MR BARTON'S ANSWERS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 – 5	
6 – 7	
8 – 9	
10 – 11	
12 – 13	
14	
TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2012

Further Mathematics 8360/1
Level 2
Paper 1 Non-Calculator

Tuesday 29 May 2012 1.30 pm to 3.00 pm

For this paper you must have:
- mathematical instruments.

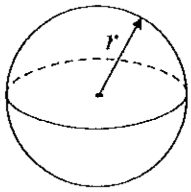
You may **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

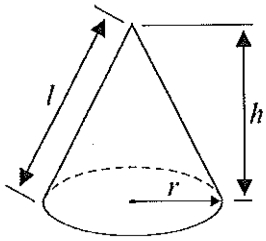
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Formulae Sheet



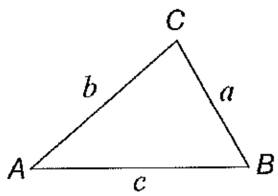
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\qquad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $\qquad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta} \qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

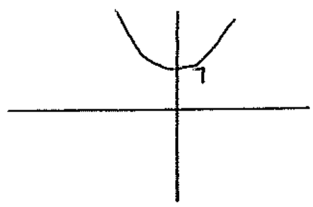
1 $f(x) = 2x^2 + 7$ for all values of x.

1 (a) What is the value of $f(-1)$?

Answer $2(-1)^2 + 7 = 2 + 7 = 9$ *(1 mark)*

1 (b) What is the range of $f(x)$?

Answer Range : $y \geq 7$ *(1 mark)*



2 $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

Work out the matrix **AB**.

$$\begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 17 \end{pmatrix}$$

AB = $\begin{pmatrix} 10 \\ 17 \end{pmatrix}$ *(2 marks)*

4

Do not write outside the box

3 Work out the greatest integer value of x that satisfies the inequality $3x + 10 < 1$

-10 $3x < -9$

$\div 3$ $x < -3$

Answer $x = -4$ *(2 marks)*

4 (a) Factorise fully $2x^2 - 2x - 40$

$(2x + 8)(x - 5)$

$2(x + 4)(x - 5)$

Answer $2(x + 4)(x - 5)$ *(3 marks)*

4 (b) Factorise fully $(x + y)^2 + (x + y)(2x + 5y)$

$(x + y)[(x + y) + (2x + 5y)$

$= (x + y)[3x + 6y]$

Answer $3(x + y)(x + 2y)$ *(3 marks)*

$3(x + y)(x + 2y)$

Do not write outside the box

5 Simplify $(2cd^4)^3$

$2^3\ c^3\ (d^4)^3$

Answer $8c^3d^{12}$ (2 marks)

6 Solve the simultaneous equations

$2y = 3x + 4 \rightarrow 2y - 3x = 4$ ① $-3x$

$2x = -3y - 7 \rightarrow 3y + 2x = -7$ ② $+3y$

Do **not** use trial and improvement.

① $\times 2 \rightarrow 4y - 6x = 8$

② $\times 3 \rightarrow 9y + 6x = -21$ (+)

$13y = -13$

$\rightarrow y = -1$

② $3y + 2x = -7$

$3(-1) + 2x = -7 \rightarrow -3 + 2x = -7$

$2x = -4$

$x = -2$

Answer (4 marks)

$x = -2, \quad y = -1$

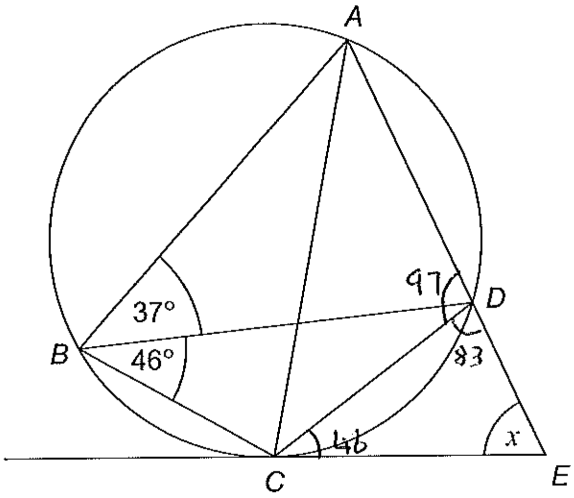
14

Turn over ▶

Do not write outside the box

7 The diagram shows a cyclic quadrilateral *ABCD*.

ADE is a straight line.
CE is a tangent to the circle.



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Work out the size of angle x.

ADC = (180 − (37+46)) = 97° (opposite angles in cyclic quad add to 180°)

∴ CDE = 180 − 97 = 83°

ECD = 46° (alternate segment theorem)

∴ x = 180 − 83 − 46

$x =$ 51° degrees *(3 marks)*

Do not write outside the box

8 A curve has equation $y = x^3 + 5x^2 + 1$

8 (a) When $x = -1$, show that the value of $\frac{dy}{dx}$ is -7.

$dy/dx = 3x^2 + 10x$

when $x = -1$, $dy/dx = 3(-1)^2 + 10(-1)$

$= 3 - 10 = -7$

(2 marks)

8 (b) Work out the equation of the tangent to the curve $y = x^3 + 5x^2 + 1$ at the point where $x = -1$

Need point + gradient

$x = -1 \rightarrow y = (-1)^3 + 5(-1)^2 + 1 = -1 + 5 + 1 = 5$

$x_1 = -1$ $\quad y - y_1 = m(x - x_1)$

$y_1 = 5$ $\quad y - 5 = -7(x - (-1))$

$m = -7$ $\quad y - 5 = -7x - 7$

from part a)

Answer $y = -7x - 2$ *(4 marks)*

Turn over for the next question

9

Do not write outside the box

9 Write this ratio in its simplest form

$$\sqrt{12} : \sqrt{48} : \sqrt{300}$$

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

$\sqrt{300} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$

$\rightarrow \quad 2\sqrt{3} : 4\sqrt{3} : 10\sqrt{3} \quad \div\sqrt{3}$

$2 : 4 : 10 \quad \div 2$

Answer 1 : 2 : 5 *(3 marks)*

10 The n^{th} term of the linear sequence 2 7 12 17 ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that **all** the terms in the new sequence are multiples of 5.

nth term $= (5n - 3)^2 + 1$

$= (5n - 3)(5n - 3) + 1$

$= 25n^2 - 15n - 15n + 9 + 1$

$= 25n^2 - 30n + 10$

$= 5(5n^2 - 6n + 2)$

Anything multiplied by 5 must be a multiple of 5

(4 marks)

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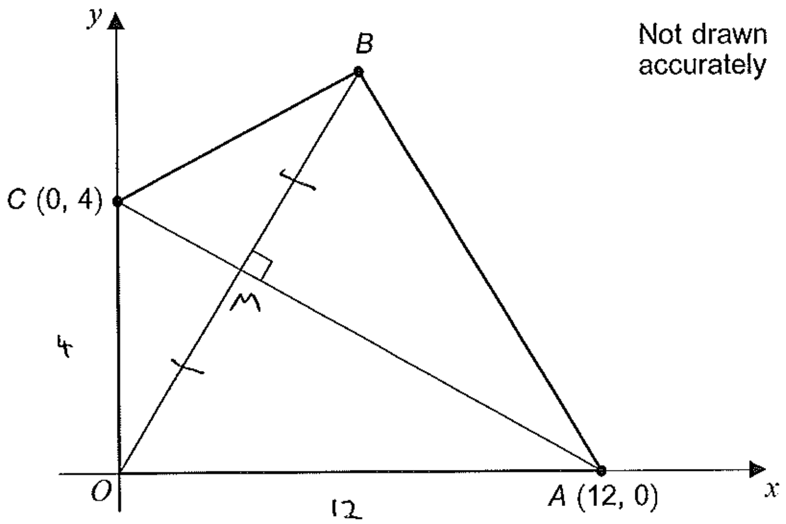
11 *OABC* is a kite.



11 (a) Work out the equation of *AC*.

gradient = $-4/12 = -1/3$

y-intercept = (0,4)

Answer $y = -1/3x + 4$ (2 marks)

11 (b) Work out the coordinates of *B*.

OB is perpendicular to AC

∴ gradient = 3

y-intercept = (0,0) → Equation of OB = $y = 3x$

M = crossing point of OB & AC

At M : $3x = -1/3x + 4$

$+1/3x$ → $3\frac{1}{3}x = 4$

→ $10/3\ x = 4$

$x = 12/10 = 6/5 = 1.2$

$y = 3x \rightarrow 3(6/5) = 18/5 = 3.6$

B must be 2 × M

→ $x = 2 \times 1.2 = 2.4$

$y = 2 \times 3.6 = 7.2$

Answer (2.4 , 7.2) (6 marks)

15

Turn over ▶

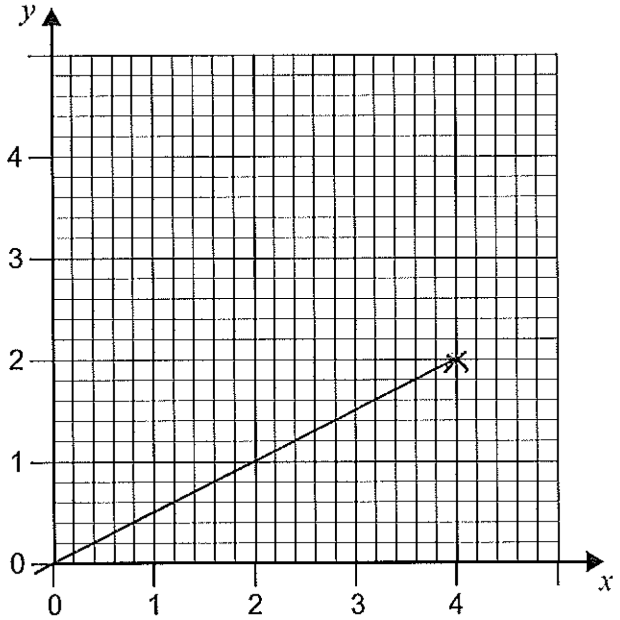
12 (a) A graph passes through (0, 0).

The rate of change of y with respect to x is always $\frac{1}{2}$.

← gradient

$y = \frac{1}{2}x$

Draw the graph of y for values of x from 0 to 4.



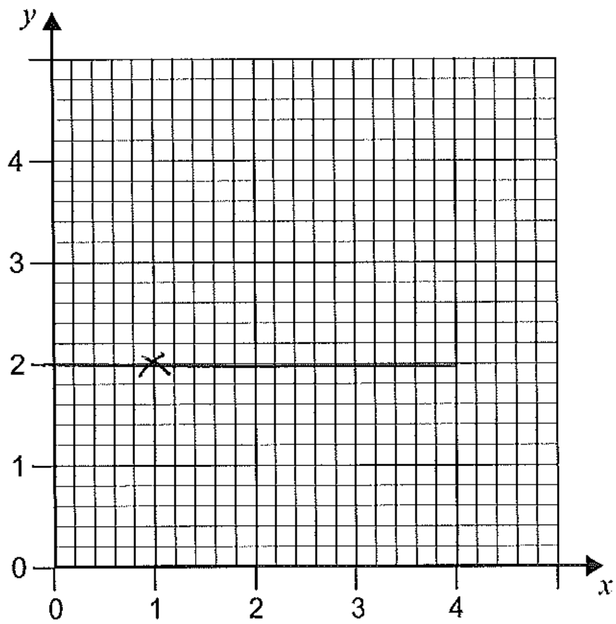
(1 mark)

12 (b) A graph passes through (1, 2).

The rate of change of y with respect to x is always 0. → gradient = 0

Draw the graph of y for values of x from 0 to 4.

$y = 2$



(1 mark)

Do not write outside the box

12 (c) $y = 2x^3 + ax$, where a is a constant.

The value of $\frac{dy}{dx}$ when $x = 2$ is twice the value of $\frac{dy}{dx}$ when $x = -1$

Work out the value of a.

$y = 2x^3 + ax$

$dy/dx = 6x^2 + a$

when $x = 2 \Rightarrow dy/dx = 6(2)^2 + a = 24 + a$

when $x = -1 \Rightarrow dy/dx = 6(-1)^2 + a = 6 + a$

$2(6 + a) = 24 + a$

$\Rightarrow 12 + 2a = 24 + a$

$\Rightarrow 2a = 12 + a$

$\Rightarrow a = 12$

$a =$ .... *(5 marks)*

Turn over for the next question

7

Turn over ▶

Do not write outside the box

13 Simplify $\dfrac{x^2+4x-12}{x^2-25} \div \dfrac{x+6}{x^2-5x}$

$= \dfrac{x^2+4x-12}{x^2-25} \times \dfrac{x^2-5x}{x+6}$

$= \dfrac{\cancel{(x+6)}(x-2)}{(x+5)\cancel{(x-5)}} \times \dfrac{x\cancel{(x-5)}}{\cancel{(x+6)}}$

Answer $\dfrac{x(x-2)}{x+5}$ *(5 marks)*

14 $x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \dfrac{25}{4}$ where $y > 0$

Work out the value of $\dfrac{x}{y}$.

$x^{3/2} = 8$ $\qquad$ $y^{-2} = 25/4$

$\sqrt[3]{}$ $x^{1/2} = \sqrt[3]{8} = 2$ $\qquad$ $y^2 = 4/25$

2 $x = 2^2 = 4$ $\qquad$ $\sqrt{}$ $y = \sqrt{4}/\sqrt{25} = 2/5$

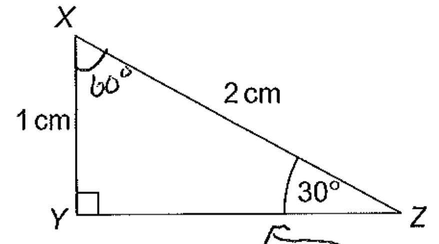
$x/y = 4 \div 2/5$

$= 4 \times 5/2 = 20/2$

$= 10$

$\dfrac{x}{y} =$ 10 *(5 marks)*

15 (a) XYZ is a right-angled triangle.



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By Pythag:

$YZ = \sqrt{2^2 - 1^2}$

$= \sqrt{3}$

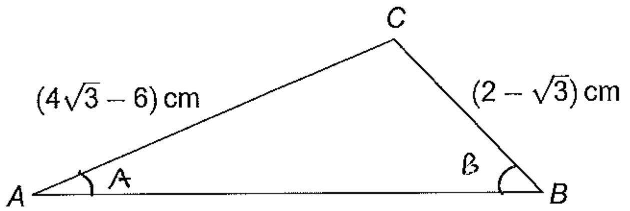
Use triangle XYZ to show that $\sin 60° = \dfrac{\sqrt{3}}{2}$

$\sin x = \text{opp}/\text{Hyp}$

$\sin(60) = \sqrt{3}/2$

(2 marks)

15 (b) Triangle ABC has an obtuse angle at C.



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Sine rule!

Given that $\sin A = \dfrac{1}{4}$, use triangle ABC to show that angle $B = 60°$

Rationalise denominator!

$\dfrac{\sin A}{2-\sqrt{3}} = \dfrac{\sin B}{4\sqrt{3}-6}$

$\times 2-\sqrt{3}$: $\sin A = \dfrac{(2-\sqrt{3})\sin B}{4\sqrt{3}-6}$

$= 1/4$

$\times 4\sqrt{3}-6$: $(4\sqrt{3}-6)\sin A = (2-\sqrt{3})\sin B$

$1/4(4\sqrt{3}-6) = (2-\sqrt{3})\sin B$

$\sqrt{3} - 1.5 = (2-\sqrt{3})\sin B$

$\div 2-\sqrt{3}$: $\dfrac{\sqrt{3}-1.5}{2-\sqrt{3}} = \sin B$

$\dfrac{\sqrt{3}-1.5}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \dfrac{2\sqrt{3} + 3 - 3 - 1.5\sqrt{3}}{4 + 2\sqrt{3} - 2\sqrt{3} - 3}$

$= \dfrac{0.5\sqrt{3}}{1} = \sqrt{3}/2 = \sin B$

$= 60°$

(6 marks)

18

Do not write outside the box

16 Prove that $\tan\theta + \dfrac{1}{\tan\theta} \equiv \dfrac{1}{\sin\theta\cos\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\sin\theta \times \sin\theta = \sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta \times \sin\theta}{\sin\theta\cos\theta} + \dfrac{\cos\theta \times \cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

(3 marks)

END OF QUESTIONS

3