Version 1.0

# Level 2 Certificate in Further Mathematics

# June 2012

## Paper 2 8360/2

**_Mark Scheme_**

Set and published by the Assessment and Qualifications Alliance.

## Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

| | |
|---|---|
| **M** | Method marks are awarded for a correct method which could lead to a correct answer. |
| **A** | Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied. |
| **B** | Marks awarded independent of method. |
| **M Dep** | A method mark dependent on a previous method mark being awarded. |
| **B Dep** | A mark that can only be awarded if a previous independent mark has been awarded. |
| **ft** | Follow through marks. Marks awarded following a mistake in an earlier step. |
| **SC** | Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth. |
| **oe** | Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$ |

# Paper 2 - Calculator

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **1** | Radius = $\sqrt{36}$ **or** 6 | B1 | Diameter = $2\sqrt{36}$ **or** 12 |
| | 2 (×) π (×) their radius | M1 | π (×) their diameter |
| | $12\pi$ **or** [37.68, 37.704] | A1 | |
| **2** | $15x^2 - 8x$ | B2 | B1 Only one term correct |
| **3** | $8^2$ **or** $4^2$ **or** 64 **or** 16 **or** 80 **or** $(-8^2)$ **or** $(-4^2)$ | M1 | |
| | $\sqrt{\text{their } 8^2 + \text{their } 4^2}$ | M1 Dep | |
| | 8.944(…) **or** $\sqrt{80}$ | A1 | oe eg $4\sqrt{5}$<br>This mark is implied by 8.94 |
| | 8.94 | B1 ft | ft From any value > 3sf seen or any value given as a surd that is rounded to 3sf |
| **4(a)** | Positive | B1 | Do not allow if more than one answer selected |
| **4(b)** | Negative | B1 | Do not allow if more than one answer selected |
| **4(c)** | One positive and one negative | B1 | Do not allow if more than one answer selected |
| **4(d)** | 0 | B1 | Do not allow if more than one answer selected |
| **4(e)** | $y = -3$ | B1 | Do not allow if more than one answer selected |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **5(a)** | Angle *ACP* = $x$ or angle *PAC* (base angles of) isosceles triangle (are equal) | M1 | |
| | Angle *APC* = 180 – 2$x$<br>angle sum of triangle (= 180°)<br>**and**<br>angle *BPC* = 2$x$<br>angles on straight line (add to 180°) | M1 Dep | *BPC* = 2$x$ external angle of triangle<br>(= sum of interior opposite angles) |
| | Angle *ABC* = 2$x$ or angle *BPC*<br>(base angles of) isosceles triangle (are equal) | A1 | SC2 'Correct' response but has reason(s) missing or incorrect |
| **5(b)** | Angle *ACB* = 2$x$ | M1 | May be implied by working |
| | $x + 2x + 2x = 180$ | M1 | oe eg 1 $5x = 180$<br>eg 2 $90 - \frac{1}{2}x = 2x$ |
| | 36 | A1 | |
| **6(a)** | $6x^2 - 15xy$ | B2 | B1 Only one correct term |
| **6(b)** | $9x^2 - 12xy + 6xy - 8y^2$ | M1 | oe<br>Must have 4 terms with at least 3 correct |
| | $9x^2 - 12xy + 6xy - 8y^2$ | A1 | All 4 terms correct |
| | $9x^2 - 6xy - 8y^2$ | A1 ft | ft From M1 A0 |
| **6(c)** | 3 : 2 | B2 ft | ft Their (a) and their (b) with $y = 0$ substituted<br>B1 ft Any equivalent unsimplified ratio eg $9x^2 : 6x^2$<br>SC1 2 : 3 |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **7(a)** | $-8 \leq m + n \leq 7$ | B2 | B1 $-8$ or 7 in correct position |
| **7(b)** | $0 \leq (m + n)^2 \leq 64$ | B2 ft | **If (a) is fully correct ft does not apply**<br>B1 For 0 or 64 in correct position<br>**If (a) is not fully correct apply ft**<br>Can only award B2 ft if their (a) has one negative value and one positive value<br>B1ft for one value in correct position<br>Can award a maximum of B1 ft if in (a) both values have the same sign or one value is zero |
| **8(a)** | *C* | B1 | Do not allow if more than one answer selected |
| **8(b)** | *A* | B1 | Do not allow if more than one answer selected |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **9(a)** | $5t + 3 = 4wt + 8w$ | M1 | |
| | $5t - 4wt = 8w - 3$ | M1 | Separation of terms in $t$ from those not in $t$ |
| | $t(5 - 4w) = 8w - 3$ | M1 | Factorisation of terms in $t$ |
| | $t = \frac{8w-3}{5-4w}$ | A1ft | oe eg $t = \frac{3-8w}{4w-5}$<br>Must have $t =$<br>**Only ft if third M1 and one other M1 gained** |
| **9(b)** | $\dfrac{8\times -\frac{1}{8} - 3}{5 - 4\times -\frac{1}{8}}$ | M1 | Substitution of $w = -\frac{1}{8}$ in their $\frac{8w-3}{5-4w}$<br>Their $\frac{8w-3}{5-4w}$ must be in terms of $w$ |
| | Numerator = – 4 **or**<br>denominator = $5\frac{1}{2}$ | A1ft | ft Their $\frac{8w-3}{5-4w}$<br>This mark can only be gained for correct evaluation of their algebraic numerator **or** their algebraic denominator |
| | $-\frac{8}{11}$ **or** $-0.\dot{7}\dot{2}$ | A1ft | ft Their $\frac{8w-3}{5-4w}$<br>This mark can only be gained for correct evaluation of their algebraic numerator **and** their algebraic denominator<br>Must be an exact value in simplest form<br>SC2 –0.72… **or** –0.73 **or** a correct evaluation of their algebraic numerator or their algebraic denominator |
| **Alt 9(b)** | $5t + 3 = -\frac{4}{8}(t + 2)$ | M1 | oe equation |
| | $44t = -32$ | A1 | oe eg $5.5t = -4$ |
| | $-\frac{8}{11}$ **or** $-0.\dot{7}\dot{2}$ | A1ft | ft from their $at = b$ if M1 A0<br>Must be an exact value in simplest form<br>SC2 –0.72… **or** –0.73 |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 10 | sin 28 chosen | B1 | cos 62 chosen |
| | $\frac{7}{\sin 28}$ | M1 | $\frac{7}{\cos 62}$ |
| | [14.9, 14.9104] | A1 | Allow 15 if correct working for M1 seen |
| 11 | $\frac{4}{3}\pi x^3$ (=) $\frac{2}{3}\pi y^3$ | M1 | oe eg 1 $\frac{4}{3}\pi \times x^3$ (=) $\frac{1}{2} \times \frac{4}{3}\pi \times y^3$<br>eg 2 $y^3 = 2x^3$ |
| | ($\frac{y^3}{x^3}$ =) $\frac{\frac{4}{3}\pi}{\frac{2}{3}\pi}$ **or** $y = \sqrt[3]{2}\,x$ | M1 Dep | oe eg $\frac{y^3}{x^3} = 2$ |
| | $2^{\frac{1}{3}}$ | A1 | $\sqrt[3]{2}$ scores M2 A0 |
| 12 | $(t + 4)(t^2 + 4t + 4t + 16)$ | M1 | oe Must be correct |
| | $t^3 + 4t^2 + 4t^2 + 16t + 4t^2 + 16t + 16t + 64$ | M1 | ft From their $(t + 4)(t^2 + 4t + 4t + 16)$<br>oe Must have at least 4 terms correct<br>M2 $t^3 + 3t^2(4) + 3t\,(4)^2 + 4^3$ oe |
| | $t^3 + 12t^2 + 48t + 64$ | A1 | |
| 13 | $\frac{16^2 + 9^2 - 20^2}{2 \times 16 \times 9}$ (= – 0.21875) | M1 | oe eg $\frac{256 + 81 - 400}{288}$ **or** $-\frac{63}{288}$<br>**or** $-288\cos x = 63$ |
| | $\cos^{-1} \frac{16^2 + 9^2 - 20^2}{2 \times 16 \times 9}$ | M1 | oe<br>This mark implies the first M1 |
| | [102.6, 102.64] | A1 | Allow 103 if correct working for M1 M1 seen |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| 14 | $x$ coordinate of centre = 2 | B1 | |
| | $y$ coordinate of centre = 5 | B1 | |
| | $(x$ – their $2)^2 + (y$ – their $5)^2$ | M1 | = 25 not needed for M1 |
| | $(x$ – their $2)^2 + (y$ – their $5)^2 = 25$ | A1 ft | oe eg Allow $5^2$ for 25<br>ft From their centre of circle<br>Ignore any attempt to expand and simplify |
| 15(a) | $3x^2 - 5$ seen | B1 | |
| | Correct step in attempt to solve their $f(x^2) = 43$ (must be a quadratic equation)<br>$3x^2 = 43 + 5$ | M1 | oe eg 1 $3x^2 - 5 - 43 = 0$<br>eg 2 $x^2 = \frac{43+5}{3}$ |
| | $x^2 = 16$ | A1 | $(3)(x + 4)(x - 4)$ |
| | 4 **and** –4 | A1 ft | ft From M1 A0 if two solutions found<br>SC2 3.56 **and** –3.56 |
| 15(b) | (gradient for $0 \le x \le 4$ =)<br>$\frac{12}{4}$ **or** 3 | M1 | oe |
| | (gradient for $4 < x \le 8$ =)<br>$\frac{12}{-4}$ **or** $-3$ | M1 | oe<br>Accept – their 3 |
| | $y$ = their $-3x + c$ **and** substitutes (8,0) or (4,12) | M1 | $y - 0$ = their $-3(x - 8)$ **or**<br>$y - 12$ = their $-3(x - 4)$ |
| | $3x$ **and** $-3x + 24$ or $-3(x - 8)$ in correct places on answer lines | A2 | A1 $3x$ **or** $-3x + 24$ or $-3(x - 8)$ in correct place on answer line **or**<br>$y = 3x$ (for $0 \le x \le 4$) **or**<br>$y = -3x + 24$ or $y = -3(x - 8)$<br>(for $4 < x \le 8$) |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **16(a)** | $1^3 - 21(1) + 20 = 0$ **or**<br>$1 - 21 + 20 = 0$ | B1 | Must have = 0 |
| | $4^3 - 21(4) + 20 = 0$ **or**<br>$64 - 84 + 20 = 0$ | B1 | Must have = 0 |
| **16(b)** | $1^3 - 10(1)^2 + 29(1) - 20 = 0$ **or**<br>$1 - 10 + 29 - 20 = 0$<br>Divides $x^3 - 10x^2 + 29x - 20$ by $(x - 1)$ and obtains answer $x^2 - 9x + 20$ | B1 | Must have = 0<br><br>B2 $(x - 1)(x - 4)(x - 5)$ **and** correct expansion of one pair of brackets<br>eg $(x - 1)(x - 4)(x - 5)$ **and** $(x^2 - 5x + 4)(x - 5)$<br>B1 $(x - 1)(x - 4)(x - 5)$ |
| | $4^3 - 10(4)^2 + 29(4) - 20 = 0$ **or**<br>$64 - 160 + 116 - 20 = 0$<br>Divides $x^3 - 10x^2 + 29x - 20$ by $(x - 4)$ and obtains answer $x^2 - 6x + 5$ | B1 | Must have = 0 |
| **16(c)** | $(x + 5)$ as 3rd factor of numerator | B1 | Implied by final answer $\frac{x+5}{ax+b}$ |
| | $(x - 5)$ as 3rd factor of denominator | B1 | Implied by final answer $\frac{cx+d}{x-5}$ |
| | $\frac{\text{their } x+5}{\text{their } x-5}$ | B1 ft | Do **not** award if further work |
| **17** | CE or EB = $2x$ **or** DF = $x$ **or**<br>FC = $3x$ **or** area ABCD = $16x^2$ | B1 | May be on diagram or implied in working |
| | Area ABE = $\frac{1}{2} \times$ their $2x \times 4x$ (= $4x^2$)<br>**and**<br>area CFE = $\frac{1}{2} \times$ their $2x \times$ their $3x$ (= $3x^2$)<br>**and**<br>area ADF = $\frac{1}{2} \times$ their $x \times 4x$ (= $2x^2$) | M2 | Attempt at all three triangle areas ABE and CFE and ADF<br>M1 Attempt at any one triangle area ABE or CFE or ADF<br>All areas must be in terms of $x$ |
| | $4x \times 4x$ – their $4x^2$ – their $3x^2$ – their $2x^2$ (= $7x^2$) | M1 Dep | Dep on at least M1<br>All areas must be in terms of $x^2$ |
| | 7 | A1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **18** | $a = 3$ **and** $b = –10$ | B3 | B2 $a = 3$ **or** $b = –10$<br>B1 $x^2 – 5x – 5x + 25$ oe |
| **19** | $(4 – x)^2 = 4x + 5$ | M1 | |
| | $16 – 4x – 4x + x^2 = 4x + 5$ | M1 Dep | Allow one error but must be a quadratic in $x$ |
| | $x^2 – 12x + 11$ (= 0) | A1 | oe Must be 3 terms |
| | $(x – 11)(x – 1)$ (= 0) | M1 | $\frac{--12 \pm \sqrt{(-12)^2 - 4(1)(11)}}{2}$ **or**<br>$(x – 6)^2 – 36 + 11 = 0$ oe |
| | $x = 11$ and $x = 1$ | A1 ft | Must have M3 to ft<br>$x = 11$ and $y = –7$ **or** $x = 1$ and $y = 3$ |
| | $x = 11$ and $y = –7$ **and**<br>$x = 1$ and $y = 3$ | A1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **Alt 19** | $y^2 = 4(4 – y) + 5$ | M1 | |
| | $y^2 = 16 – 4y + 5$ | M1 Dep | Allow one error but must be a quadratic in $y$ |
| | $y^2 + 4y – 21$ (= 0) | A1 | oe Must be 3 terms |
| | $(y + 7)(y – 3)$ (= 0) | M1 | $\frac{-4 \pm \sqrt{4^2 - 4(1)(-21)}}{2}$ **or**<br>$(y + 2)^2 – 4 – 21 = 0$ oe |
| | $y = –7$ and $y = 3$ | A1 ft | Must have M3 to ft<br>$x = 11$ and $y = –7$ **or**<br>$x = 1$ and $y = 3$ |
| | $x = 11$ and $y = –7$ **and**<br>$x = 1$ and $y = 3$ | A1 | |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **20** | $150 - 6x^2$ | B1 | |
| | their $150 - 6x^2 > 0$ **or**<br>their $150 - 6x^2 = 0$ | M1 | their $150 - 6x^2$ must be in terms of $x$<br>Must be $> 0$ or $= 0$ |
| | $\frac{150}{6} > x^2$ or $(6)(5 - x)(5 + x)$ $(> 0)$<br>**or**<br>$\frac{150}{6} = x^2$ or $(6)(5 - x)(5 + x)$ $(= 0)$ | M1 Dep | ft Their inequality **only if a quadratic** either simplified to $k > x^2$ or factorised correctly<br>**or**<br>ft Their equation **only if a quadratic** either simplified to $k = x^2$ or factorised correctly |
| | $-5 < x < 5$ | A1 | Allow $x > -5$ and $x < 5$ (must have both inequalities as well as the 'and') |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **21** | Fully correct method to eliminate a letter from *OB* and *AB*<br>$2(2x) = 11x - 7$ | M1 | oe eg 1 $2y = 11(\frac{y}{2}) - 7$<br>eg 2 $2y - 4x = 0$<br>$2y - 11x = -7$<br>**and** $7x = 7$ |
| | Coordinates of $B = (1, 2)$ | A1 | Implied by $x = 1$ and $y = 2$ |
| | Fully correct method to eliminate a letter from *OA* and *AB*<br>$2y = 11(-3y) - 7$ | M1 | oe eg 1 $x + 3(\frac{11x - 7}{2}) = 0$<br>eg 2 $2x + 6y = 0$<br>$33x - 6y = 21$<br>**and** $35x = 21$ |
| | Coordinates of $A = (0.6, -0.2)$ | A1 | oe Implied by $x = 0.6$ and $y = -0.2$ |
| | $OB^2$ = their $1^2$ + their $2^2$<br>**or**<br>$AB^2$ = (their 1 – their $0.6)^2$ + (their 2 – their $-0.2)^2$ | M1 | oe<br>eg correct attempt at *OB* **or** *AB*<br>ft Their *B* and/or their *A* |
| | $OB = \sqrt{5}$ **and** $AB = \sqrt{5}$ | A1 | oe<br>eg $OB^2 = 5$ **and** $AB^2 = 5$ |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **22** | $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-4\\ 3\end{pmatrix}$ | M1 | |
| | $\begin{pmatrix}x\\ -y\end{pmatrix} = \begin{pmatrix}-4\\ 3\end{pmatrix}$ **or** $\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-4\\ -3\end{pmatrix}$ **or**<br><br>Q (–4, –3) | A1 | oe |
| | $\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}$ = their $\begin{pmatrix}-4\\ -3\end{pmatrix}$ | M1 Dep | |
| | $\begin{pmatrix}-Y\\ -X\end{pmatrix} = \begin{pmatrix}-4\\ -3\end{pmatrix}$ | A1 ft | oe ft Their $\begin{pmatrix}-4\\ -3\end{pmatrix}$ if M2 gained |
| | (3, 4) | A1 ft | ft Their $\begin{pmatrix}-4\\ -3\end{pmatrix}$ if M2 gained<br><br>SC4 $\begin{pmatrix}3\\ 4\end{pmatrix}$ |
| **Alt 1 22** | $\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$ | M1 | This order only |
| | $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ | A1 | |
| | Their $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$ | M1 Dep | |
| | $\begin{pmatrix}-y\\ x\end{pmatrix} = \begin{pmatrix}-4\\ 3\end{pmatrix}$ | A1 ft | oe ft Their $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ if M2 gained |
| | (3, 4) | A1 ft | ft Their $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ if M2 gained<br><br>SC3 (–3, –4) SC4 $\begin{pmatrix}3\\ 4\end{pmatrix}$ |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **Alt 2 22** | $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ | M1 | |
| | $\begin{pmatrix} -y \\ -x \end{pmatrix}$ | A1 | |
| | $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ their $\begin{pmatrix} -y \\ -x \end{pmatrix}$ | M1 Dep | |
| | $\begin{pmatrix} -y \\ x \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ | A1 ft | oe ft Their $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ if M2 gained |
| | (3, 4) | A1 ft | ft Their $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ if M2 gained<br>SC4 $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ |
| **Alt 3 22** | Attempt to reflect (–4, 3) in the $x$-axis | M1 | |
| | (–4, –3) | A1 | |
| | Attempt to reflect their (–4, –3) in the line $y = -x$ | M1 Dep | |
| | (3, 4) | A2 ft | ft Their (–4, –3) if M2 gained<br>SC4 $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ |

| Q | Answer | Mark | Comments |
|---|---|---|---|
| **23** | Trials values either side of $x = 0$<br>$x = -1$ $\frac{dy}{dx} = 9$ **and**<br>$x = 1$ $\frac{dy}{dx} = -1$ | M1 | oe<br>Allow statements that $\frac{dy}{dx}$ is positive/negative but any evaluations seen must be correct |
| | Maximum $(0, \frac{4}{3})$ | A1 | Can only be awarded with correct method seen |
| | Trials values either side of $x = 2$<br>$x = 1$ $\frac{dy}{dx} = -1$<br>(may have been seen earlier)<br>$x = 3$ $\frac{dy}{dx} = -3$ | M1 | oe<br>Allow statements that $\frac{dy}{dx}$ is negative but any evaluations seen must be correct |
| | (Point of) inflection (2, 0) | A1 | Can only be awarded with correct method seen |
| **Alt 23** | $\frac{d^2y}{dx^2} = -3x^2 + 8x - 4$ **and**<br>substitutes $x = 0$ **and** $\frac{d^2y}{dx^2} = -4$ | M1 | Second derivative must be correct<br>Allow statement that $\frac{d^2y}{dx^2}$ is negative but if evaluation seen it must be correct |
| | Maximum $(0, \frac{4}{3})$ | A1 | Can only be awarded with correct method seen |
| | Trials values either side of $x = 2$<br>$x = 1$ $\frac{dy}{dx} = -1$<br>$x = 3$ $\frac{dy}{dx} = -3$ | M1 | oe eg uses second and third derivatives<br>Allow statements that $\frac{dy}{dx}$ is negative but any evaluations seen must be correct |
| | (Point of) inflection (2, 0) | A1 | Can only be awarded with correct method seen |