Level 2 Certificate in Further Mathematics

June 2013

Paper 1 8360/1

Final

Mark Scheme

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Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Q	Answer	Mark	Comments
1a	$9 - (-1)^3$ or $9 - -1$	M1	
	10	A1	SC1 for 8
1b	$9 - x^3 = 1$ or $x^3 = 8$	M1	oe
	2	A1	SC1 for −2

2a	$\left(\frac{-4+2}{2}, \frac{3+11}{2}\right)$	M1	oe
	(–1, 7)	A1	SC1 for one coordinate correct
2b	$(r^2 =)\ 3^2 + 4^2$ or $(r^2 =)\ 25$ or $(d^2 =)\ 6^2 + 8^2$ or $(d^2 =)\ 100$	M1	oe ft their centre
	$(r = 5)$	A1ft	SC1 for 10
2c	$(x + 1)^2 + (y - 7)^2 = 25$	B1ft	oe ft their centre and radius
2d	$-\frac{1}{2}$ or -0.5	B1	Accept $\frac{-1}{2}$, $\frac{1}{-2}$ or $-.5$

3a	$\frac{4}{BC} = \frac{2}{3}$	M1	oe
	$(BC =)\ 6$	A1	
3b	$\frac{\text{their } 6}{AB} = \frac{2}{3}$	M1	oe eg follow through their 6 using a similar triangles/scale factor method
	$(AB =)\ 9$	A1ft	
	$(AP =)\ 5$	A1ft	

4	6^2 (= 36)	M1	
	$\sqrt{x}$ = their 36 − 33	M1	oe
	9	A1	

Q	Answer	Mark	Comments
5a	$(x + 7 + x - 3)(x + 7 - x + 3)$	M1	Allow one sign error
	$(2x + 4) \times 10$	A1	oe
	$10 \times 2(x + 2)$ or $20x + 40$	A1	
	Alternative method		
	$x^2 + 7x + 7x + 49$ $(-)\ x^2 - 3x - 3x + 9$	M1	oe Allow one error
	$x^2 + 7x + 7x + 49$ $-\ (x^2 - 3x - 3x + 9)$	A1	oe All terms correct
	$x^2 + 7x + 7x + 49$ $-\ x^2 + 3x + 3x - 9 = 20x + 40$	A1	oe
5b	$20(100 + 2)$ or 204×10	M1	11449 or 9409 seen
	2040	A1	

6	$81x^4y^{20}$	B2	B1 for two components correct

7	$2y^3 - 10y^2 + 4y - 3y^2 + 15y - 6$	M1	Must have at least five terms with at least four correct
	$2y^3 - 10y^2 + 4y - 3y^2 + 15y - 6$	A1	
	$2y^3 - 13y^2 + 19y - 6$	A1ft	ft from M1 A0

8a	$4x^3 - 10x\ (+ 0)$	B2	Accept $4 \times x^3 - 10 \times x$ B1 for $4x^3$ or $4 \times x^3$ B1 for $-10x$ or $-10 \times x$ $4x^3 - 10x$ + something extra scores B1 eg $4x^3 - 10x + 9$
8b	(when $x = 2$) (gradient =) 12	B1ft	ft their answer to (a)
	(when $x = 2$) (y =) 5	B1	
	their 5 = their 12 × 2 + c or $y - 5 = 12(x - 2)$	M1	oe
	$y = 12x - 19$	A1ft	ft their m and their 5

Q	Answer	Mark	Comments
9	$x = \frac{-6 \pm \sqrt{\{6^2 - 4(1)(7)\}}}{2(1)}$	M1	Allow one substitution or sign error
	$x = \frac{-6 \pm \sqrt{8}}{2}$	A1	
	$\sqrt{8} = 2\sqrt{2}$	A1ft	For simplifying their surd (if possible to do so)
	$x = -3 \pm \sqrt{2}$	A1	
	Alternative method		
	$(x + 3)^2$	M1	
	$(x + 3)^2 - 9 + 7$ (= 0) or $(x + 3)^2 - 2$ (= 0) or $(x + 3)^2 = 2$	M1dep	
	$x + 3 = (\pm)\sqrt{2}$	M1dep	
	$x = -3 \pm \sqrt{2}$	A1	

10	$a + 2x = n(a - x)$	M1	
	$a + 2x = na - nx$	A1	oe
	$nx + 2x = na - a$ or $x(n + 2) = na - a$ or $x(n + 2) = a(n - 1)$	M1	oe for collecting the x terms on one side and the other terms on the opposite side Allow one sign error
	$x = \frac{a(n-1)}{n+2}$ or $x = \frac{na - a}{n+2}$	A1	oe

11a	$x + 5$, x and $x - 3$	B2	Any order B1 for any two of x, $x + 5$ or $x - 3$ B1 for x, $x - 5$ **and** $x + 3$
11b	$f(x) = x(x + 5)(x - 3)$	M1	ft their three factors
	$f(x) = x^3 + 2x^2 - 15x$ or $b = 2$ and $c = -15$	A1ft	ft their three factors, one of which **must** be x
	Alternative method		
	$(-5)^3 + b(-5)^2 + c(-5) = 0$ and $(3)^3 + b(3)^2 + c(3) = 0$	M1	oe eg $25b - 5c = 125$ and $9b + 3c = -27$ Allow one error in total
	$b = 2$ and $c = -15$ or $f(x) = x^3 + 2x^2 - 15x$	A1	

Q	Answer	Mark	Comments
12	$x^2 - 12$ or $x - 4y$	M1	
	$x^2 - 12 = 4x$ and $x - 4y = 8$	M1	These can still be in matrix form
	$(x - 6)(x + 2)$ (= 0)	A1	$x = \dfrac{-(-4) \pm \sqrt{\{(-4)^2 - 4 \times 1 \times (-12)\}}}{2(1)}$
	$x = 6$ and -2	A1ft	ft their quadratic if possible or $x = 6$ and $y = -\frac{1}{2}$
	$y = -\frac{1}{2}$ and $-2\frac{1}{2}$ or $-\frac{5}{2}$	A1ft	ft from their x values or $x = -2$ and $y = -2\frac{1}{2}$
	Alternative method		
	$x^2 - 12$ or $x - 4y$	M1	
	$x^2 - 12 = 4x$ and $x - 4y = 8$	M1	These can still be in matrix form
	$(4)(2y + 5)(2y + 1)$ (= 0) or $(8y + 20)(2y + 1)$ (= 0) or $(2y + 5)(8y + 4)$ (= 0)	A1	$y = \dfrac{-12 \pm \sqrt{12^2 - 4 \times 4 \times 5}}{2(4)}$ or $y = \dfrac{-48 \pm \sqrt{48^2 - 4 \times 16 \times 20}}{2(16)}$
	$y = -\frac{1}{2}$ and $-2\frac{1}{2}$ or $-\frac{5}{2}$	A1ft	ft their quadratic if possible or $y = -\frac{1}{2}$ and $x = 6$
	$x = 6$ and -2	A1ft	ft from their y values or $y = -2\frac{1}{2}$ and $x = -2$

13	$(y =) \dfrac{8}{\sqrt{3} - 1}$	M1	oe
	$(y =) \dfrac{8}{(\sqrt{3} - 1)} \times \dfrac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$	M1	
	$(y =) \dfrac{8\sqrt{3} + 8}{3 - 1}$	A1	
	$(y =)\ 4\sqrt{3} + 4$	A1	$2\sqrt{3} + 2$ from $\dfrac{8\sqrt{3} + 8}{3 + 1}$ and $\sqrt{3} + 1$ from $\dfrac{8\sqrt{3} + 8}{9 - 1}$ both score SC3
	Alternative method 1		
	$y\sqrt{3} = 8 + y$ and $3y^2 = 64 + 16y + y^2$	M1	Re-arrange and square both sides, Allow one error
	$y^2 - 8y - 32 = 0$ or $2y^2 - 16y - 64 = 0$ **and** $(y =) \dfrac{8 \pm \sqrt{\{8^2 - 4(1)(-32)\}}}{2(1)}$ or $(y =) \dfrac{16 \pm \sqrt{\{16^2 - 4(2)(-64)\}}}{2(2)}$	M1	Allow one substitution or sign error
	$(y =)\ 4 \pm 4\sqrt{3}$	A1	
	$(y =)\ 4 + 4\sqrt{3}$	A1	Solution with negative sign must be discounted
	Alternative method 2		
	$(a + b\sqrt{3})(\sqrt{3} - 1)\ (=8)$	M1	
	$a\sqrt{3} + 3b - a - b\sqrt{3}$	M1	
	$a = b$	A1	
	$(y =)\ 4 + 4\sqrt{3}$	A1	

Q	Answer	Mark	Comments
14	Join *BD*		
	Angle *BDC* = $2x$	M1	Alternate segment theorem
	Angle *BDO* = x	M1	
	Angle *DBO* = x	M1	Isosceles triangle *BOD*
	Angle *BOD* = 180 − $2x$	M1	Angle sum of triangle *BOD*
	$y = 360 - 90 - 90 - (180 - 2x)$ $y = 2x$	A1	Angle sum of quadrilateral *ABOD* $y = 2x$ clearly shown from simplification
	Must have at least two different reasons stated in the proof	B1ft	
	Alternative method 1		
	Angle *OBC* = $90 - 2x$	M1	Tangent-radius property
	Angle *OCB* = $90 - 2x$	M1	Isosceles ∆ *OBC*
	Angle *OCD*= x	M1	Isosceles ∆ *OCD*
	Angle *BCD* = $90 - 2x + x$ = $90 - x$ hence Angle *BOD* = $180 - 2x$	M1	Angle at centre = 2 × angle at circumference
	$y = 360 - 90 - 90 - (180 - 2x)$ $y = 2x$	A1	Angle sum of quadrilateral *ABOD* $y = 2x$ clearly shown from simplification
	Must have at least two different reasons stated in the proof	B1ft	

Q	Answer	Mark	Comments
14	**Alternative method 2**		
	Angle *OBC* = 90 − 2x	M1	Tangent-radius property
	Angle *OCB* = 90 − 2x	M1	Isosceles ∆ *OBC*
	Angle *OCD*= x	M1	Isosceles ∆ *OCD*
	Angle *BCD* = 90 − 2x + x = 90 − x hence Angle *BOD* = 180 − 2x	M1	Angle at centre = 2 × angle at circumference
	Angle *BOD* = 360 − 90 − 90 − y = 180 − y hence y = 2x	A1	Angle sum of quadrilateral *ABOD* y = 2x clearly shown from comparison
	Must have at least two different reasons stated in the proof	B1ft	
	Alternative method 3		
	Angle *OBC* = 90 − 2x	M1	Tangent-radius property
	Angle *OCB* = 90 − 2x	M1	Isosceles ∆ *OBC*
	Angle *OCD*= x	M1	Isosceles ∆ *OCD*
	Angle *BCD* = 90 − 2x + x = 90 − x	M1	
	y = 360 − 90 − (90 − 2x) − (90 − x) − x − 90 hence y = 2x	A1	Angle sum of quadrilateral *ABCD* y = 2x clearly shown from simplification
	Must have at least two different reasons stated in the proof	B1ft	
	Alternative method 4		
	Angle *BOD* = 180 − y	M1	Angle sum of quadrilateral *ABOD*
	Angle *OCD* = x	M1	Isosceles ∆ *OCD*
	Angle *OBC* = 90 − 2x	M1	Tangent-radius property
	Angle *BCO*= 90 − 2x hence Angle *BOD* reflex = 360 − (90 − 2x) − (90 − 2x) − x − x = 180 + 2x	M1	Isosceles ∆ *OBC* Angle sum of quadrilateral *BODC* ... this can also come from Angle *BOC* (4x) + Angle *DOC* (180 − 2x)
	180 − y + 180 + 2x = 360 hence y = 2x	A1	Angles round a point y = 2x clearly shown from rearranging
	Must have at least two different reasons stated in the proof	B1ft	

Q	Answer	Mark	Comments
15	$2(x^2 - 6x)$	M1	
	$2(x - 3)^2$	M1dep	
	$2((x - 3)^2 - 9\ (-3.5))$ or $2(x - 3)^2 - 18\ (-7)$	M1dep	
	$2(x - 3)^2 - 25$	A1	
	Alternative method		
	$x^2 + bx + bx + b^2$	M1	
	$a = 2$	M1	
	$-12 = 2ab$ or $-12 = 4b$ **and** $-7 = ab^2 + c$ or $-7 = 2b^2 + c$	M1	
	$2(x - 3)^2 - 25$	A1	

16	$7\frac{1}{9} = \frac{64}{9}$	B1	Can be done at any stage
	$x^{\frac{2}{3}} = \frac{9}{64}$ or $(\sqrt[3]{x})^2 = \frac{9}{64}$ or $\sqrt[3]{(x^2)} = \frac{9}{64}$	M1	oe or the reciprocals $1 \div x^{\frac{2}{3}} = \frac{64}{9}$ or $\frac{1}{(\sqrt[3]{x})^2} = \frac{64}{9}$ or $\frac{1}{\sqrt[3]{(x^2)}} = \frac{64}{9}$
	$x = \left(\frac{9}{64}\right)^{\frac{3}{2}}$ or $\sqrt[3]{x} = \sqrt{\left(\frac{9}{64}\right)}$ or $x^2 = \left(\frac{9}{64}\right)^3$	M1	oe or the reciprocals $\frac{1}{x} = \left(\frac{64}{9}\right)^{3/2}$ or $\frac{1}{\sqrt[3]{x}} = \sqrt{\left(\frac{64}{9}\right)}$ or $\frac{1}{x^2} = \left(\frac{64}{9}\right)^3$
	$x = \left(\frac{3}{8}\right)^3$ or $\frac{1}{x} = \left(\frac{8}{3}\right)^3$	A1	
	$(x =) \pm \frac{27}{512}$ or $\frac{27}{512}$ or $-\frac{27}{512}$	A1	SC3 for $\frac{512}{27}$