(Written Solutions)

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Further Mathematics 8360/1

Level 2

Paper 1 Non-Calculator

Wednesday 19 June 2013 1.30 pm to 3.00 pm

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14 – 15	
16	
TOTAL	

For this paper you must have:
• mathematical instruments.
You may **not** use a calculator.
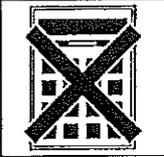
Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Draw diagrams in pencil.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
• Do all rough work in this book. Cross through any work that you do not want to be marked.
• In all calculations, show clearly how you work out your answer.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 70.
• You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Formulae Sheet

**Volume of sphere** $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
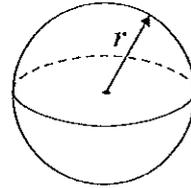
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
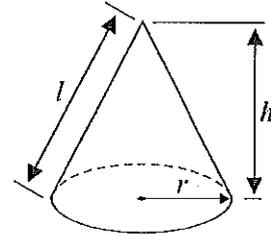
In any triangle ABC

Area of triangle $= \frac{1}{2}ab \sin C$

Sine rule $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\qquad a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
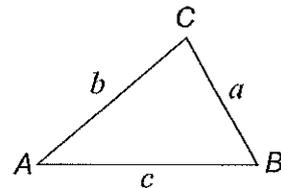
The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \qquad \sin^2 \theta + \cos^2 \theta \equiv 1$$

Answer **all** questions in the spaces provided.

1 A curve has gradient function $\dfrac{dy}{dx} = 9 - x^3$

1 (a) Work out the gradient of the curve when $x = -1$

$dy/dx = 9 - (-1)^3$

$= 9 - -1 = 10$

Answer.............. 10 .... *(2 marks)*

dy/dx

1 (b) Work out the value of x where the rate of change of y with respect to x is 1.

$9 - x^3 = 1$ $x = \sqrt[3]{8}$

$+x^3 \Big\{ \ 9 = 1 + x^3$ $= 2$

$-1 \ \Big\} \ 8 = x^3$

Answer.............. 2 .... *(2 marks)*

Turn over for the next question

2 A is $(-4, 3)$ and B is $(2, 11)$

AB is a diameter of the circle.

Not drawn
accurately
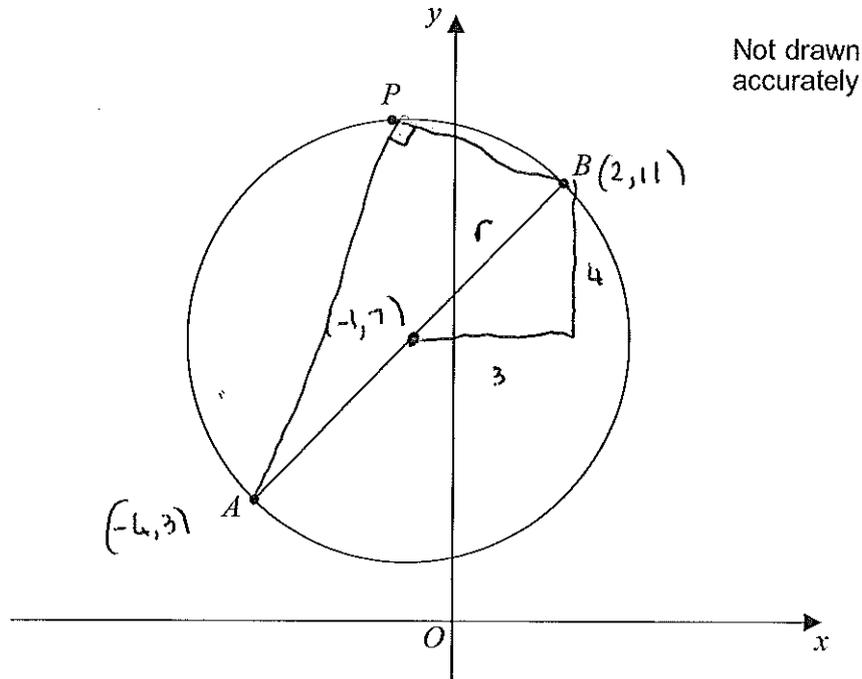
2 (a)   Work out the coordinates of the centre of the circle.

Midpoint of AB

$\boxed{x}$ $\dfrac{-4+2}{2} = \dfrac{-2}{2} = -1$

$\boxed{y}$ $\dfrac{3+11}{2} = \dfrac{14}{2} = 7$

Centre = (.......-1......., ...7.......)

(2 marks)

2 (b) Work out the radius of the circle.

$$r^2 = 3^2 + 4^2$$

$$r^2 = 9 + 16 = 25$$

$$r = \sqrt{25} = 5$$

Radius = ... 5 .... *(2 marks)*

2 (c) Write down the equation of the circle.

Answer ......$(x+1)^2 + (y-7)^2 = 25$...... *(1 mark)*

2 (d) P is another point on the circle.
The gradient of the line AP is 2.

angles in semi-circle = 90°

Write down the gradient of the line PB.

Answer......$-\frac{1}{2}$...... *(1 mark)*

Turn over for the next question

6

3 ABC is a right-angled triangle.
P is a point on AB.
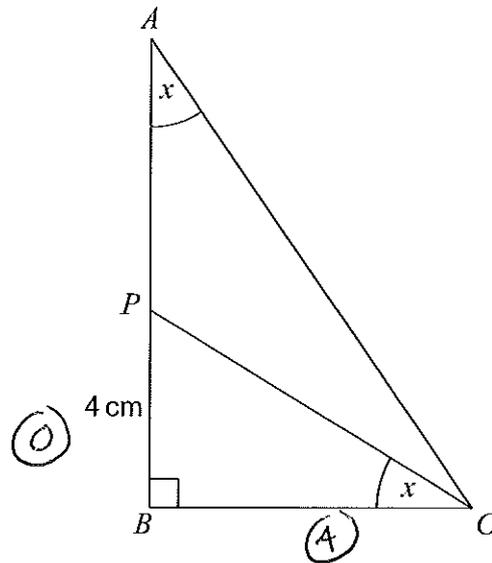
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$BP = 4\,\text{cm}$ and $\tan x = \dfrac{2}{3}$

3 (a)   Work out the length of BC.

$\text{Tan}\; x = \text{opp}/\text{Adj}$

$\rightarrow \quad 2/3 = 4/BC$

$\times BC \quad \{ \quad 2/3\,(BC) = 4$

$\div\, 2/3 \quad \} \quad BC = 4 \div 2/3 = 4 \times \dfrac{3}{2} = 12/2 = 6$

Answer .... cm *(2 marks)*

⑥

3 (b)   Work out the length of AP.

$\text{Tan}\; x = \text{opp}/\text{Adj}$

$\rightarrow \quad 2/3 = 6/AB$

$\rightarrow \quad AB = 9$

$AP = AB - 4$

$= 9 - 4 = 5$

Answer .... 5 .... cm *(3 marks)*

4 Solve $\sqrt{(33 + \sqrt{x})} = 6$

$33 + \sqrt{x} = 36$

$\sqrt{x} = 3$

$x = 9$
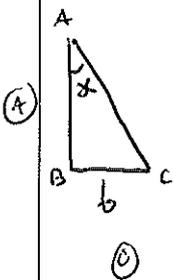
$x = $ .... 9 .... (3 marks)

5 (a) Show that $(x+7)^2 - (x-3)^2$ simplifies to $20(x+2)$

$[(x+7)(x+7)] - [(x-3)(x-3)]$

$x^2 + 14x + 49 - [x^2 - 6x + 9]$

$20x + 40$

$= 20(x+2)$

(3 marks)

5 (b) Hence, or otherwise, work out $107^2 - 97^2$

$(x+7)^2 - (x-3)^2 \Rightarrow x = 100$

$\therefore = 20(100+2) = 20 \times 102$

Answer .... $= 2040$ .... (2 marks)

6 Simplify $(3xy^5)^4$

$$3^4 x^4 y^{20}$$

Answer $81x^4y^{20}$ *(2 marks)*

7 Expand and simplify $(y^2 - 5y + 2)(2y - 3)$

$$2y^3 - 3y^2 - 10y^2 + 15y + 4y - 6$$

$$2y^3 - 13y^2 + 19y - 6$$

Answer .... *(3 marks)*

8 A curve has equation $y = x^4 - 5x^2 + 9$

8 (a) Work out $\dfrac{dy}{dx}$

$$\dfrac{dy}{dx} = \underline{\quad 4x^3 \quad -10x \quad} \qquad \text{(2 marks)}$$

8 (b) Work out the equation of the tangent to the curve at the point where $x = 2$

Give your answer in the form $y = mx + c$

$\bullet$ Gradient: $x = 2, \quad \frac{dy}{dx} = 4(2)^3 - 10(2)$

$= 4(8) - 20 = 12$

$\bullet$ Find y: $x = 2, \quad y = (2)^4 - 5(2)^2 + 9$

$= 16 - 20 + 9 = 5$

$x_1 = 2$ $\qquad y - y_1 = m(x - x_1)$

$y_1 = 5$

$m = 12$ $\qquad y - 5 = 12(x - 2)$

$y - 5 = 12x - 24$

$y = 12x - 19$

Answer.. (4 marks)

Turn over ▷

9 Solve $x^2 + 6x + 7 = 0$

Give your answer in the form $a \pm \sqrt{b}$, where a and b are integers.

Complete the square!

$(x+3)^2 - 9 + 7 = 0$

$(x+3)^2 - 2 = 0$

$+2 \quad (x+3)^2 = 2$

$\sqrt{} \quad x+3 = \pm\sqrt{2}$

$-3 \quad x = -3 \pm \sqrt{2}$

Answer.. *(4 marks)*

10 Make x the subject of the formula $\dfrac{a + 2x}{a - x} = n$

$\times (a-x) \quad a + 2x = n(a-x)$

$a + 2x = an - nx$

$-a \quad 2x = an - a - nx$

$+nx \quad 2x + nx = an - a$

Factorise $\quad x(2 + n) = an - a$

$\div (2+n) \quad x = \dfrac{an - a}{2 + n}$

Answer.. *(4 marks)*

11   Here is a sketch of a cubic function $y = f(x)$
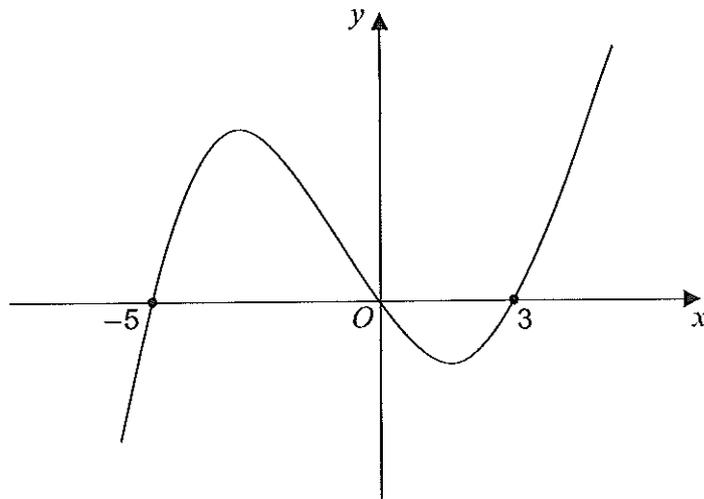
11 (a)   Use the sketch to write down the **three** linear factors of $f(x)$.

Answer $(x + 5)$, x , $(x - 3)$ *(2 marks)*

11 (b)   You are given that $f(x) = x^3 + bx^2 + cx$

Work out the values of b and c.

$f(x) = x(x + 5)(x - 3)$

$= x[x^2 - 3x + 5x - 15]$

$= x[x^2 + 2x - 15]$

$= x^3 + 2x^2 - 15x$

$b =$ 2 , $c =$ -15 *(2 marks)*

Turn over ▷

12 Work out **all** solutions for x and y if $\begin{pmatrix} x & 3 \\ 1 & y \end{pmatrix}\begin{pmatrix} x \\ -4 \end{pmatrix} = \begin{pmatrix} 4x \\ 8 \end{pmatrix}$

$$\begin{pmatrix} x \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} x & 3 \\ 1 & y \end{pmatrix}\begin{pmatrix} 4x \\ 8 \end{pmatrix}$$

① $x^2 - 12 = 4x$

② $x - 4y = 8$

→ $x = 8 + 4y$ or $x - 8 = 4y$

→ $y = \dfrac{x-8}{4}$ ⓧ

① $x^2 - 12 = 4x$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6$ $x = -2$

② $y = \dfrac{x-8}{4}$ $y = \dfrac{-2-8}{4} = \dfrac{-10}{4} = -2.5$

→ $y = \dfrac{6-8}{4}$

$= -\dfrac{1}{2}$

Answer.. *(5 marks)*

$x = 6, \; y = -\dfrac{1}{2}$

AND

$x = -2, \; y = -2.5$

13   Solve $y(\sqrt{3} - 1) = 8$

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

$$\div (\sqrt{3}-1) \quad y = \frac{8}{\sqrt{3}-1}$$

RATIONALISE
$$= \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{8\sqrt{3} + 8}{3 + \sqrt{3} - \sqrt{3} - 1} = \frac{8\sqrt{3} + 8}{2}$$

$$= 4\sqrt{3} + 4$$

$y = \underline{\quad 4 + 4\sqrt{3} \quad}$ *(4 marks)*

Turn over for the next question

9

Turn over ▶

14 ABP and ADQ are tangents to the circle, centre O.

C lies on the circumference of the circle.
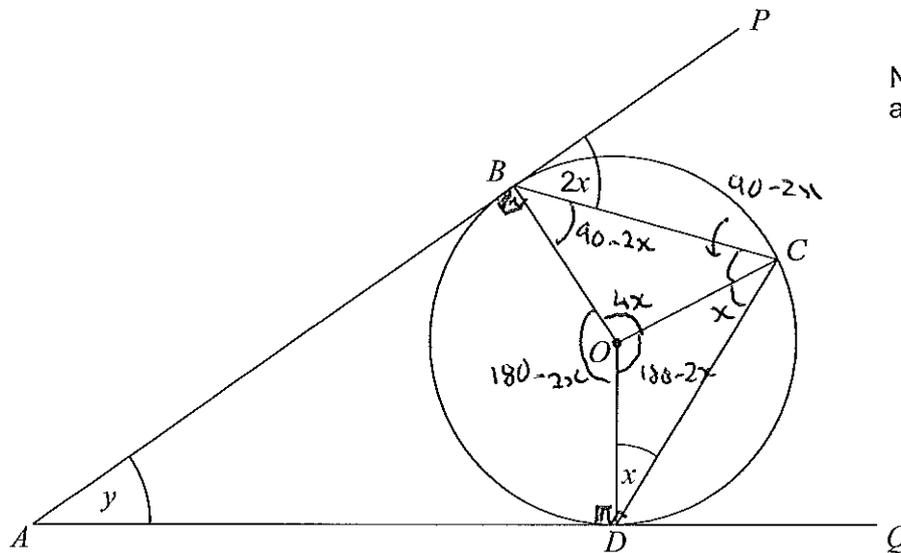
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Prove that $y = 2x$
Give reasons for any statements you make.

$PBO = 90°$ (tangent meets radius at $90°$)

$\therefore CBO = 90 - 2x$

$OCB = 90 - 2x$ (isosceles triangle)

$\therefore COB = 180 - (90 - 2x) - (90 - 2x) = 4x$ (angles in a $\triangle$)

$OCB = x$ (isosceles triangle)

$\therefore COD = 180 - 2x$ (angles in a triangle add to $180°$)

$\therefore BOD = 360 - 4x - (180 - 2x) = 180 - 2x$
        (angles round a point $= 360°$)

$ABO = ADO = 90°$ (tangent meets radius at $90°$)

$\therefore y = 360 - (180 - 2x) - 90 - 90$

$= 2x$ (angles round a point $= 360°$)

....

....

....

....

....

....

(6 marks)

15   Express $2x^2 - 12x - 7$ in the form $a(x+b)^2 + c$

$$= 2\left[x^2 - 6x - 3.5\right]$$

$$= 2\left[(x-3)^2 - 9 - 3.5\right]$$

$$= 2\left[(x-3)^2 - 12.5\right]$$

$$= 2(x-3)^2 - 25$$

Answer.. *(4 marks)*

Turn over for the next question

16   Solve $x^{-\frac{2}{3}} = 7\frac{1}{9}$

Write your answer as a proper fraction.

$$7\frac{1}{9} = \frac{64}{9} \; , \qquad x^{-\frac{2}{3}} = \frac{1}{x^{2/3}}$$

$$\frac{1}{x^{3/3}} = \frac{64}{9}$$

$\times x^{2/3} \left\{ \rule{0pt}{12pt}\right.$ $1 = \frac{64\,x^{2/3}}{9}$

$\times 9 \left\{ \rule{0pt}{10pt}\right.$ $9 = 64\,x^{2/3}$

$\div 64 \left\{ \rule{0pt}{10pt}\right.$ $9/64 = x^{2/3}$

$\sqrt{} \left\{ \rule{0pt}{10pt}\right.$ $\pm\, 3/8 = x^{1/3}$

$3 \left\{ \rule{0pt}{10pt}\right.$ $\pm\, 27/512 = x$

$x = \underline{\quad 27/512 \quad}$ or $\underline{\; -27/512 \;}$ *(5 marks)*

END OF QUESTIONS