MR BARTON'S ANSWERS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
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16 – 17	
18 – 19	
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TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2012

Further Mathematics 8360/2
Level 2
Paper 2 Calculator

Friday 1 June 2012 1.30 pm to 3.30 pm

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

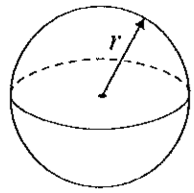
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

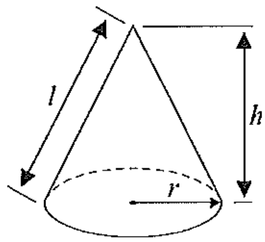
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

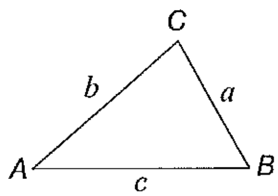


Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*



Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

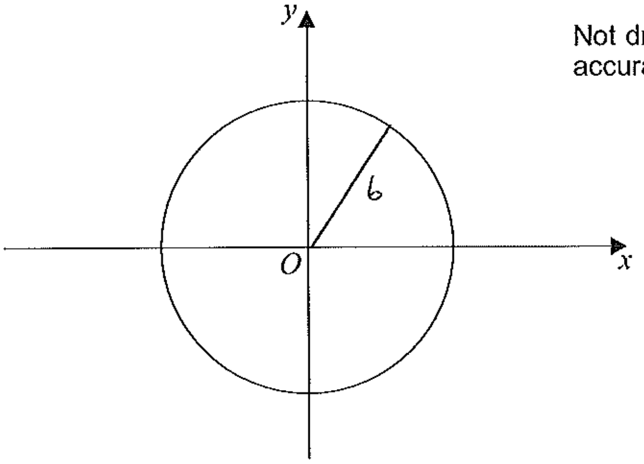
Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Here is a sketch of the circle $x^2 + y^2 = 36$



Not drawn accurately

Work out the circumference of the circle.

Radius = $\sqrt{36}$ = 6 → Diameter = 12

C = π × d = π × 12

Answer 37.6991... cm (3 marks)

Turn over for the next question

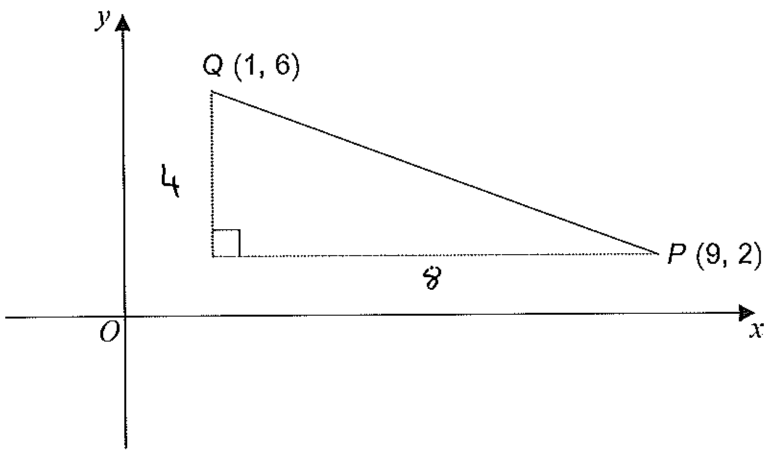
3

2 $y = 5x^3 - 4x^2$

Work out $\frac{dy}{dx}$.

$\frac{dy}{dx} =$ $15x^2 - 8x$ *(2 marks)*

3



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Work out the length of PQ.
Give your answer to 3 significant figures.

$PQ = \sqrt{8^2 + 4^2}$

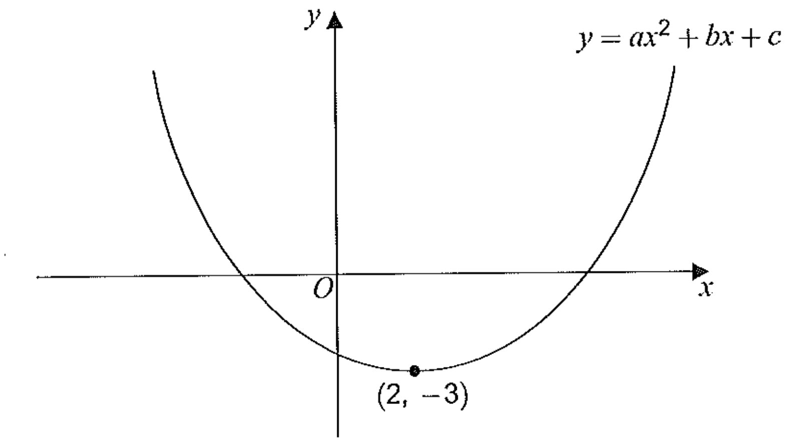
$= \sqrt{80}$

$= 8.94427...$

$PQ =$ 8.94 (3sf) *(4 marks)*

Do not write outside the box

4 A sketch of $y = ax^2 + bx + c$ is shown.
The minimum point is (2, −3).



For the sketch shown, circle the correct answer in each of the following.

4 (a) The value of a is

U shape

zero positive negative *(1 mark)*

4 (b) The value of c is

y intercept

zero positive negative *(1 mark)*

4 (c) The solutions of $ax^2 + bx + c = 0$ are

both zero both positive both negative one positive and one negative *(1 mark)*

cross x-axis

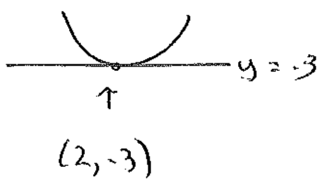
4 (d) The **number** of solutions of $ax^2 + bx + c = -6$ is

0 1 2 3 *(1 mark)*

Draw line y = -6 !

4 (e) The equation of the tangent to $y = ax^2 + bx + c$ at (2, −3) is

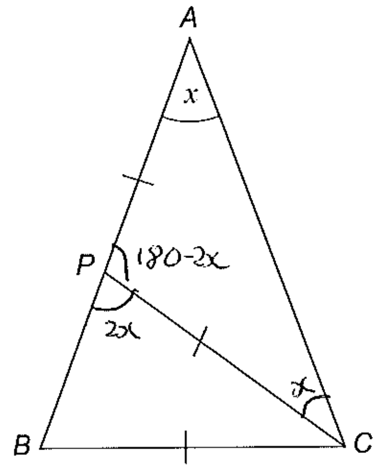
$x = 2$ $y = 2$ $x = -3$ $y = -3$ *(1 mark)*



11

Turn over ▶

5 *ABC* is a triangle.
P is a point on *AB* such that $AP = PC = BC$
Angle $BAC = x$



Not drawn accurately

5 (a) Prove that angle $ABC = 2x$

ACP = x (isosceles Δ)

APC = 180 − 2x (angles in Δ = 180)

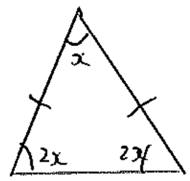
BPC = 180 − (180 − 2x) = ~~2x~~ 2x (Angles on straight line = 180)

ABC = 2x (isosceles triangle)

(3 marks)

5 (b) You are also given that $AB = AC$

Work out the value of x.



x + 2x + 2x
= 5x

5x = 180

x = 180/5

$x =$ 36° degrees *(3 marks)*

Do not write outside the box

6 (a) Expand $3x(2x - 5y)$

Answer $6x^2 - 15xy$ *(2 marks)*

6 (b) Expand and simplify $(3x + 2y)(3x - 4y)$

$9x^2 - 12xy + 6xy - 8y^2$

Answer $9x^2 - 6xy - 8y^2$ *(3 marks)*

6 (c) Work out the ratio $(3x + 2y)(3x - 4y) : 3x(2x - 5y)$ when $y = 0$

Give your answer as simply as possible.

$y = 0 \rightarrow (3x + 0)(3x - 0) : 3x(2x - 0)$

$\rightarrow (3x)(3x) : (3x)(2x)$

$\div 3x$ $\quad 3x : 2x$

$\div x$

Answer 3 : 2 *(2 marks)*

7 $1 \leqslant m \leqslant 5$ and $-9 \leqslant n \leqslant 2$

7 (a) Work out an inequality for $m + n$. Add together!

~~m~~ $1 + -9 = -8$

$5 + 2 = 7$

Answer $-8 \leqslant m + n \leqslant 7$ *(2 marks)*

7 (b) Work out an inequality for $(m + n)^2$.

$(-8)^2 = 64$, $7^2 = 49$

BUT smallest value is if $m = 1$ and $n = -1$

$\rightarrow (m+n)^2 = 0^2 = 0$

Answer $0 \leqslant (m + n)^2 \leqslant 64$ *(2 marks)*

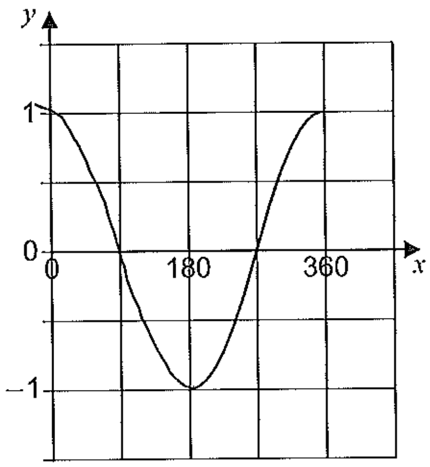
17

Turn over ▶

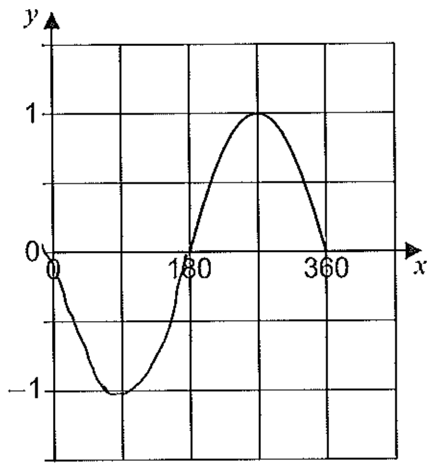
Do not write outside the box

8 Four graphs are shown for $180° \leqslant x \leqslant 360°$

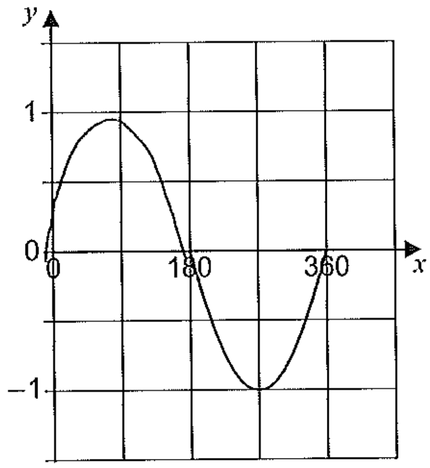
Graph A



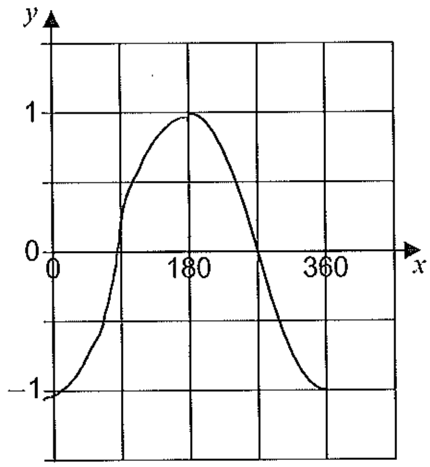
Graph B



Graph C



Graph D



8 (a) Which graph is $y = \sin x$?

Graph ......C...... *(1 mark)*

8 (b) Which graph is $y = \cos x$?

Graph ......A...... *(1 mark)*

9 Here is a formula.

$$5t + 3 = 4w(t + 2)$$

9 (a) Rearrange the formula to make t the subject.

$5t + 3 = 4wt + 8w$

$-4wt$ $5t - 4wt + 3 = 8w$

-3 $5t - 4wt = 8w - 3$

Factorise! $t(5 - 4w) = 8w - 3$

$\div (5-4w)$ $t = \dfrac{8w - 3}{5 - 4w}$

Answer .... *(4 marks)*

9 (b) Work out the exact value of t when $w = -\frac{1}{8}$

Give your answer in its simplest form.

$t = \dfrac{8(-1/8) - 3}{5 - 4(-1/8)}$

$= \dfrac{-1 - 3}{5 + 1/2} = \dfrac{-4}{5\frac{1}{2}} = \dfrac{-8}{11}$

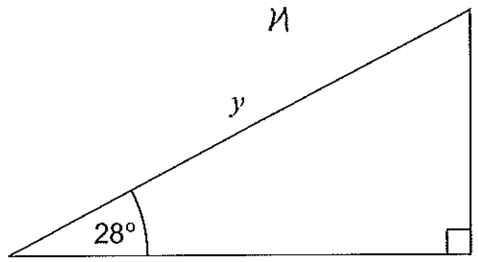
$t =$ $-8/11$ *(3 marks)*

9

Turn over ▶

Do not write outside the box

10 An aircraft flies y kilometres in a straight line at an angle of elevation of 28°.
The gain in height is 7 kilometres.



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$\sin\theta = \frac{Opp}{Hyp}$

Work out the value of y.

$\sin(28) = 7/y$

$y\sin(28) = 7$

$y = 7/\sin(28)$

$= 14.9103\ldots$

$y =$ 14.9 (1dp) km *(3 marks)*

11 A sphere has radius x centimetres.
A hemisphere has radius y centimetres.
The shapes have equal volumes.

Work out the value of $\frac{y}{x}$.

Give your answer in the form $a^{\frac{1}{3}}$ where a is an integer.

$\frac{4}{3} \div 2 = \frac{2}{3}$

$\boxed{x}$ $V = \frac{4}{3}\pi x^3$ $\boxed{y}$ $V = \frac{2}{3}\pi y^3$

Volumes are equal $\rightarrow$ $\frac{4}{3}\pi x^3 = \frac{2}{3}\pi y^3$

$\div\pi$ $\frac{4}{3}x^3 = \frac{2}{3}y^3$

$\times 3$ $4x^3 = 2y^3$

$\div x^3$ $4 = \frac{2y^3}{x^3}$

$\div 2$ $2 = \frac{y^3}{x^3}$

$\sqrt[3]{\ }$ $\sqrt[3]{2} = \frac{y}{x}$

$\rightarrow \frac{y}{x} = 2^{\frac{1}{3}}$

$\frac{y}{x} =$ $\frac{y}{x} = 2^{\frac{1}{3}}$ *(3 marks)*

Do not write outside the box

12 Expand and simplify $(t+4)^3$

$= (t+4)(t+4)(t+4)$

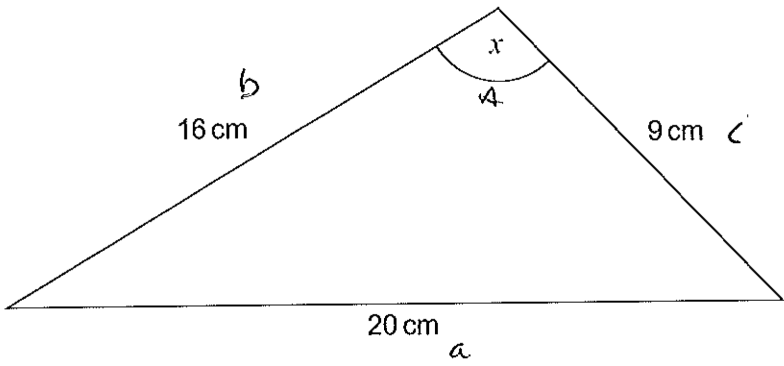
$= (t^2+4t+4t+16)(t+4)$

$= (t^2+8t+16)(t+4)$

$= t^3 + 8t^2 + 16t + 4t^2 + 32t + 64$

Answer $t^3 + 12t^2 + 48t + 64$ *(3 marks)*

13



Not drawn accurately

Work out angle x.

Cosine rule!

$$\cos A = \frac{16^2 + 9^2 - 20^2}{2 \times 16 \times 9}$$

$\rightarrow \cos A = -0.21875...$

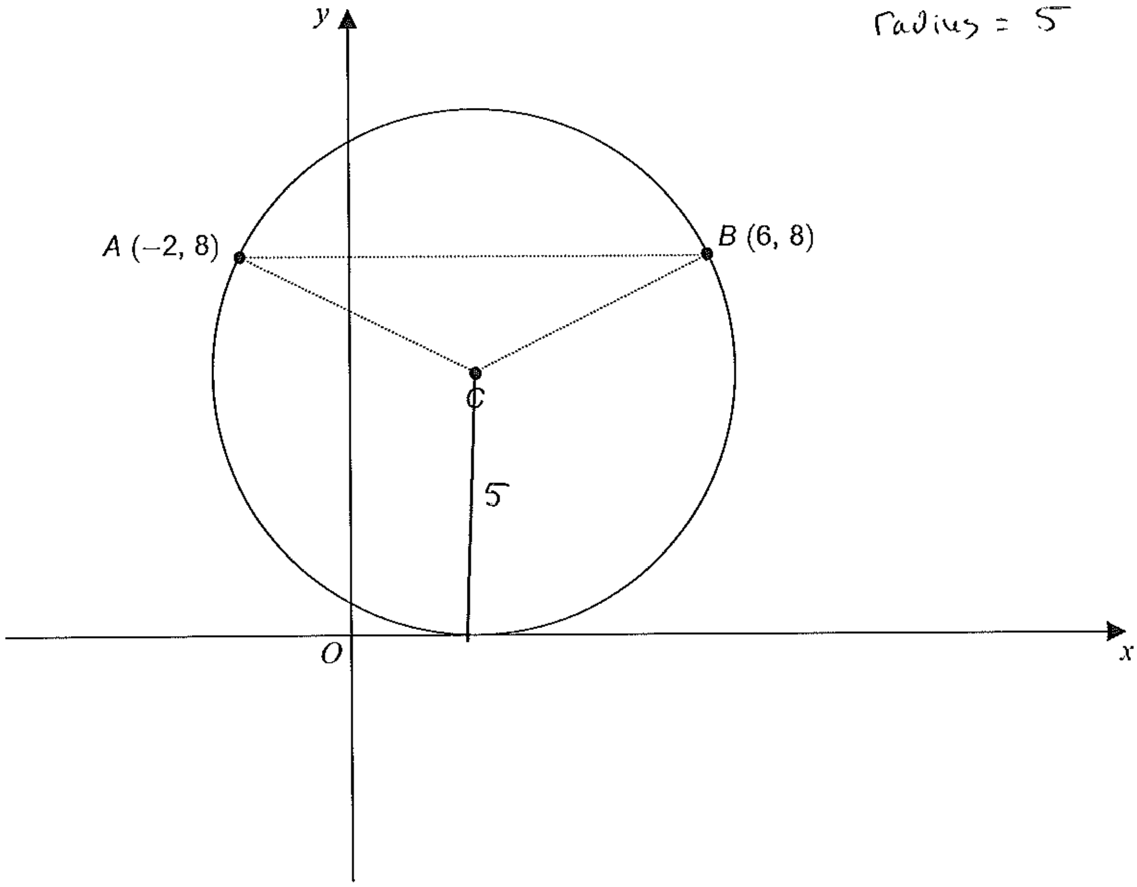
$\rightarrow A = \cos^{-1}(-0.21875...)$

$x =$ 102.6 (1dp) degrees *(3 marks)*

12

Do not write outside the box

14 The sketch shows a circle, centre C, radius 5.
The circle passes through the points A $(-2, 8)$ and B $(6, 8)$.
The x-axis is a tangent to the circle.



Work out the equation of the circle.

radius = 5 → y co-ordinate of centre = 5

midpoint of AB = (2, 8) → x - co-ordinate = 2

radius = 5, $5^2 = 25$

∴ equation is $(x-2)^2 + (y-5)^2 = 25$

Answer .... *(4 marks)*

Do not write outside the box

15 (a) $f(x) = 3x - 5$ for all values of x.

Solve $f(x^2) = 43$

$f(x^2) = 3(x^2) - 5 = 43$

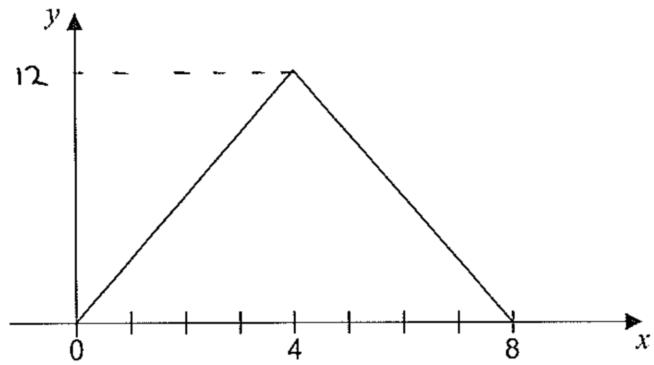
$+5 \rightarrow 3x^2 - 5 = 43$

$\div 3 \rightarrow 3x^2 = 48$

$\sqrt{\ } \rightarrow x^2 = 16$

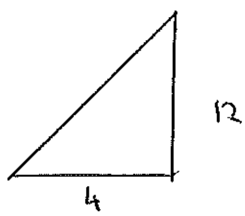
Answer $x = \pm\sqrt{16} \rightarrow x = 4$ ~~or~~ AND $x = -4$ (4 marks)

15 (b) A sketch of $y = g(x)$ for domain $0 \leqslant x \leqslant 8$ is shown.



The graph is symmetrical about $x = 4$
The range of $g(x)$ is $0 \leqslant g(x) \leqslant 12$

Work out the function $g(x)$.



gradient $= 12/4 = 3$

y-intercept $= (0,0)$

$\rightarrow$ equation is $y = 3x$

Between 4 & 8, gradient $= -3$

Co-ordinate $= (4, 12)$ (*)

(*)
$x_1 = 4$
$y_1 = 12$
$m = -3$

$y - y_1 = m(x - x_1)$

$y - 12 = -3(x - 4)$

$y - 12 = -3x + 12$

$y = -3x + 24$

$g(x) =$ $3x$ $0 \leqslant x \leqslant 4$

$-3x + 24$ $4 < x \leqslant 8$

(5 marks)

13

Do not write outside the box

16 (a) Use the factor theorem to show that $(x - 1)$ and $(x - 4)$ are factors of $x^3 - 21x + 20$

$f(x) = x^3 - 21x + 20$

$f(1) = (1)^3 - 21(1) + 20 = 0 \rightarrow (x-1)$ is a factor

$f(4) = (4)^3 - 21(4) + 20 = 0 \rightarrow (x-4)$ is a factor

(2 marks)

16 (b) Show that $(x - 1)$ and $(x - 4)$ are also factors of $x^3 - 10x^2 + 29x - 20$

$f(1) = (1)^3 - 10(1)^2 + 29(1) - 20 = 0 \rightarrow (x-1)$ is factor

$f(4) = (4)^3 - 10(4)^2 + 29(4) - 20 = 0 \rightarrow (x-4)$ is factor

(2 marks)

16 (c) Hence, simplify fully $\dfrac{x^3 - 21x + 20}{x^3 - 10x^2 + 29x - 20}$

$x^3 - 21x + 20 = (x-1)(x-4)(\quad)$

must be $x+5$ as

$-1 \times -4 \times 5 = 20$

$x^3 - 10x^2 + 29x - 20 = (x-1)(x-4)(\quad)$

must be $x-5$ as

$-1 \times -4 \times -5 = -20$

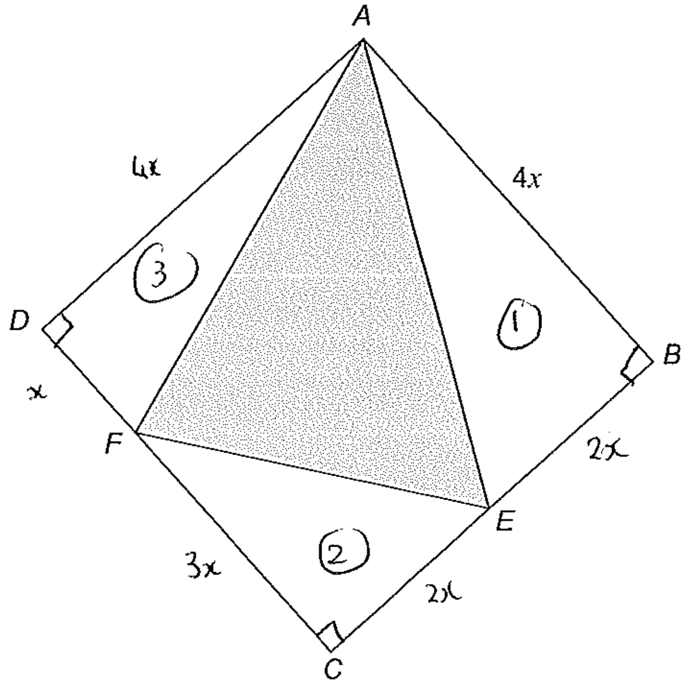
$\rightarrow \dfrac{(x-1)(x-4)(x+5)}{(x-1)(x-4)(x-5)}$

Answer $\dfrac{x+5}{x-5}$ *(3 marks)*

Do not write outside the box

17 $ABCD$ is a square of side length $4x$.

E is the midpoint of BC.
$DF : FC = 1 : 3$



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You are given that

area of triangle $AEF = kx^2$

Work out the value of k.

Area of square $= 4x \times 4x = 16x^2$

Area of ① $= \frac{1}{2} \times 2x \times 4x = 4x^2$

Area of ② $= \frac{1}{2} \times 2x \times 3x = 3x^2$

Area of ③ $= \frac{1}{2} \times x \times 4x = 2x^2$

Area of ① + ② + ③ $= 9x^2$

$\therefore$ Area of shaded triangle $= 16x^2 - 9x^2$

$= 7x^2$

$k =$ 7 *(5 marks)*

12

Turn over ▶

18 $(x-5)^2 + a \equiv x^2 + bx + 28$

Work out the values of a and b. (Complete the square or expand)

$(x-5)^2 + a$

$(x-5)(x-5) + a$

$= x^2 - 10x + 25 + a = x^2 + bx + 28$

$\rightarrow b = -10$

AND $25 + a = 28 \rightarrow a = 3$

$a =$ 3 $b =$ -10 *(3 marks)*

19 Solve the simultaneous equations

$x + y = 4 \rightarrow y = 4 - x$ ①

$y^2 = 4x + 5$ ②

Do **not** use trial and improvement.

Sub ① into ②

$\rightarrow (4-x)^2 = 4x + 5$

$(4-x)(4-x) = 4x + 5$

$16 - 4x - 4x + x^2 = 4x + 5$

$x^2 - 8x + 16 = 4x + 5$

$-4x$, -5

$x^2 - 12x + 16 = 5$

$x^2 - 12x + 11 = 0$

$(x-1)(x-11) = 0$

$x = 1$ $x = 11$

$\boxed{x=1}$ $y = 4 - 1 = 3$

$\boxed{x=11}$ $y = 4 - 11 = -7$

$x = 1, y = 3$

AND

$x = 11, y = -7$

Answer .... *(6 marks)*

Do not write outside the box

20 For what values of x is $y = 150x - 2x^3$ an increasing function?

$dy/dx > 0$ (positive!)

$y = 150x - 2x^3$

$dy/dx = 150 - 6x^2$

Find turning points when $dy/dx = 0$

$\rightarrow 150 - 6x^2 = 0$

$+6x^2$ $150 = 6x^2$

$\div 6$ $25 = x^2$

$\surd$ $x = \pm\sqrt{25} = 5$ or -5

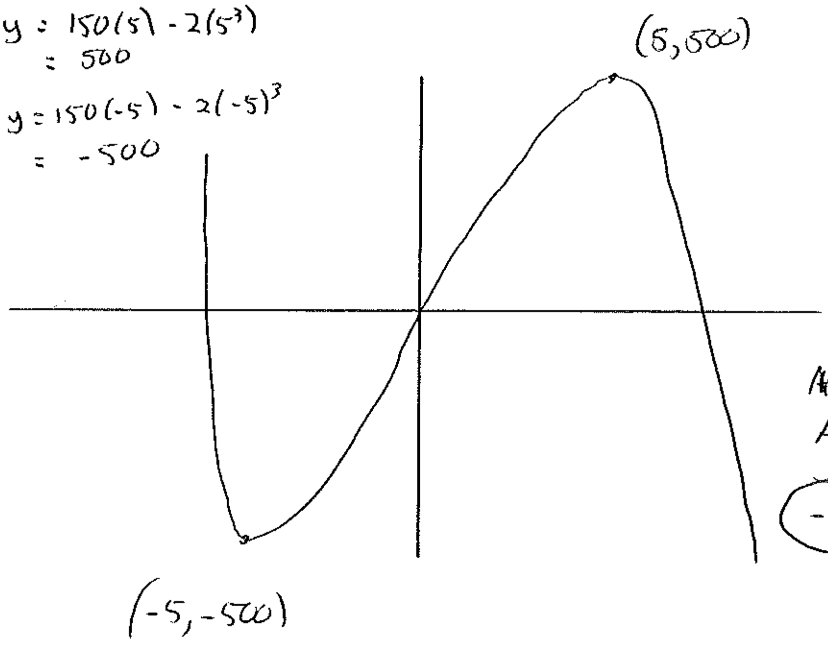
Answer .... (4 marks)

Turn over for the next question

$\boxed{\text{Negative cubic!}}$

$x = 5,\ y = 150(5) - 2(5^3)$
$= 500$

$x = -5$ $y = 150(-5) - 2(-5)^3$
$= -500$



Function increases between -5 and 5

~~Ans~~ Answer:

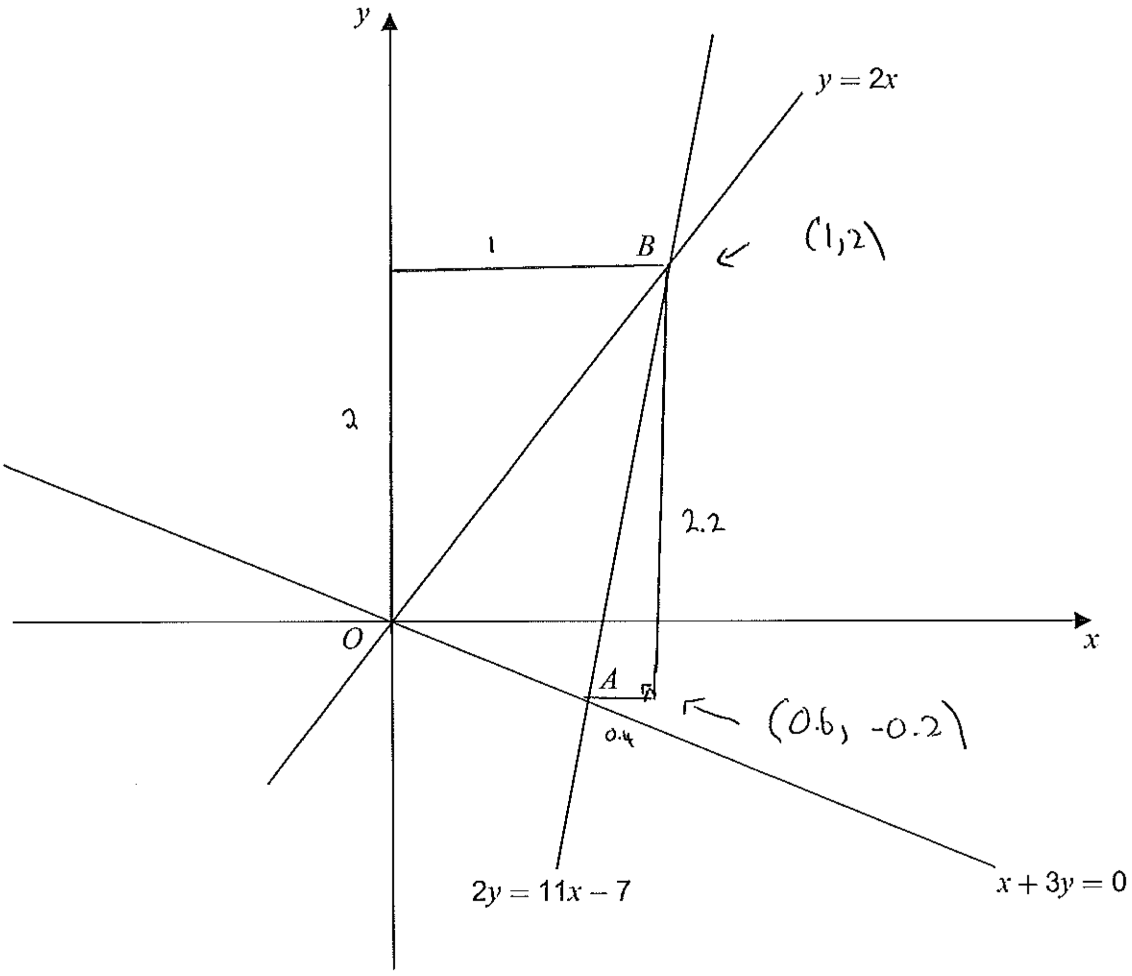
$-5 < x < 5$

13

21 The equations of three straight lines are

$$y = 2x \qquad x + 3y = 0 \qquad 2y = 11x - 7$$

The lines intersect at the points O, A and B as shown on this sketch.



See next page for working out

Do not write outside the box

Show that length OB = length AB

$\boxed{\text{FIND A}}$ where $2y = 11x - 7$ (2) meets $x + 3y = 0$

$\rightarrow x = -3y$ (1)

Sub (1) into (2) $\rightarrow 2y = 11(-3y) - 7$

$\rightarrow 2y = -33y - 7$

$\rightarrow 35y = -7$

$\rightarrow y = -7/35 = -0.2$

(1) $x = -3y = -3(-0.2) = 0.6$

$\therefore A = (0.6, -0.2)$

$\boxed{\text{Find B}}$ where $y = 2x$ meets $2y = 11x - 7$

$\hookrightarrow 2y = 4x$ (1) $\quad 2y = 11x - 7$ (2)

Put (1) = (2) $\rightarrow 4x = 11x - 7$

$\rightarrow 0 = 7x - 7$

$\rightarrow 7 = 7x \rightarrow x = 1$

(1) $y = 2x \rightarrow y = 2(1) = 2$

$\therefore B = (1, 2)$

$\boxed{\text{See Diagram!}}$ $OB = \sqrt{1^2 + 2^2} = \sqrt{5}$

$AB = \sqrt{0.4^2 + 2.2^2} = \sqrt{5}$

(6 marks)

$\therefore OB = AB$

Turn over for the next question

6

Turn over ▶

22 The transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps point P to point Q. T1

The transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ maps point Q to point R. T2

Point R is $(-4, 3)$.

Work out the coordinates of point P.

$\boxed{\text{Work backwards}}$

$(T_2)(Q) = (R)$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \quad \rightarrow x = -4 \quad \rightarrow y = -3$$

$\rightarrow Q = (-4, -3)$

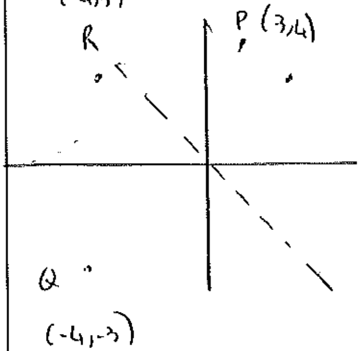
$(T_1)(P) = (Q)$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad \rightarrow y = 4 \quad \rightarrow x = 3$$

$\rightarrow P = (3, 4)$

OR Do it by T_1 = reflection in $y = -x$

T_2 = reflection in x - axis



Answer (3 , 4) **(5 marks)**

Do not write outside the box

23 The curve $y = f(x)$ is such that $\frac{dy}{dx} = -x(x-2)^2$

The stationary points of the curve are at $\left(0, \frac{4}{3}\right)$ and (2, 0).

Determine the nature of each stationary point.
You **must** show your working.

$\boxed{\text{Test } x = 0}$ $x = -1$, $dy/dx = -(-1)(-1-2)^2 = 1(-3)^2 = \cancel{8}\ 9$

$x = 1$ $dy/dx = -(1)(1-2)^2 = (-1)(-1)^2 = -1$

Gradient Goes : Positive – 0 – negative

∴ $(0, 4/3)$ is MAXIMUM

$\boxed{\text{Test } x = 2}$ $x = 1$ $dy/dx = -1$ (from before)

$x = 3$ $dy/dx = -(3)(3-2)^2 = -3(1) = -3$

Gradient goes : negative – 0 – negative

(4 marks)

∴ (2, 0) is a POINT OF INFLECTION.

END OF QUESTIONS

9