MR BARTON'S ANSWERS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
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22 – 23	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
January 2013

Further Mathematics 8360/2

Level 2

Paper 2 Calculator

Tuesday 29 January 2013 1.30 pm to 3.30 pm

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

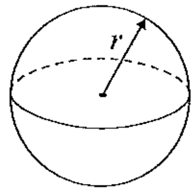
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

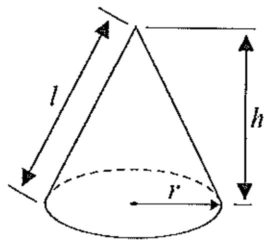
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



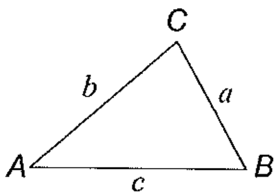
In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

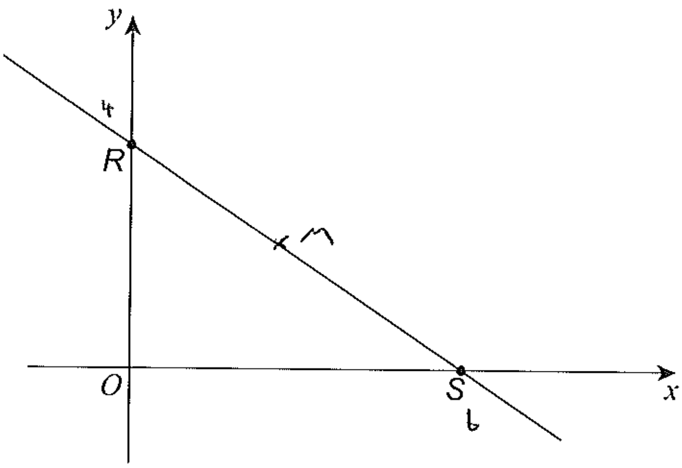
Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A sketch of $2x + 3y = 12$ is shown.



1 (a) Work out the coordinates of *R*. $x = 0$

$\Rightarrow 2(0) + 3y = 12 \Rightarrow 3y = 12 \Rightarrow y = 4$

Answer (0 , 4) *(1 mark)*

1 (b) Work out the coordinates of the midpoint of *RS*.

Need S : $y = 0 \Rightarrow 2x + 3(0) = 12$

$\Rightarrow 2x = 12 \Rightarrow x = 6$

Answer (3 , 2) *(2 marks)*

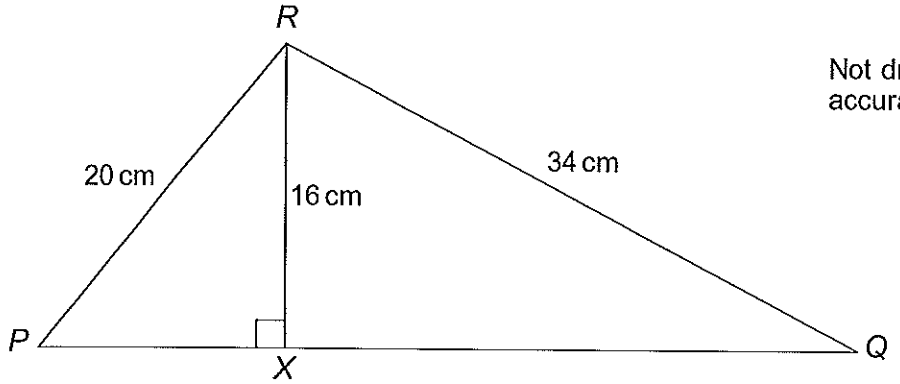
midpoint $\boxed{x} = 6/2 = 3$

$\boxed{y} = 4/2 = 2$

3

Do not write outside the box

2 In triangle PQR, X is a point on PQ.
RX is perpendicular to PQ.



Not drawn accurately

Work out the ratio $PX : XQ$
Give your answer in its simplest form.

$\boxed{PX}$ $\sqrt{20^2 - 16^2} = \sqrt{144} = 12$

$\boxed{XQ}$ $\sqrt{34^2 - 16^2} = \sqrt{900} = 30$

RATIO = 12 : 30

= 2 : 5

Answer 2 : 5 *(4 marks)*

Do not write outside the box

3 Solve $5d - 3 > d + 17$

$-d$ $4d - 3 > 17$

$+3$ $4d > 20$

$\div 4$ $d > 5$

Answer $d > 5$ *(2 marks)*

4 Match each statement with an equation.
You will **not** use all of the equations.

One has been done for you.

Statement
A curve passing through (0, 0)
A curve passing through (1, 0)
A circle centre (2, −1)
A circle passing through (3, 1)

$x=1,\ y=0$

$x=3,\ y=1$

Equation
$x^2 + y^2 = 10$
$(x+2)^2 + (y-1)^2 = 1$
$y = x^3$
$y = x^3 + x - 2$
$(x-2)^2 + (y+1)^2 = 1$
$y = x^2 - 2$

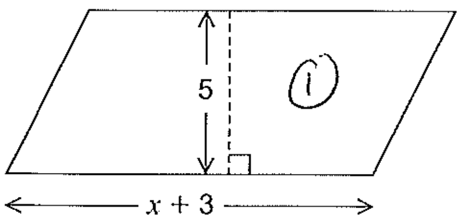
(3 marks)

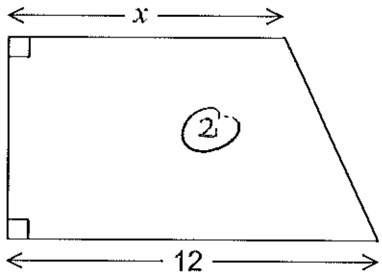
9

Turn over ▶

Do not write outside the box

5 A parallelogram and a trapezium are shown.
All lengths are in centimetres.





Not drawn accurately

The area of the parallelogram is equal to the area of the trapezium.

Work out the value of x.

1 Area $= 5(x+3) = 5x + 15$

2 Area $= \frac{1}{2}(12+x) \times 6 = 3(12+x) = 36 + 3x$

1 = 2 $\rightarrow$ $5x + 15 = 36 + 3x$

$-3x$ $2x + 15 = 36$

-15 $2x = 21$

$\div 2$ $x = 10.5$

$x =$ 10.5 cm *(4 marks)*

Do not write outside the box

6 A function f(x) is defined as

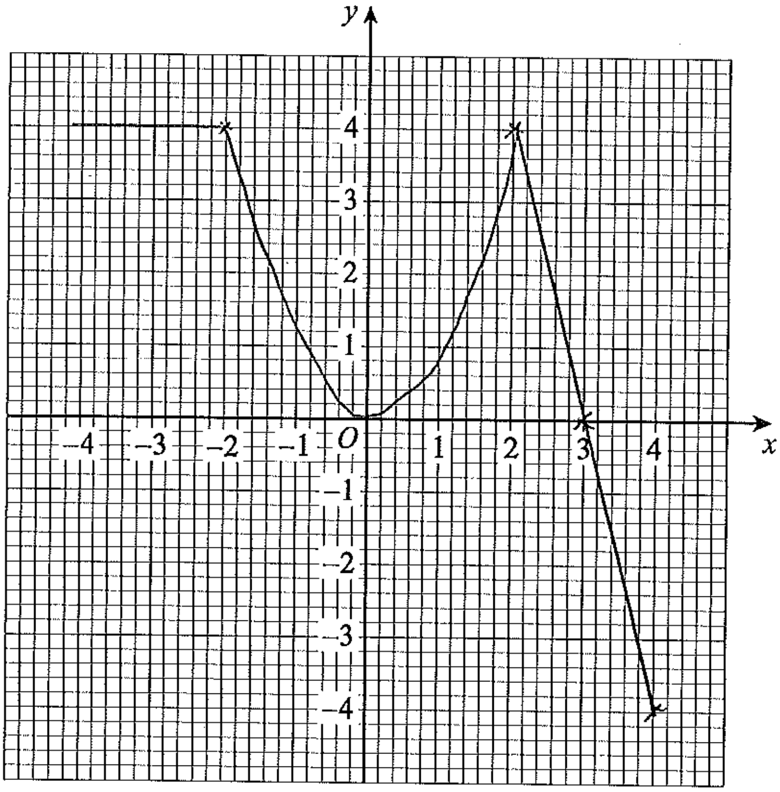
$y = 4$ $\quad f(x) = 4 \qquad x < -2$

$y = x^2$ $\quad = x^2 \qquad -2 \leqslant x \leqslant 2$

$y = 12 - 4x$ $\quad = 12 - 4x \qquad x > 2$

x	2	3	4
y	4	0	-4

6 (a) Draw the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 4$



(3 marks)

6 (b) Use your graph to write down **how many** solutions there are to $f(x) = 3$

crosses $y = 3$, 3 times

Answer .... 3 .... *(1 mark)*

6 (c) Solve $f(x) = -10$

Graph = -10 for 12 - 4x section

$\Rightarrow 12 - 4x = -10$

$+4x$ $\quad 12 = 4x - 10$

$+10$ $\quad 22 = 4x$

$\div 4$ $\quad 5.5 = x$ $\quad x =$ .... 5.5 .... *(2 marks)*

10

Turn over ▶

Do not write outside the box

7 Here are the first four terms of a sequence.

$4a \qquad 9a \qquad 14a \qquad 19a$

The nth term of the sequence is $\dfrac{10n - 2}{3}$

Work out the value of a.

$\boxed{\text{1st term}}$ $n = 1 \rightarrow \dfrac{10(1) - 2}{3} = 8/3$

$\therefore 4a = 8/3$

$\times 3 \quad 12a = 8$

$\div 12 \quad a = 8/12 = 2/3$

$a =$ 2/3 (2 marks)

8 (a) Factorise fully $5m^2 - 20p^2$

$5(m^2 - 4p^2)$ ✓ diff of 2 squares

$5(m + 2p)(m - 2p)$

Answer .... (3 marks)

8 (b) You are given that $p = 15$ and $5m^2 - 20p^2 = 0$

Using your answer to part (a), or otherwise, work out the values of m.

$5m^2 - 20p^2 = 0 \rightarrow 5(m+2p)(m-2p) = 0$

$\div 5 \quad (m + 2p)(m - 2p) = 0$

$p = 15 \quad (m + 30)(m - 30) = 0$

$m = -30$ AND $m = 30$

Answer .... (2 marks)

Do not write outside the box

9 (a) Expand $(x+m)(x+n)$

$x^2 + nx + mx + mn$

Answer .... *(1 mark)*

9 (b) $x^2 + qx + r \equiv (x+m)(x+n)$

Use your answer to part (a) to write q and r in terms of m and n.

sum of xs $\rightarrow$ $q =$ $m + n$

number at end $\rightarrow$ $r =$ mn *(2 marks)*

9 (c) r is an odd integer.

Use your answer to part (b) to explain why q is an even integer.

For r to be an odd integer, both m and n need to be odd integers, as $m \times n$ is odd

$q = m + n$

$= \text{odd} + \text{odd} = \text{even integer}$

$\Rightarrow q$ is even integer

(2 marks)

12

Turn over ▶

Do not write outside the box

10 $S = \dfrac{a}{1-r}$

10 (a) Show that $r = \dfrac{S-a}{S}$

$\times(1-r)$ $S(1-r) = a$

$S - Sr = a$

$+Sr$ $S = a + Sr$

$-a$ $S - a = Sr$

$\div S$ $\dfrac{S-a}{S} = r$

(3 marks)

10 (b) Work out the value of r when $S = 10a$

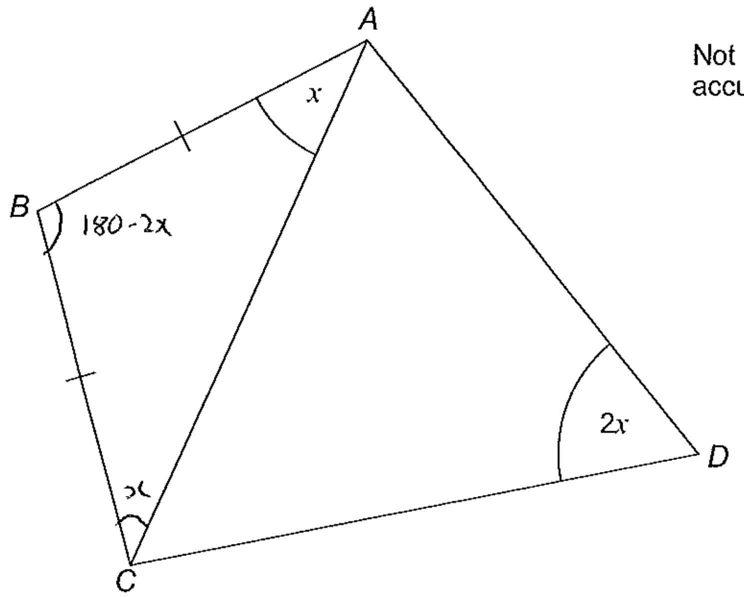
$r = \dfrac{S-a}{S}$

$= \dfrac{10a - a}{10a} = \dfrac{9a}{10a} = 9/10$

$r =$ 9/10 or 0.9 *(2 marks)*

Do not write outside the box

11 In the diagram, $AB = BC$



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Prove that $ABCD$ is a cyclic quadrilateral.
Give reasons for any statements you make.

BCA = x (isosceles triangle)

CBA = 180 - 2x (angles in a triangle add to 180°)

ABC + ADC = 180 - 2x + 2x

= 180

Opposite angles add to 180°

∴ cyclic quadrilateral

(3 marks)

8

Turn over ▶

Do not write outside the box

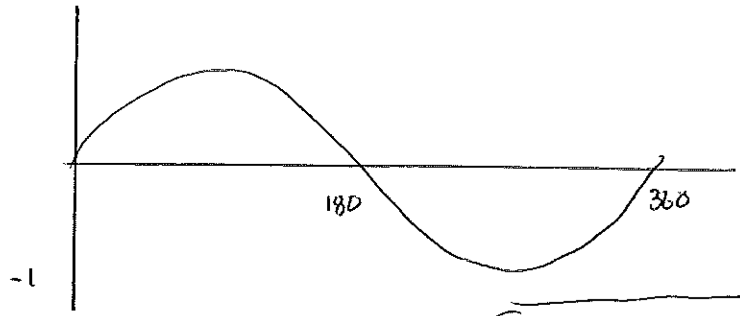
12 $f(x) = \sin x \qquad 180° \leqslant x \leqslant 360°$

$g(x) = \cos x \qquad 0° \leqslant x \leqslant \theta$

12 (a) Calculate the value of f(210°).

Answer sin (210) = -0.5 *(1 mark)*

12 (b) Complete this inequality for the range of $f(x)$.



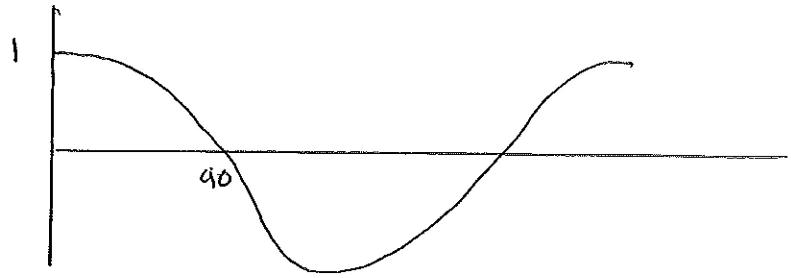
Just 180 - 360

Answer -1 $\leqslant f(x) \leqslant$ 0 *(2 marks)*

12 (c) You are given that $0 \leqslant g(x) \leqslant 1$

Work out the value of θ.

cos must lie between 0 and 1



∴ only happens between 0 and 90

$0 \leqslant x \leqslant \theta$

$\theta =$ 90 degrees *(1 mark)*

Do not write outside the box

13 (a) Show that $\frac{4}{x} + \frac{2}{x-1}$ simplifies to $\frac{6x-4}{x(x-1)}$

$$\frac{4(x-1)}{x(x-1)} + \frac{2x}{x(x-1)} = \frac{4x-4}{x(x-1)} + \frac{2x}{x(x-1)}$$

$$= \frac{6x-4}{x(x-1)}$$

(2 marks)

13 (b) Hence, or otherwise, solve $\frac{4}{x} + \frac{2}{x-1} = 3$

Give your solutions to 3 significant figures.

$$\frac{6x-4}{x(x-1)} = 3$$

$$6x-4 = 3x(x-1)$$

$$6x-4 = 3x^2-3x$$

$$+4 \quad 6x = 3x^2-3x+4$$

$$-6x \quad 0 = 3x^2-9x+4$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad a=3,\ b=-9,\ c=4$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2-4(3)(4)}}{2(3)} \rightarrow x = \frac{9 \pm \sqrt{\cancel{4}33}}{6}$$

Answer 2.46 AND 0.543 *(5 marks)*

2.45742... 0.54257...

11

Turn over ▶

Do not write outside the box

14 The value of x is 50% **more** than the value of t. ①
The value of y is 10% **less** than the value of w. ②

$x = y$

Work out $\frac{t}{w}$

Give your answer as a decimal.

~~tw~~ ① $1.5t = x$

② $0.9w = y$

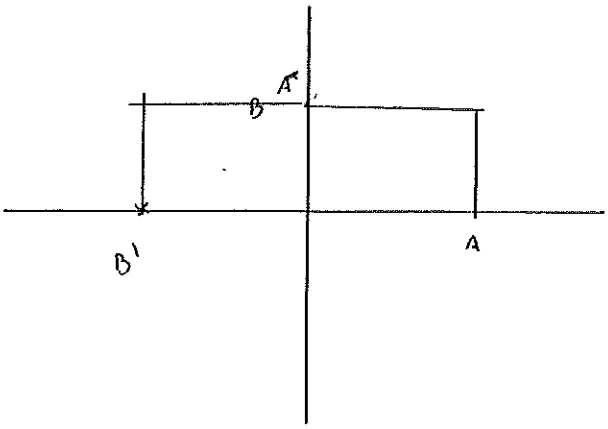
As $x = y \Rightarrow 1.5t = 0.9w$

$\div 1.5$ $t = 0.6w$

$\div w$ $t/w = 0.6$

$\frac{t}{w} =$ 0.6 *(4 marks)*

15 Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



A → (0,1)

B → (-1,0)

= Rotation

90° anti-clockwise

about (0,0)

(3 marks)

Do not write outside the box

16 $y = (x^3 - 1)^2 + (\sqrt{x})^8$

Work out $\frac{dy}{dx}$.

$y = (x^3-1)(x^3-1) \quad + (x^{1/2})^8$

$= x^6 - x^3 - x^3 + 1 \quad + x^4$

$= x^6 - 2x^3 + 1 \quad + x^4$

$dy/dx = 6x^5 - 6x^2 \quad + 4x^3$

$\frac{dy}{dx} = $ .... *(5 marks)*

Turn over for the next question

12

Do not write outside the box

17 ① $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y-axis.

② $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$

Work out the matrix that represents a reflection in the y-axis followed by a reflection in the line $y = x$

Need ② × ①

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Answer $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ *(2 marks)*

Do not write outside the box

18 Express $1 - \tan\theta \sin\theta \cos\theta$ in terms of $\cos\theta$.

$\tan\theta = \sin\theta / \cos\theta$ $\qquad \sin^2\theta + \cos^2\theta = 1$

$\rightarrow \quad 1 - \left(\sin\theta / \cos\theta\right) \sin\theta \cos(\theta)$

$= 1 - \dfrac{\sin\theta \sin\theta \cos\theta}{\cos\theta} = 1 - \sin^2\theta$

$= \cos^2\theta$

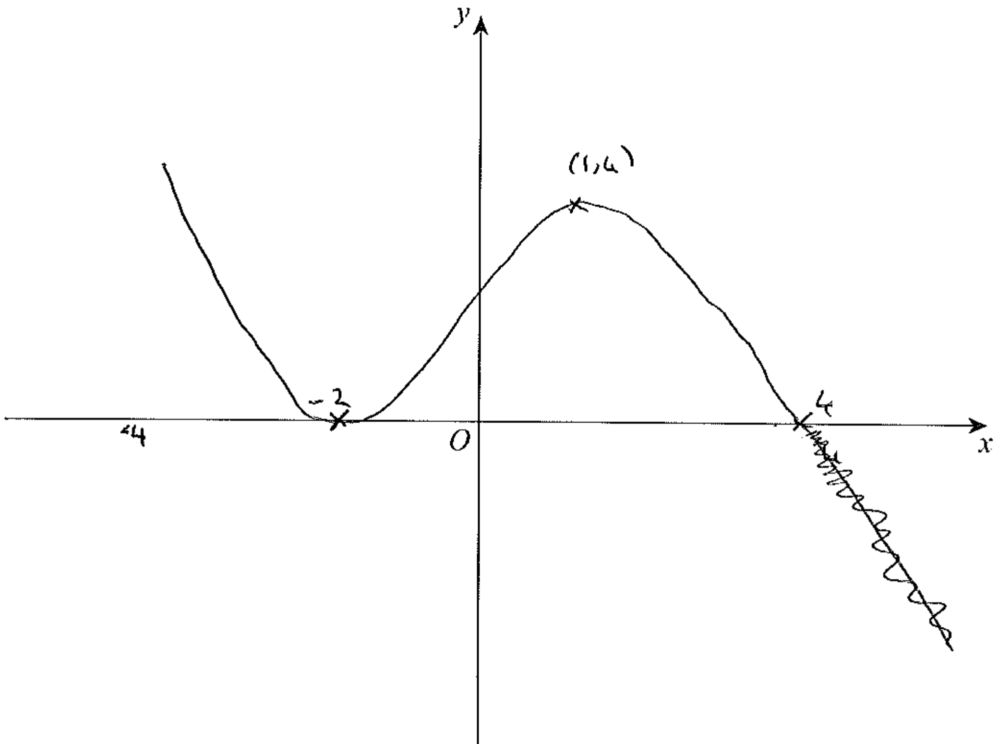
Answer $\cos^2\theta$ *(3 marks)*

19 A cubic function $f(x)$ has domain $-4 \leqslant x \leqslant 4$

The curve $y = f(x)$

- has a minimum point at $(-2, 0)$
- has a maximum point at $(1, 4)$
- meets the x-axis at $(4, 0)$.

Sketch the graph of $y = f(x)$ on these axes.
Label any points where the graph meets the x-axis.



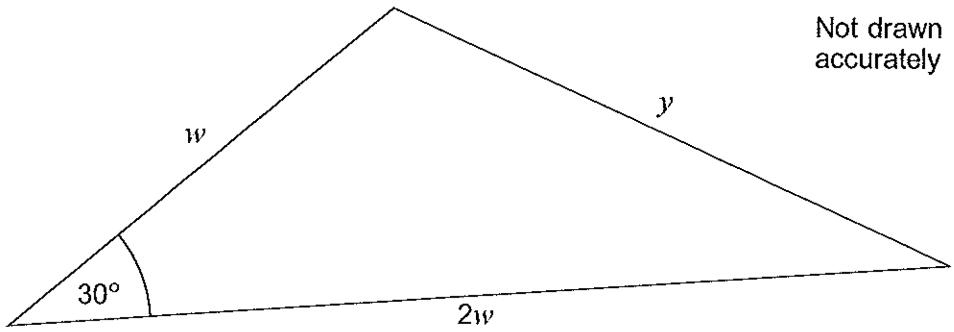
(4 marks)

9

Turn over ▶

Do not write outside the box

20 The area of this triangle is 18 cm^2.



Work out y.

$Area = \frac{1}{2} ab \sin(c)$

$18 = \frac{1}{2}(w)(2w) \sin(30)$

$18 = \frac{1}{2}(2w^2) \frac{1}{2}$

$18 = \frac{1}{2} w^2$

$36 = w^2$

$\rightarrow w = 6$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos(A)$

$y^2 = 12^2 + 6^2 - 2(12)(6)\cos(30)$

$y^2 = 180 - 72\sqrt{3}$

$y = \sqrt{180 - 72\sqrt{3}}$

$= 7.43588\ldots$

$y =$ 7.44 (2dp) cm *(5 marks)*

Do not write outside the box

21 Work out the equation of the normal to the curve $y = x^2 + 4x + 5$ at the point where $x = -3$

$dy/dx = 2x + 4$ $\qquad y = (-3)^2 + 4(-3) + 5 = 2$

when $x = -3 \rightarrow dy/dx = 2(-3) + 4 = -2$

-2 is gradient of tangent, so gradient of normal $= 1/2$

$x_1 = -3 \qquad y - y_1 = m(x - x_1)$

$y_1 = 2 \qquad y - 2 = 1/2(x + 3)$

$m = 1/2 \qquad y - 2 = 1/2 x + 1.5$

$y = 1/2 x + 3.5$

Answer .... *(5 marks)*

22 $f(x) = x^3 + ax^2 + bx + 24$ for all values of x.

Two of the factors of $f(x)$ are $(x - 2)$ and $(x + 3)$. Factor Theorem!

Work out the values of a and b.

$f(2)$ must $= 0 \rightarrow (2)^3 + a(2)^2 + b(2) + 24 = 0$

$\rightarrow 8 + 4a + 2b + 24 = 0$

① $4a + 2b = -32$

$f(-3)$ must $= 0 \rightarrow (-3)^3 + a(-3)^2 + b(-3) + 24 = 0$

$\rightarrow -27 + 9a - 3b + 24 = 0$

② $9a - 3b = 3$

① $\times 3 \rightarrow 12a + 6b = -96$

② $\times 2 \rightarrow 18a - 6b = 6$ (+) $\qquad$ ① $4a + 2b = -32$

$30a = -90 \qquad 4(-3) + 2b = -32$

$\rightarrow a = -3 \qquad -12 + 2b = -32$

$2b = -20$

$b = -10$

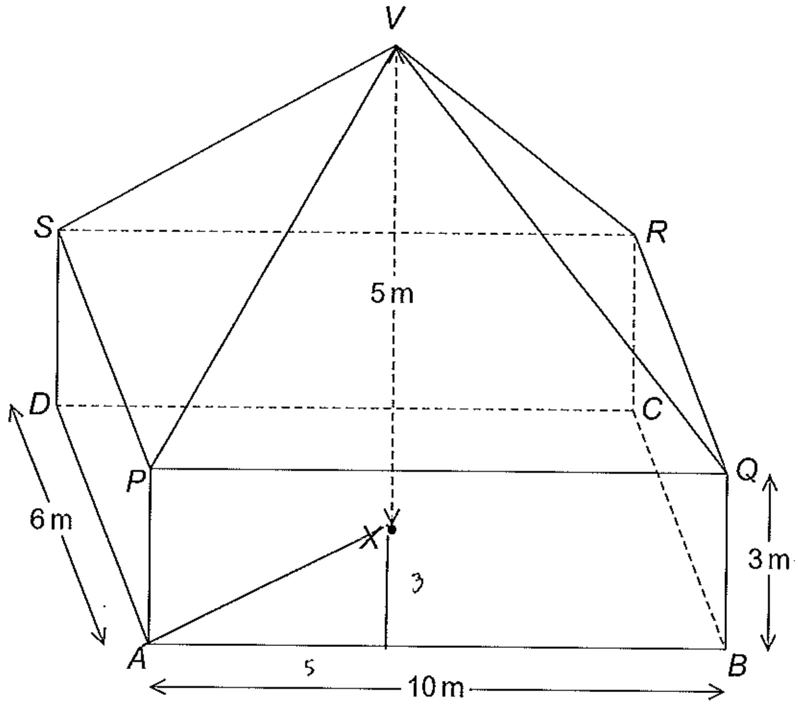
$a =$ -3 $\qquad b =$ -10 *(5 marks)*

15

Turn over ▶

Do not write outside the box

23 The diagram shows a cuboid *ABCDPQRS* and a pyramid *PQRSV*.
V is directly above the centre, *X*, of *ABCD*.

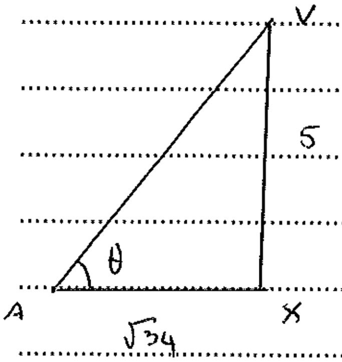


The total height, *VX*, is 5 metres.

Do not write outside the box

23 (a) Work out the angle between the line *VA* and the plane *ABCD*.

Need AX $= \sqrt{5^2 + 3^2} = \sqrt{34}$



$\tan\theta = 5/\sqrt{34}$

$\theta = \tan^{-1}(5/\sqrt{34})$

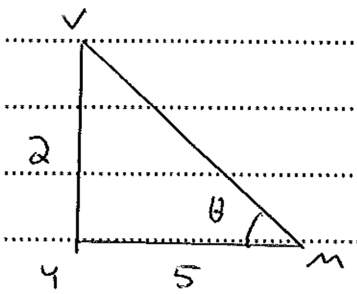
$= 40.6128...$

Answer 40.6° (1dp) degrees *(4 marks)*

23 (b) Work out the angle between the planes *VQR* and *PQRS*.

M: midpoint of RQ

Y: centre of PQRS



$\tan\theta = 2/5$

$\theta = \tan^{-1}(2/5)$

$= 21.8014...$

Answer 21.8° (1dp) degrees *(2 marks)*

6

Turn over ▶

24 Solve $3\cos^2\theta - 1 = 0$ for $0° \leqslant \theta \leqslant 180°$

$3\cos^2\theta = 1$

$\cos^2\theta = 1/3$

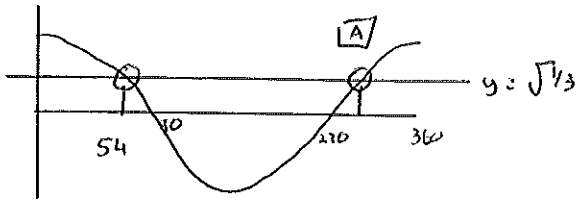
$\cos\theta = \pm\sqrt{1/3}$

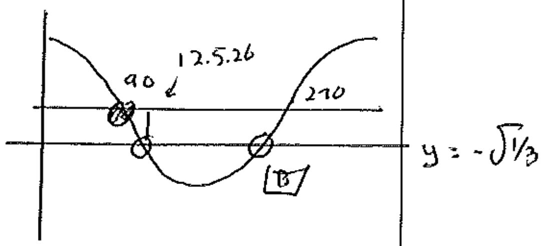
① $\cos\theta = \sqrt{1/3}$

$\theta = 54.74$

② $\cos\theta = -\sqrt{1/3}$

$\theta = 125.26°$





$\boxed{A}$ $\theta = 360 - 54.74$

$= 305.26$

↑

out of the domain!

$\boxed{B}$ $\theta = 360 - 125.26$

$= 234.74$

↑

out of Domain!

Answer $\theta = 54.74°, \theta = 125.26°$ *(4 marks)*

(2dp)

Do not write outside the box

25 Here are two number machines.

(1) Input a → -4 → $\times k$ → Output b

(2) Input $a+2$ → $\times 3$ → $-k$ → Output $b+2$

Work out a in terms of k.

(1) $k(a-4) = b \rightarrow ak - 4k = b$

(2) $3(a+2) - k = b+2$

$\rightarrow 3a + 6 - k = b + 2$

$-2 \; \{ \; 3a + 4 - k = b$

Both equal to b $\rightarrow 3a + 4 - k = ak - 4k$

$+k \; \{ \; 3a + 4 = ak - 3k$

$-4 \; \{ \; 3a = ak - 3k - 4$

$-ak \; \{ \; 3a - ak = -3k - 4$

Factorix $\{ \; a(3-k) = -3k - 4$

$\div(3-k) \; \{ \; a = \dfrac{-3k-4}{3-k}$

$a =$ $\dfrac{-3k-4}{3-k}$ or $\dfrac{3k+4}{k-3}$ **(6 marks)**

END OF QUESTIONS

10