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Level 2 Certificate in Further Mathematics

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Paper 2 8360/2

Final

Mark Scheme

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Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q	Answer	Mark	Comments
1(a)	(0, 4)	B1	
1(b)	(3, 2)	B2 ft	ft their (0, 4) **and** their S (= (6, 0) if correct) B1ft One coordinate correct SC1 (2, 3)

2	$20^2 - 16^2$ (= 144) **or** $20^2 = PX^2 + 16^2$ **or** $34^2 - 16^2$ (= 900) **or** $34^2 = XQ^2 + 16^2$	M1	Uses trigonometry eg $\cos^{-1}\frac{16}{20}$ (= [36.86989765, 36.9])	3, 4, 5 triangle **or** 8, 15, 17 triangle identified
	$\sqrt{20^2 - 16^2}$ **or** $\sqrt{34^2 - 16^2}$	M1Dep	eg 20 × sin their 36.9	3×4 **or** 15×2
	12 **and** 30	A1		
	2 : 5	A1 ft	Allow 1 : 2.5 ft from M2 A0 if their ratio needs simplifying	

3	$5d - d > 17 + 3$	M1	Allow one sign or arithmetic error eg $4d > 21$ or $5d - d > 17 - 3$
	$d > 5$	A1	

4	Box 2 → $y = x^3 + x - 2$	B1	
	Box 3 → $(x - 2)^2 + (y + 1)^2 = 1$	B1	
	Box 4 → $x^2 + y^2 = 10$	B1	

5	$5(x + 3)$ (=) $\frac{1}{2} \times 6 \times (x + 12)$	B2	oe eg $5x + 15$ (=) $\frac{1}{2} \times 6 \times (12 - x) + 6x$ B1 $5(x + 3)$ **or** $\frac{1}{2} \times 6 \times (x + 12)$ oe
	$5x - 3x = 36 - 15$	M1	Collects like terms ft their areas but must have at least B1 and have both areas in terms of x
	$10\frac{1}{2}$	A1 ft	oe ft from B1 M1 Do not ft if negative solution

Q	Answer	Mark	Comments
6(a)	Line from (–4, 4) to (–2, 4)	B1	
	Curve through (–2, 4) (–1, 1) (0, 0) (1, 1) and (2, 4)	B1	$\pm \frac{1}{2}$ square
	Line from (2, 4) to (4, –4)	B1	
6(b)	3	B1 ft	ft their graph
6(c)	$12 - 4x = -10$	M1	oe
	5.5	A1	oe

7	$(n = 1) \quad 4a = \frac{10 \times 1 - 2}{3}$	M1	$(n = 2) \quad 9a = \frac{10 \times 2 - 2}{3}$ **or** $(n = 3) \quad 14a = \frac{10 \times 3 - 2}{3}$ **or** $(n = 4) \quad 19a = \frac{10 \times 4 - 2}{3}$
	$\frac{2}{3}$	A1	oe
	Alternative method		
	$5an - a = \frac{10n - 2}{3}$	M1	oe
	$\frac{2}{3}$	A1	oe

8(a)	$5(m + 2p)(m - 2p)$	B3	B2 $(5m + 10p)(m - 2p)$ **or** $(5m - 10p)(m + 2p)$ B1 $5(m^2 - 4p^2)$ **or** $(5m + ap)(m + bp)$ where $ab = \pm 20$
8(b)	Their $(m + 2p) = 0$ **or** Their $(m - 2p) = 0$	M1	oe eg $m = -2p$ **or** $m = 2p$ May substitute for p at this stage
	–30 **and** 30	A1	
	Alternative method		
	$5m^2 - 20 \times 15 \times 15 = 0$	M1	oe eg $5m^2 = 4500$
	–30 **and** 30	A1	

Q	Answer	Mark	Comments
9(a)	$x^2 + mx + nx + mn$	B1	oe
9(b)	$(q =)\ m + n$	B1 ft	oe ft their (a)
	$(r =)\ m\ (\times)\ n$	B1 ft	oe ft their (a)
9(c)	Any complete explanation eg m and n are both odd (integers) **and** odd (integer) + odd (integer) = even (integer) (so q is even)	B2	B1 Any partial explanation eg1 m and n are odd (integers) eg2 odd × odd = odd **and** odd + odd = even
10(a)	$S\ (1 - r) = a$	B1	$\frac{a}{S} = 1 - r$
	$S - Sr = a$	M1	Any valid correct step from their first step
	$S - a = Sr \quad (\frac{S - a}{S} = r)$	A1	Clearly shown with no errors
10(b)	$\frac{10a - a}{10a} \quad (= \frac{9a}{10a})$	M1	$10a = \frac{a}{1 - r}$ oe
	$\frac{9}{10}$	A1	oe
11	$\angle ACB = x$ **and** (Triangle ABC is) isosceles	M1	oe
	$\angle ABC = 180 - 2x$ **and** Angle sum of triangle (is 180°)	M1	oe $\angle CAD + \angle ACD = 180 - 2x$ **and** Angle sum of triangle (is 180°)
	$180 - 2x + 2x = 180$ **and** Opposite angles of cyclic quadrilateral (add up to 180°)	A1	Must have seen working for both M marks oe eg $\angle ABC + \angle ADC = 180$ **and** Opposite angles of cyclic quadrilateral SC2 'Correct' solution with one reason missing SC1 'Correct' solution with >1 reason missing

Q	Answer	Mark	Comments
12(a)	–0.5	B1	oe
12(b)	$-1 \leq f(x) \leq 0$	B2	B1 $-1 \leq f(x) \leq c$ $c \neq 0$ $c > -1$ **or** $d \leq f(x) \leq 0$ $d \neq -1$ $d < 0$ SC1 Correct sketch of $y = \sin x$ with 180, 360 and –1 labelled on axes
12(c)	90	B1	

13(a)	$\frac{4(x-1)+2x}{x(x-1)}$	M1	oe eg two separate fractions Condone absence of brackets only if recovered
	$\frac{4x-4+2x}{x(x-1)}$ $(=\frac{6x-4}{x(x-1)})$	A1	Do not condone absence of brackets even if recovered
13(b)	$6x - 4 = 3x(x-1)$	M1	oe eg $4(x-1) + 2x = 3x(x-1)$
	$3x^2 - 9x + 4 \ (= 0)$	A1	$-3x^2 + 9x - 4 \ (= 0)$
	$\frac{--9 \pm \sqrt{(-9)^2 - 4 \times 3 \times 4}}{2 \times 3}$ $(=\frac{9 \pm \sqrt{33}}{6})$	M2	Correct use of formula for their quadratic M1 Allow one sign error (must have square root and numerator all over $2a$) Allow M2 for correct factorisation of their quadratic M2 $(x-\frac{3}{2})^2 = \frac{9}{4} - \frac{4}{3}$ oe M1 $(x-\frac{3}{2})^2 - \frac{9}{4} + \frac{4}{3} = 0$ oe
	2.46 **and** 0.543	A1	Must both be to 3 significant figures

Q	Answer	Mark	Comments
14	$1.5t$	M1	oe eg1 $t + \frac{50}{100}t$ eg2 $2x = 3t$ eg3 $x : t = 3 : 2$
	$0.9w$	M1	oe eg1 $w - \frac{10}{100}w$ eg2 $10y = 9w$ eg3 $w : y = 10 : 9$
	Their $1.5t$ = their $0.9w$	M1Dep	Dep on at least one M mark gained oe eg1 $\frac{\text{their } 0.9}{\text{their } 1.5}$ eg2 $15t = 9w$ eg3 w (: x) : t = 10 (: 9) : 6
	0.6	A1 ft	ft from M1 M0 M1 or M0 M1 M1 SC2 $1.\dot{6}$ or 1.67 SC1 $\frac{5}{3}$ oe fraction
	Alternative Method		
	Chooses an appropriate pair of values for x and t eg $x = 90$ and $t = 60$	M1	Chooses an appropriate pair of values for y and w eg $y = 180$ and $w = 200$
	Their $90 = 0.9w$ ($w = 100$)	M1	Their $180 = 1.5t$ ($t = 120$)
	$\frac{\text{their } 60}{\text{their } 100}$	M1Dep	Dep on at least one M mark gained $\frac{\text{their } 120}{\text{their } 200}$
	0.6	A1 ft	ft from M1 M0 M1 or M0 M1 M1 SC2 $1.\dot{6}$ or 1.67 SC1 $\frac{5}{3}$ oe fraction

Q	Answer	Mark	Comments
15	Rotation, through 90° (anticlockwise), about O **or** Rotation, through 270° clockwise, about O	B3	B1 for each part SC1 $\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix}$ **or** $\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}-1\\0\end{pmatrix}$ **or** $\begin{pmatrix}\cos 90 & -\sin 90\\ \sin 90 & \cos 90\end{pmatrix}$
16	At least two terms correct from $x^6 - x^3 - x^3 + 1$	M1	Condone omission of terms
	$x^6 - x^3 - x^3 + 1$	A1	oe
	x^4	B1	
	$6x^5 - 3x^2 - 3x^2 + 4x^3$	M1	One of their terms in x differentiated correctly
	$6x^5 - 6x^2 + 4x^3$	A1	
17	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	M1	$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}-1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\-1\end{pmatrix}$ **or** $\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$
	$\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$	A1	SC1 $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$
18	Use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$	M1	eg $1 - \frac{\sin\theta}{\cos\theta}\sin\theta\cos\theta$
	$1 - \sin^2\theta$	M1Dep	oe eg $\sin^2\theta + \cos^2\theta - \sin\theta\sin\theta$
	$\cos^2\theta$	A1	Condone $(\cos\theta)^2$ but do not allow $\cos\theta^2$

Q	Answer	Mark	Comments
19	Cubic curve with exactly one minimum and exactly one maximum	M1	
	Minimum at (–2, 0) labelled with x value –2	M1Dep	Dep on first M1
	Maximum in correct quadrant	M1Dep	Dep on first M1
	Fully correct curve	A1	Must be for the correct domain SC3 'Correct' curve over correct domain but (–2, 0) not labelled with x value –2

20	$\frac{1}{2} \times w \times 2w \times \sin 30$ (= 18)	M1	oe eg1 $2w^2 \sin 30 = 36$ eg2 $\sin 30 = \frac{18}{w^2}$
	$w^2 = 36$ **or** $w = 6$ **or** $2w = 12$	A1	
	their 6^2 + their 12^2 $-2 \times$ their $6 \times$ their $12 \times \cos 30$ (= [55.29, 55.3])	M1	their $36 + 4 \times$ their 36 $-4 \times$ their $36 \times \cos 30$ (= [55.29, 55.3])
	$\sqrt{\text{their [55.29, 55.3]}}$	M1 Dep	Dep on previous M1 Do not allow if from incorrect working eg $\sqrt{36\cos 30}$ is M0 Dep
	[7.4, 7.44]	A1 ft	ft their w if 2nd and 3rd M1 gained

21	$2x + 4$	M1	
	-2	A1	
	$\frac{1}{2}$	M1	$\frac{-1}{\text{their } -2}$
	$y = 2$	B1	
	$y - 2 = \frac{1}{2}(x + 3)$	A1 ft	oe eg $y = \frac{1}{2}x + \frac{7}{2}$ ft their $\frac{1}{2}$ and their 2 if M2 gained

Q	Answer	Mark	Comments
22	$2^3 + a(2)^2 + b(2) + 24$	M1	oe eg $8 + 4a + 2b + 24$
	$(-3)^3 + a(-3)^2 + b(-3) + 24$	M1	oe eg $-27 + 9a - 3b + 24$
	$4a + 2b = -32$ **and** $9a - 3b = 3$	A1	oe Must be 2 correct equations
	Multiplies equation(s) to have the same coefficient for one variable **and** attempts to eliminate by addition or subtraction eg $12a + 6b = -96$ $18a - 6b = 6$ **and** $30a = -90$	M1	Allow two errors in first stage and one error in second stage (must use the appropriate operation for elimination for their equations) oe eg substitution method used
	$a = -3$ **and** $b = -10$	A1	
	Alternative method		
	$(x - 4)$	M1	
	$x^2 - 2x + 3x - 6$ **or** $x^2 - 2x - 4x + 8$ **or** $x^2 + 3x - 4x - 12$	M1	$x^2 + x - 6$ **or** $x^2 - 6x + 8$ **or** $x^2 - x - 12$ ft their $(x - 4)$
	$x^3 + x^2 - 6x - 4x^2 - 4x + 24$ **or** $x^3 - 6x^2 + 8x + 3x^2 - 18x + 24$ **or** $x^3 - x^2 - 12x - 2x^2 + 2x + 24$	M1	their $(x - 4) \times$ their $(x^2 + x - 6)$ **or** $(x + 3) \times$ their $(x^2 - 6x + 8)$ **or** $(x - 2) \times$ their $(x^2 - x - 12)$ Allow two errors or omissions
	$x^3 + x^2 - 6x - 4x^2 - 4x + 24$ **or** $x^3 - 6x^2 + 8x + 3x^2 - 18x + 24$ **or** $x^3 - x^2 - 12x - 2x^2 + 2x + 24$	A1	oe eg $x^3 - 3x^2 - 10x + 24$ Must be fully correct
	$a = -3$ **and** $b = -10$	A1	

Q	Answer	Mark	Comments
23(a)	(*AC* =) $\sqrt{10^2 + 6^2}$ (= $\sqrt{136}$)	M1	[11.66, 11.7]
	(*AX* =) their AC ÷ 2 (= [5.8, 5.85])	M1	(*AX* =) $\sqrt{5^2 + 3^2}$ (= $\sqrt{34}$) is M2 Do **not** allow their AC to be 10
	tan (*VAX*) = $\frac{5}{\text{their } AX}$	M1Dep	Dep on at least one M mark gained (*AV*=) $\sqrt{5^2 + \text{their } AX^2}$ (= $\sqrt{59}$) **and** sin (*VAX*) = $\frac{5}{\text{their AV}}$ (× sin 90) or cos (*VAX*) = $\frac{\text{their AX}}{\text{their AV}}$ or correct use of cosine rule in triangle VAX Do **not** allow their AX to be their AC
	[40.5, 40.8]	A1	Allow 41 if correct method seen SC3 Answer [0.707, 0.7115] SC3 Answer [45.02, 45.293]
23(b)	tan *VMY* = $\frac{2}{5}$	M1	oe (*M* is midpoint of *RQ, Y* is the centre of *PQRS)*)
	[21.8, 21.80141]	A1	Allow 22 if correct method seen SC1 Angle *VMY* clearly marked on a diagram SC1 Answer [0.38, 0.381] SC1 Answer [24.2, 24.224]

Q	Answer	Mark	Comments
24	$\cos^2\theta = \frac{1}{3}$	B1	May be implied in working $\sin^2\theta = \frac{2}{3}$ or $\tan^2\theta = 2$
	$\cos\theta = (\pm)\sqrt{\frac{1}{3}}$	M1	oe eg $\cos\theta = (\pm)$ [0.57(7), 0.6] $\sin\theta = (\pm)\sqrt{\frac{2}{3}}$ oe or $\tan\theta = (\pm)\sqrt{2}$ oe
	[54.7, 54.7602]	A1	
	[125.2398, 125.3]	A1 ft	ft 180 – their [54.7, 54.7602] if M1 gained Correct or ft A0 if an incorrect solution [0, 180] also seen

25	$b = k(a - 4)$ **or** $b + 2 = 3(a + 2) - k$	M1	oe
	$b = ka - 4k$ **or** $b + 2 = 3a + 6 - k$	M1	Correctly expands brackets First M1 implied if this mark gained
	$ka - 4k + 2 = 3a + 6 - k$	M1	oe Attempt to eliminate b from their two equations Allow one error or omission
	$ka - 3a = 6 - k + 4k - 2$	M1	Correctly separates terms in a for their equation
	$a(k - 3) = 3k + 4$	M1	Correctly factorises terms in a for their equation
	$a = \frac{3k + 4}{k - 3}$	A1	oe eg $a = \frac{-3k - 4}{3 - k}$