

$(5x+10y)(5x-6y)$

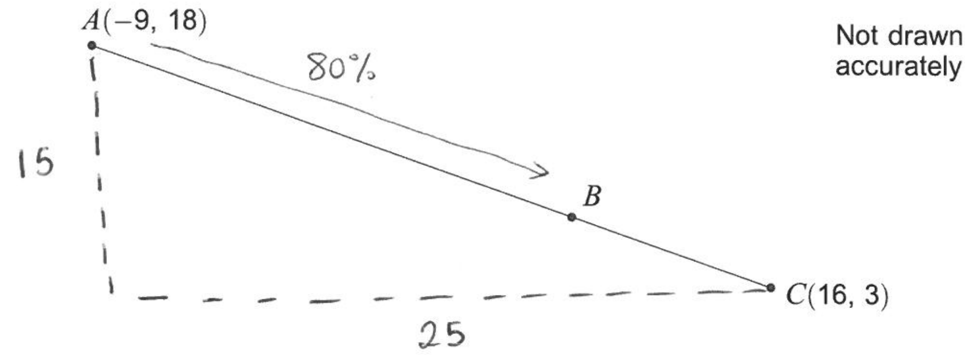
$\downarrow$

$(x+2y)(5x-6y)$

Answer $(x+2y)(5x-6y)$

Do not write outside the box

9 ABC is a straight line.
BC is 20% of AC.



Work out the coordinates of B.

[4 marks]

80% of 15 = 12   80% of 25 = 20

12 down , 20 right

$(-9+20,\ 18-12) = (11, 6)$

Answer (11 , 6)

Turn over for the next question

9

Do not write outside the box

10 Rationalise the denominator of $\dfrac{8}{3-\sqrt{5}}$

Give your answer in the form $a + b\sqrt{5}$ where a and b are integers.

[3 marks]

$$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{8(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{24+8\sqrt{5}}{4}$$

$$= 6 + 2\sqrt{5}$$

$$8(3+\sqrt{5}) = 24 + 8\sqrt{5}$$

$$(3-\sqrt{5})(3+\sqrt{5}) = 9 + \cancel{3\sqrt{5}} - \cancel{3\sqrt{5}} - \sqrt{25}$$

$$= 9 - 5 = 4$$

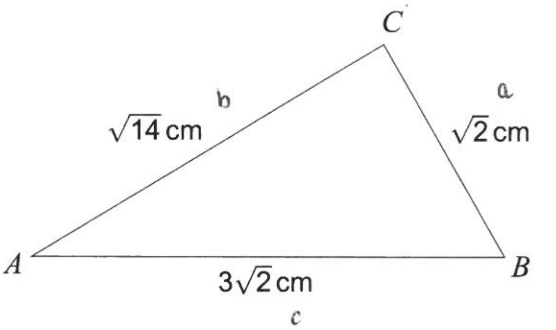
Answer $6 + 2\sqrt{5}$

Do not write outside the box

11 (a) Here is triangle ABC.

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$



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Show that angle $B = 60°$

[3 marks]

$\cos B = \dfrac{(\sqrt{2})^2 + (3\sqrt{2})^2 - (\sqrt{14})^2}{2(\sqrt{2})(3\sqrt{2})}$ $\quad (3\sqrt{2})^2 = 3 \times \sqrt{2} \times 3 \times \sqrt{2} = 18$

$\cos B = \dfrac{2 + 18 - 14}{6\sqrt{4}}$

$\cos B = \dfrac{6}{12} = \dfrac{1}{2}$

$B = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

11 (b) Hence work out the area of triangle ABC.

[3 marks]

$\frac{1}{2} ab \sin C \Rightarrow \frac{1}{2}(\sqrt{2})(3\sqrt{2}) \sin 60$ $\quad \frac{\sqrt{3}}{2}$

$\left(\frac{1}{2} ac \sin B\right)$ in this case $\quad = \frac{1}{2}(3\sqrt{4})\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}(6)\left(\frac{\sqrt{3}}{2}\right)$

$= 3\left(\frac{\sqrt{3}}{2}\right)$

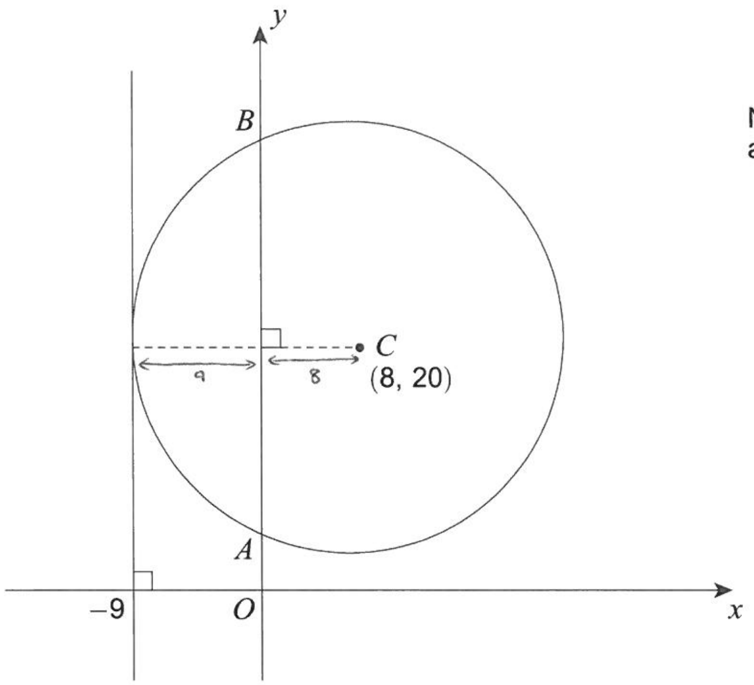
Answer $\dfrac{3\sqrt{3}}{2}$ cm^2

9

Turn over ▶

Do not write outside the box

12 The line $x = -9$ is a tangent to the circle, centre $C(8, 20)$



12 (a) Show that the radius of the circle is 17.

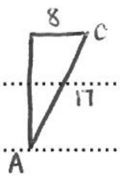
[1 mark]

$8 + 9 = 17$

12 (b) The circle intersects the y-axis at A and B.

Show that the length AB is 30.

[3 marks]

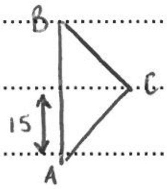


$a^2 + b^2 = c^2$

$8^2 + b^2 = 17^2$

$b^2 = 289 - 64 = 225$

$b = \sqrt{225} = 15$



so $AB = 15 + 15 = 30$

Do not write outside the box

13 A curve has equation $y = x^3 - 3x^2 + 5$

13 (a) Show that the curve has a minimum point when $x = 2$

[4 marks]

$\frac{dy}{dx} = 3x^2 - 6x$

when $x = 2$: $3(2)^2 - 6(2) = 12 - 12 = 0$

$\frac{dy}{dx} = 0 \therefore$ stationary point when $x = 2$

$\frac{d^2y}{dx^2} = 6x - 6$

when $x = 2$: $6(2) - 6 = 12$

$\frac{d^2y}{dx^2} = 12 > 0 \therefore$ minimum point when $x = 2$

13 (b) Show that the tangent at the minimum point meets the curve again when $x = -1$

[3 marks]

Sub $x=2$ into equation to find y:

$y = 2^3 - 3(2)^2 + 5 = 8 - 12 + 5 = 1$

Sub $x = -1$ into equation to see if $y = 1$ again:

$y = (-1)^3 - 3(-1)^2 + 5 = -1 - 3 + 5 = 1$

11

Turn over ▶

Do not write outside the box

14 $(x - a)$ is a factor of $x^3 + 2ax^2 - a^2x - 16$

14 (a) Show that $a = 2$

[2 marks]

$a^3 + 2a(a^2) - a^2(a) - 16 = 0$

$a^3 + 2a^3 - a^3 - 16 = 0$

$2a^3 - 16 = 0$

$2a^3 = 16$

$a^3 = 8$

$a = \sqrt[3]{8} = 2$ as required

14 (b) Solve $x^3 + 4x^2 - 4x - 16 = 0$

[4 marks]

$2 \times ? = -16$ so use factor theorem to find 2 numbers to find -8

$f(1) = 1^3 + 4(1)^2 - 4(1) - 16 = 1 + 4 - 4 - 16 = -15$

$f(-1) = (-1)^3 + 4(-1)^2 - 4(-1) - 16 = -1 + 4 + 4 - 16 = -9$

$f(-2) = (-2)^3 + 4(-2)^2 - 4(-2) - 16 = -8 + 16 + 8 - 16 = 0$

$\therefore (x+2)$ is a factor

$f(-4) = (-4)^3 + 4(-4)^2 - 4(-4) - 16 = -64 + 64 + 16 - 16 = 0$

$\therefore (x+4)$ is a factor

$(x-2)(x+2)(x+4) = 0$

so $x = 2, -2, -4$

Answer $x = 2, -2, -4$

Do not write outside the box

15 Prove that $\dfrac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \tan\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

[3 marks]

$$\frac{\sin\theta - \sin^3\theta}{\cos^3\theta} = \frac{\sin\theta(1-\sin^2\theta)}{\cos^3\theta}$$

$$= \frac{\sin\theta(\cos^2\theta)}{\cos^3\theta}$$

$$= \frac{\sin\theta\,\cancel{\cos\theta}\,\cancel{\cos\theta}}{\cos\theta\,\cancel{\cos\theta}\,\cancel{\cos\theta}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

as required.

Turn over for the next question

9

Do not write outside the box

16 $2x^2 - 2bx + 7a \equiv 2(x - a)^2 + 3$

Work out the **two** possible pairs of values of a and b.

[6 marks]

when $x = 0$: $2(0)^2 - 2b(0) + 7a = 2((0) - a)^2 + 3$

$7a = 2a^2 + 3$

$2a^2 - 7a + 3 = 0$ × to make 6, + to make −7

$(2a - 6)(2a - 1) = 0$

$(a - 3)(2a - 1) = 0$

$a - 3 = 0$ $\quad$ $2a - 1 = 0$

$a = 3$ $\quad$ $2a = 1 \Rightarrow a = \frac{1}{2}$

when $x = 1$: $2(1)^2 - 2b(1) + 7a = 2(1 - a)^2 + 3$

$2 - 2b + 7a = 2(1 - a)^2 + 3$

$-2b = 2(1 - a)^2 - 7a + 1$

$2b = 7a - 1 - 2(1 - a)^2$

~~when~~ sub in $a = 3$:

$2b = 7(3) - 1 - 2(1 - 3)^2$

$2b = 21 - 1 - 8$

$2b = 12$

$b = 6$

sub in $a = \frac{1}{2}$:

$2b = 7(\frac{1}{2}) - 1 - 2(1 - \frac{1}{2})^2$

$2b = \frac{7}{2} - 1 - 2(\frac{1}{4})$

$2b = \frac{5}{2} - \frac{2}{4} = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2$

$b = 1$

$a =$ 3, $b =$ 6

and

$a =$ $\frac{1}{2}$, $b =$ 1

END OF QUESTIONS

6