AQA Level 2 Certificate in Further Mathematics

Specimen Assessment Materials 8360

For exams June 2012 onwards
For certification June 2012 onwards

AQA Level 2 Certificate in Further Mathematics - May 2011

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Background Information

Introduction

This Level 2 Certificate in Further Mathematics qualification **fills the gap** for high achieving students by assessing their higher order mathematical skills, particularly in algebraic reasoning, in greater depth without infringing upon AS Level mathematics, thus preparing them fully to maximise their potential in further studies at Level 3. It offers the opportunity for stretch and challenge that builds on the Key Stage 4 curriculum and is intended as an additional qualification to the GCSE Mathematics, rather than as a replacement.

The content assumes prior knowledge of the Key Stage 4 Programme of Study and covers the areas of algebra and geometry, which are crucial to further study in the subject, in greater depth and breadth. This new qualification places an emphasis on higher order technical proficiency, rigorous argument and problem solving skills. It also gives an introduction to calculus and matrices and develops further skills in trigonometry, functions and graphs.

The AQA Level 2 Certificate in Further Mathematics is an untiered Level 2 linear qualification for learners who

- either already have, or are expected to achieve grades A and A* in GCSE mathematics
- are likely to progress to A-Level study in mathematics and possibly further mathematics.

It will be graded on a five-grade scale: A* with Distinction (A^), A*, A, B and C.

The qualification is designed to be assessed as a full Level 2 mathematics qualification in its own right and is therefore not dependent on GCSE mathematics.

Therefore there are no prior learning requirements but there is the expectation that candidates have some assumed knowledge.

The specification content is set out in six distinct topic areas although questions will be asked that range across these topics.

- Number
- Algebra
- Co-ordinate Geometry (2 dimensions only)
- Calculus
- Matrix Transformations
- Geometry

Papers

These specimen papers have been designed to exemplify the question papers, to be set for our Level 2 Certificate in Further Mathematics Specification, for first qualification in June 2012. The associated mark scheme follows each paper.

The question papers should be read in conjunction with AQA Level 2 Certificate in Further Mathematics Specification 2011 onwards. This specification is available on the website http://web.aqa.org.uk/qual/igcse/maths.php

The question papers are intended to represent the length and balance of the papers that will be set for the examination and to indicate the types of questions that will be used. It must be emphasised, however, that the questions have not been subjected to the rigorous review that would take place with questions before use in examination.

Mark schemes

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

Certificate in Further Mathematics
Level 2

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
TOTAL	

Further Mathematics 8360/1

Level 2

Specimen Paper 1

Non-Calculator

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

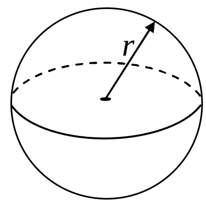
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

8360/1

Formulae Sheet

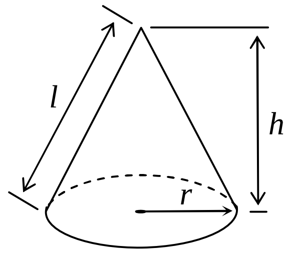
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



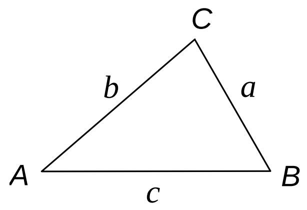
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*

Area of triangle $= \frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ $\sin^2 \theta + \cos^2 \theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) Solve $7(3x - 1) + 2(x + 7) = 3(6x - 1)$

Answer $x =$ .... *(4 marks)*

1 (b) Solve $\sqrt{3x + 10} = 4$

Answer $x =$ .... *(2 marks)*

Turn over for the next question

6

2 (a) The nth terms of two sequences are $4n + 13$ and $6n - 21$

Which term has the same value in each sequence?

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Answer ………………………………………………………… *(3 marks)*

2 (b) The first five terms of a quadratic sequence are 4 10 18 28 40

Work out an expression for the nth term.

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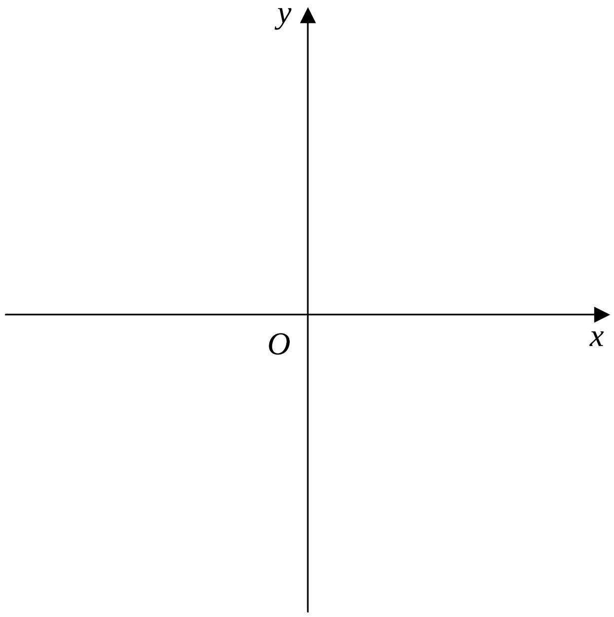
Answer ………………………………………………………… *(5 marks)*

Do not write outside the box

3 (a) On the axes below sketch the graph of $y = x^2 - 9$

Label clearly any points of intersection with the x-axis.



(2 marks)

3 (b) Write down all the integer solutions to $x^2 - 9 < 0$

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Answer .... *(2 marks)*

Turn over for the next question

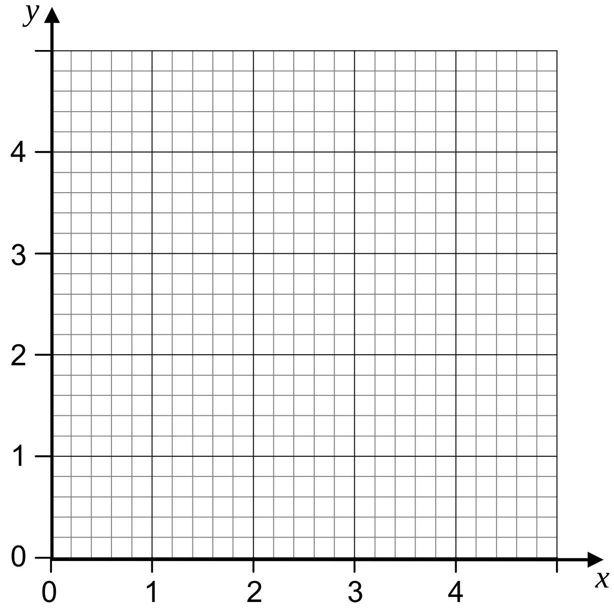
12

Do not write outside the box

4 A function $f(x)$ is defined as

$$f(x) = \begin{cases} 3x & 0 \leqslant x < 1 \\ 3 & 1 \leqslant x < 3 \\ 12 - 3x & 3 \leqslant x \leqslant 4 \end{cases}$$

Calculate the area enclosed by the graph of $y = f(x)$ and the x-axis.



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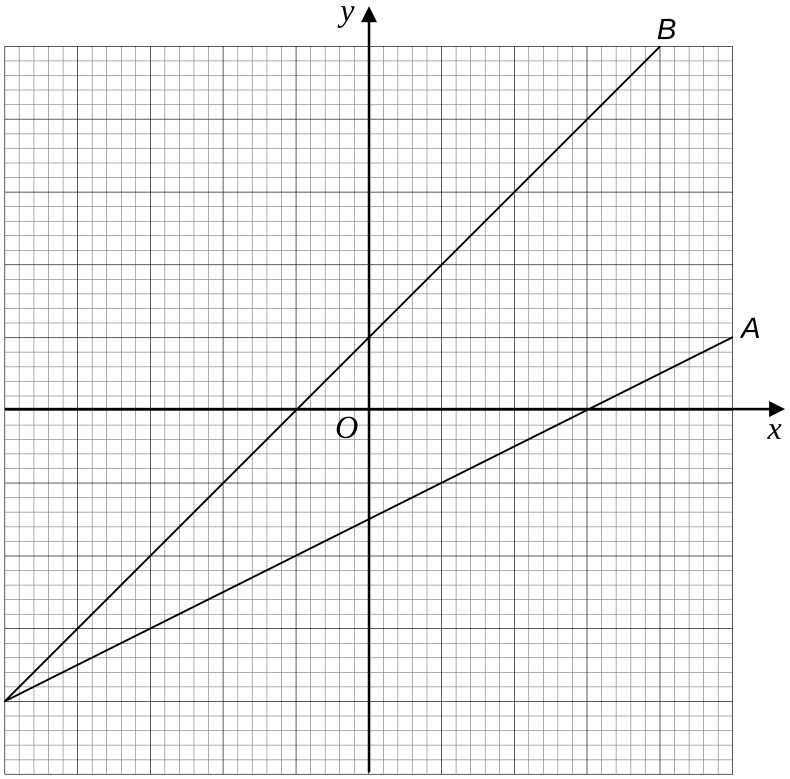
Answer .... units^2 *(5 marks)*

Do not write outside the box

5 The graph shows two lines *A* and *B*.

The equation of line *B* is $y = 2x + 2$



Work out the equation of line *A*.

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Answer .... *(4 marks)*

9

Turn over ▶

8360/1

6 Work out $2\frac{2}{3} - 1\frac{3}{4} \div 1\frac{1}{8}$

Give your answer as a fraction in its simplest form.

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Answer .... *(5 marks)*

7 (a) Solve $x^{\frac{2}{3}} = 9$

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Answer $x =$ ……... *(2 marks)*

7 (b) The reciprocal of $y^{\frac{1}{2}}$ is 5

Work out the value of y.

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Answer …….. *(2 marks)*

Do not write outside the box

8 Make d the subject of $c = \dfrac{8(c - d)}{d}$

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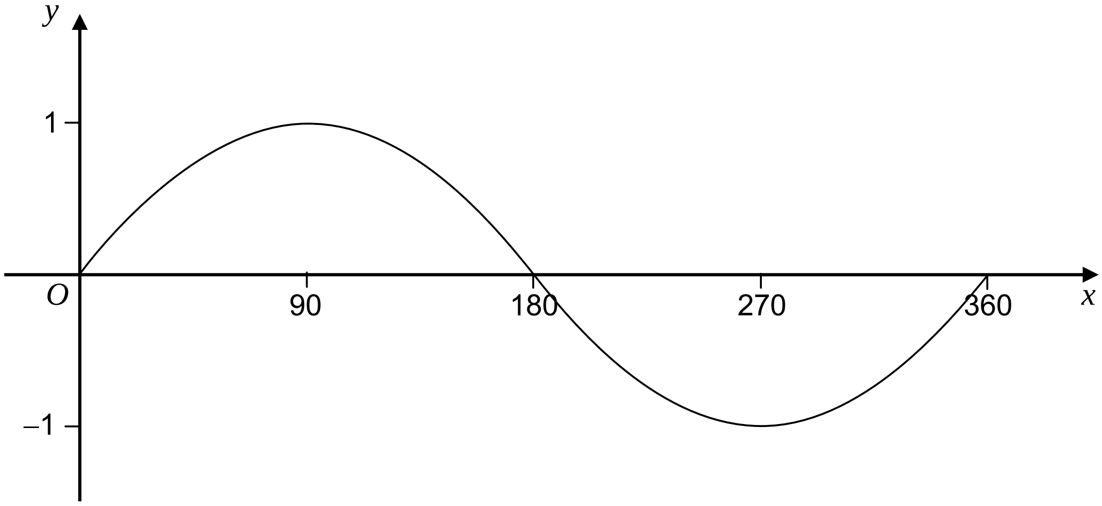
Answer .... *(4 marks)*

9 The sketch shows $y = \sin x$ for $0° \leqslant x \leqslant 360°$



The value of sin 73° = 0.956 to 3 significant figures.

Use the sketch to find **two** angles between 0° and 360° for which $\sin x = -0.956$

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Answer .... and .... *(2 marks)*

15

Do not write outside the box

10 (a) Write $\sqrt{75} + \sqrt{12}$ in the form $a\sqrt{b}$ where a and b are integers.

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Answer .... *(2 marks)*

10 (b) Rationalise and simplify $\dfrac{2\sqrt{2}+1}{\sqrt{2}-3}$

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Answer .... *(5 marks)*

11 The points $A\,(-1, -7)$ and $B\,(24, 23)$ are on a straight line ACB.

$AC : CB = 2 : 3$

Work out the coordinates of C.

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Answer (...................., ...) *(4 marks)*

12 Prove that $\tan^2 x - 1 \equiv \dfrac{1 - 2\cos^2 x}{\cos^2 x}$

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(3 marks)

13 (a) Work out the coordinates of the stationary point for the curve $y = x^2 + 3x + 4$

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Answer (...................., ...) *(4 marks)*

13 (b) Explain why the equation $x^2 + 3x + 4 = 0$ has no real solutions.

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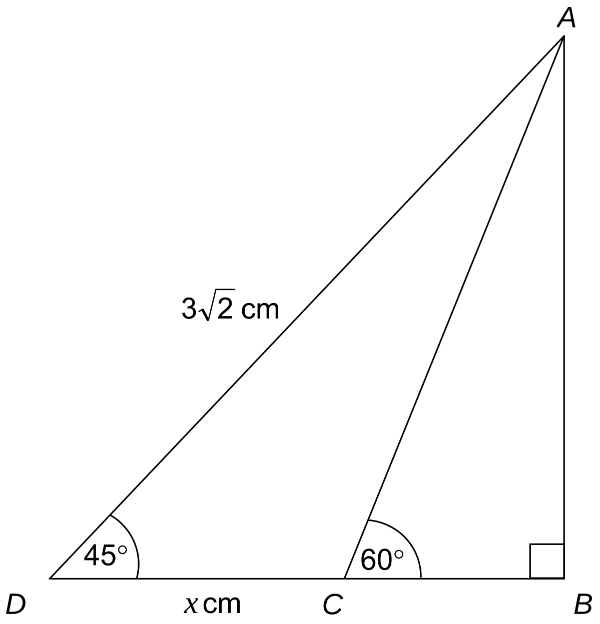
(2 marks)

20

Do not write outside the box

14 In the diagram, *DCB* is a straight line.



Work out the length of *DC*, marked x on the diagram.

Write your answer in the form $a - \sqrt{b}$

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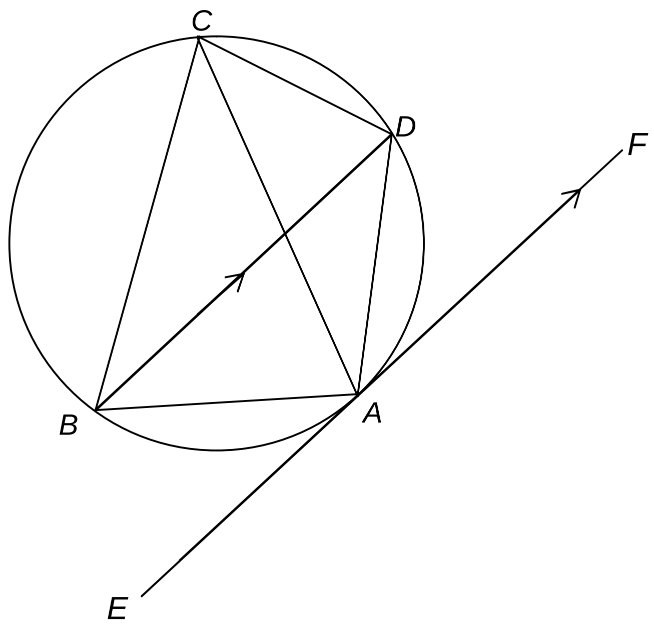
Answer .... cm *(4 marks)*

Do not write outside the box

15 A, B, C and D are points on the circumference of a circle such that BD is parallel to the tangent to the circle at A.



Prove that AC bisects angle BCD.

Give reasons at each stage of your working.

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(4 marks)

END OF QUESTIONS

8

Level 2 Certificate in Further Mathematics

Specimen Paper 1 8360/1

Mark Scheme

Mark Schemes

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

It is not possible to indicate all the possible approaches to questions that would gain credit in a 'live' examination. The principles we work to are given in the glossary on page 3 of this mark scheme.

- Evidence of any method that would lead to a correct answer, if applied accurately, is generally worthy of credit.
- Accuracy marks are awarded for correct answers following on from a correct method. The correct method may be implied, but in this qualification there is a greater expectation that method will be appropriate and clearly shown.

Set and published by the Assessment and Qualifications Alliance.

Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Paper 1 - Non-Calculator

Q	Answer	Mark	Comments
1(a)	$21x - 7 + 2x + 14 = 18x - 3$	M1	Allow one error
	Their $21x + 2x - 18x = -3 + 7 - 14$	M1	Allow one rearrangement error
	$5x = -10$	A1ft	
	$x = -2$	A1ft	Must have gained M2 for ft
1(b)	$3x + 10 = 16$	M1	
	$x = 2$	A1	

Q	Answer	Mark	Comments
2(a)	$4n + 13 = 6n - 21$	M1	List terms in both sequences with 81 appearing in both lists
	$6n - 4n = 13 + 21$	M1	$4n + 13 = 81$ or $6n - 21 = 81$
	17	A1	
2(b)	Attempt at first differences (at least three) 6 8 10 12	M1	Alternative - Works with $an^2 + bn + c$ Attempt to find at least two of the three equations in a, *b* and *c* eg, any two of a + b + c = 4 4a + 2b + c = 10 9a + 3b + c = 18
	Attempt at second differences (at least two) and divides their second difference by 2 to obtain coefficient of n^2 2 2 2 and $1n^2$	M1	Eliminates one letter from any two of their equations eg, 3a + b = 6 or 5a + b = 8 or 8a + 2b = 14
	Subtracts n^2 from original sequence 4 – 1 10 – 4 18 – 9 28 – 16 40 – 25 (= 3 6 9 12 15)	M1	Eliminates the same letter from a different pair of their equations
	Attempt at differences of their 3 6 9 12 15 or $3n$	M1	Attempt at solving their two equations in two variables
	$n^2 + 3n$	A1	(a = 1, b = 3, c = 0) $n^2 + 3n$

Q	Answer	Mark	Comments
3(a)	U shape crossing x-axis in two places	B1	
	–3 and 3 marked	B1	
3(b)	–2, –1, 0, 1, 2	B2	Any 3 of these B1 These 5 plus -3 and 3 B1
4	Graph drawn	B3	B1 For each part Accept vertices of trapezium clearly marked
	$\frac{1}{2}$ (4 + 2) × 3	M1	Attempt to find their area
	9	A1 ft	
5	Attempt to work out the scale on the y-axis eg, 0, 2, 4, seen as labels or statement that y-axis goes up in 2s or evidence that y intercept is 2 for given line	M1	
	Attempt to work out the scale on the x-axis eg, 0, 1, 2, seen as labels or evidence of using gradient of 2 for given line and scale on y-axis to work out horizontal scale	M1	
	Evidence of working out gradient eg, triangle drawn on graph or 2 ÷ 2 or 1	M1	
	$y = x - 3$	A1	oe

Q	Answer	Mark	Comments
6	Attempts division before subtraction	B1	
	$\frac{7}{4} \div \frac{9}{8}$	M1	Allow one error in numerators
	$\frac{14}{9}$	A1	oe fraction
	$\frac{24}{9}$ – their $\frac{14}{9}$	M1	
	$\frac{10}{9}$	A1ft	oe ft $2\frac{2}{3}$ – their $\frac{14}{9}$
7(a)	$x = 9^{\frac{3}{2}}$	M1	oe
	27	A1	
7(b)	$\frac{1}{5^2}$ or $y^{-1} = 25$ or $y^{\frac{1}{2}} = \frac{1}{5}$ or $\frac{1}{y^{\frac{1}{2}}} = 5$	M1	
	$\frac{1}{25}$	A1	oe
8	$cd = 8(c - d)$	M1	or $c = \frac{8c - 8d}{d}$
	$cd = 8c - 8d$	M1	
	$cd + 8d = 8c$	M1	
	$d = \frac{8c}{(c + 8)}$	A1	
9	270 – 17 (= 253) or 270 + 17 (= 287)	M1	
	253 and 287	A1	

Q	Answer	Mark	Comments
10(a)	$5\sqrt{3}$ (+) $2\sqrt{3}$	M1	
	$7\sqrt{3}$	A1	
10(b)	$\dfrac{(2\sqrt{2}+1)(\sqrt{2}+3)}{(\sqrt{2}-3)(\sqrt{2}+3)}$	M1	
	Num $2 \times 2 + \sqrt{2} + 6\sqrt{2} + 3$	M1	
	$7 + 7\sqrt{2}$	A1	
	Denom $2 - 9$	A1	
	$-1 - \sqrt{2}$	A1ft	Allow $-(1 + \sqrt{2})$ ft If both Ms awarded
11	$24 - -1$ (= 25) or $23 - -7$ (= 30)	M1	
	$\frac{2}{5}$ × their 25 (= 10) or $\frac{2}{5}$ × their 30 (=12)	M1	$\frac{3}{5}$ × their 25 (= 15) or $\frac{3}{5}$ × their 30 (= 18)
	-1 + their 10 (= 9) or -7 + their 12 (= 5)	M1	24 – their 15 (= 9) or 23 – their 18 (= 5)
	(9, 5)	A1	
12	$\dfrac{\sin^2 x}{\cos^2 x} - 1$	M1	Use of $\tan x \equiv \dfrac{\sin x}{\cos x}$
	$\dfrac{\sin^2 x - \cos^2 x}{\cos^2 x}$	M1	
	$\dfrac{1 - \cos^2 x - \cos^2 x}{\cos^2 x}$	A1	

Q	Answer	Mark	Comments
13(a)	$\left(\frac{dy}{dx} =\right)$ $2x + 3$	M1	$(x + 1\frac{1}{2})^2 - 1\frac{1}{2}^2 + 4$
	$x = -1\frac{1}{2}$	A1	oe
	$y = (-1\frac{1}{2})^2 + 3(-1\frac{1}{2}) + 4$	M1	$(x + 1\frac{1}{2})^2 + 1.75$
	$y = 1\frac{3}{4}$	A1ft	oe turning points at $(-1\frac{1}{2}, 1\frac{3}{4})$ Allow follow through if first M1 awarded
13(b)	Sketch showing turning point above x-axis and statement that curve never crosses x-axis so no solution (B1 For sketch showing turning point above x-axis with statement not made)	B2	B2 A complete valid explanation using correct mathematical language eg, stating that $b^2 - 4ac = -7$ which is < 0 so implies no real solution due to a negative number not having a real square root B1 For a partially correct explanation using correct mathematical language eg, stating that $b^2 - 4ac = -7$ which is < 0 so implies no real solution

Q	Answer	Mark	Comments
14	$BD = 3\sqrt{2}\ \cos 45\ (= 3)$ or $AB = 3\sqrt{2}\ \sin 45\ (= 3)$	M1	
	$BC =$ their $\frac{AB}{\tan 60} = \left(\frac{3}{\sqrt{3}}\right)$	M1	
	$BC = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	M1	
	$3 - \sqrt{3}$	A1	

Q	Answer	Mark	Comments
15	$\angle BCA = \angle BAE$ Alternate segment theorem	B1	oe Correct geometrical reasons must be given
	$\angle BAE = \angle DBA$ Alternate angles equal	B1	oe Correct geometrical reasons must be given
	$\angle DBA = \angle ACD$ Angles in the same segment are equal	B1	oe Correct geometrical reasons must be given
	So $\angle BCA = \angle ACD$ AC bisects $\angle BCD$	B1	SC2 For correct argument without reasons

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
14 - 15	
16	
TOTAL	

Certificate in Further Mathematics
Level 2

Further Mathematics 8360/2

Level 2

Specimen Paper 2

Calculator

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
2 hours

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

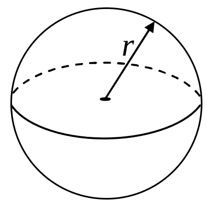
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

8360/2

Formulae Sheet

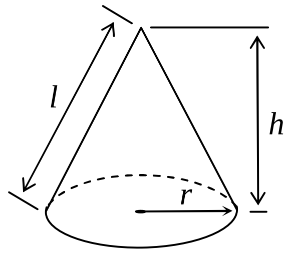
Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$



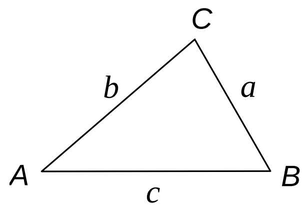
Volume of cone = $\frac{1}{3}\pi r^2 h$

Curved surface area of cone = $\pi r l$



In any triangle *ABC*

Area of triangle = $\frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 a, b, c and d are consecutive integers.

Explain why $ab + cd$ is always even.

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(3 marks)

2 Work out the distance between the point $A(1, 4)$ and the point $B(7, 12)$.

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Answer .... units *(2 marks)*

3 The nth term of a sequence is given by $\dfrac{3n+1}{6n-5}$

3 (a) Write down the first, tenth and hundredth terms of the sequence.

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Answer .... , ... , ... *(2 marks)*

3 (b) Show that the limiting value of $\dfrac{3n+1}{6n-5}$ is $\dfrac{1}{2}$ as $n \to \infty$

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(2 marks)

9

Turn over ▶

Do not write outside the box

4 The function $f(x)$ is defined as $f(x) = x^2 + x$

4 (a) Write down the value of $f(7)$

....

Answer .... *(1 mark)*

4 (b) Solve $f(x) = 0$

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....

Answer .... *(2 marks)*

4 (c) Write an expression for $f(x+1) - f(x)$

Give your answer in its simplest form.

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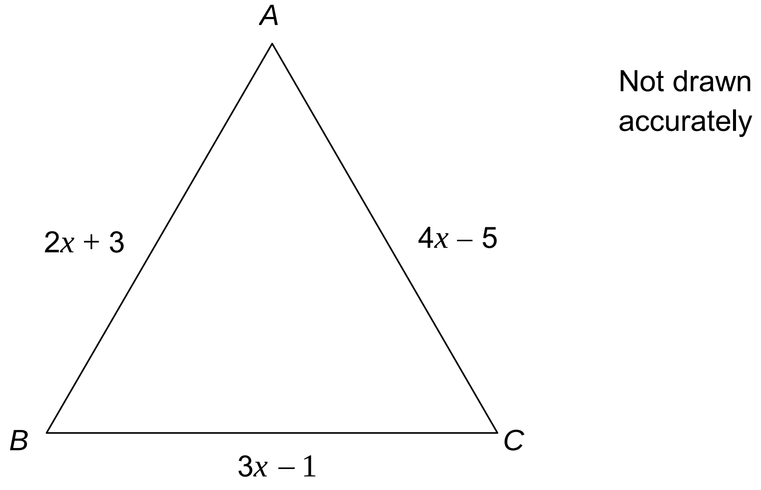
Answer .... *(3 marks)*

Do not write outside the box

5 The diagram shows triangle ABC with $AB = AC$.



Show that triangle ABC is equilateral.

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(5 marks)

6 x, y and z are three quantities such that

$x : y = 3 : 2$ and $y : z = 5 : 4$

Express the ratio $x : z$ in its simplest form.

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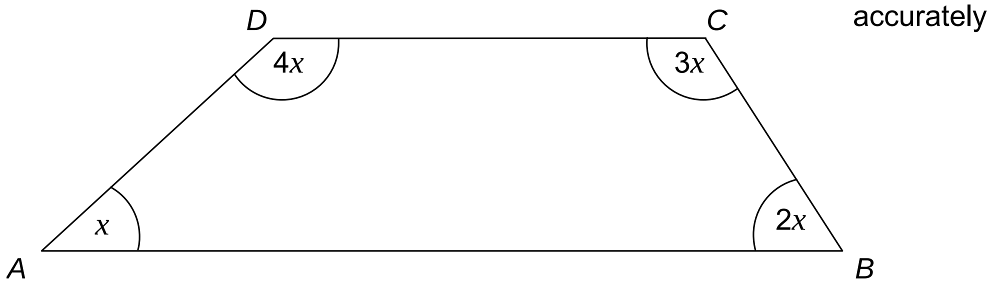
Answer .... : .... *(3 marks)*

14

Do not write outside the box

7 *ABCD* is a quadrilateral.



Prove that *AB* is parallel to *DC*.

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(5 marks)

Do not write outside the box

8 The function f(x) is defined as $f(x) = \dfrac{1}{x^2 - 3x - 10}$

f(x) has domain all x except $x = a$ and $x = b$

Work out a and b.

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Answer .... *(3 marks)*

9 (a) Expand and simplify $(x - 5)(x^2 + 4x - 2)$

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Answer .... *(4 marks)*

9 (b) Factorise fully $(x^2 - 16) - (x - 4)(3x + 5)$

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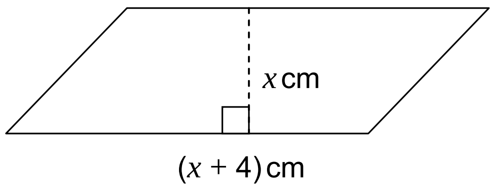
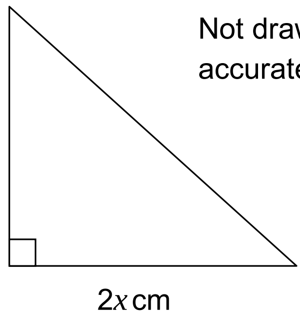
Answer .... *(4 marks)*

16

Do not write outside the box

10 Here are a parallelogram and an isosceles triangle.





Not drawn accurately

10 (a) The area of the triangle is greater than the area of the parallelogram.

Show that $x^2 - 4x > 0$

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(3 marks)

10 (b) Work out the least integer value for x.

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Answer .... *(3 marks)*

11 Write $\dfrac{a^{\frac{1}{2}} \times a^{\frac{3}{2}}}{(a^3)^4}$ as a single power of a.

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Answer .... *(3 marks)*

Do not write outside the box

12 n is an integer.

Prove that $(n-2)^2 + n(8-n)$ is always a multiple of 4.

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(3 marks)

13 Solve the simultaneous equations $y^2 = x + 3$ and $y = 2x$

Do **not** use trial and improvement.

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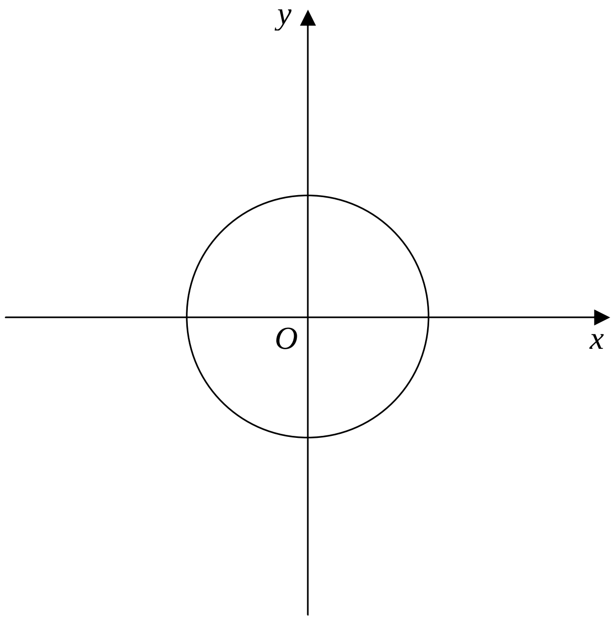
Answer .... *(5 marks)*

17

Turn over ▶

Do not write outside the box

14 On the axes below is a circle centre (0, 0) and passing through the point (3, 0).



14 (a) Write down the equation of the circle.

Answer .... *(1 mark)*

14 (b) Decide whether the point (2, 2) is inside or outside the circle.
Show how you decide.

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Answer .... *(2 marks)*

14 (c) The circle above is translated so that the image of (3, 0) is (5, 4).

Write down the equation of the new circle.

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Answer .... *(2 marks)*

Do not write outside the box

15 A triangle has sides 10.2 cm, 6.8 cm and 5.7 cm.

Work out the area of the triangle.

Answer .... cm^2 *(5 marks)*

16 Work out the equation of the perpendicular bisector of $P(3, -1)$ and $Q(5, 7)$.

Give your answer in the form $y = ax + b$

Answer .... *(5 marks)*

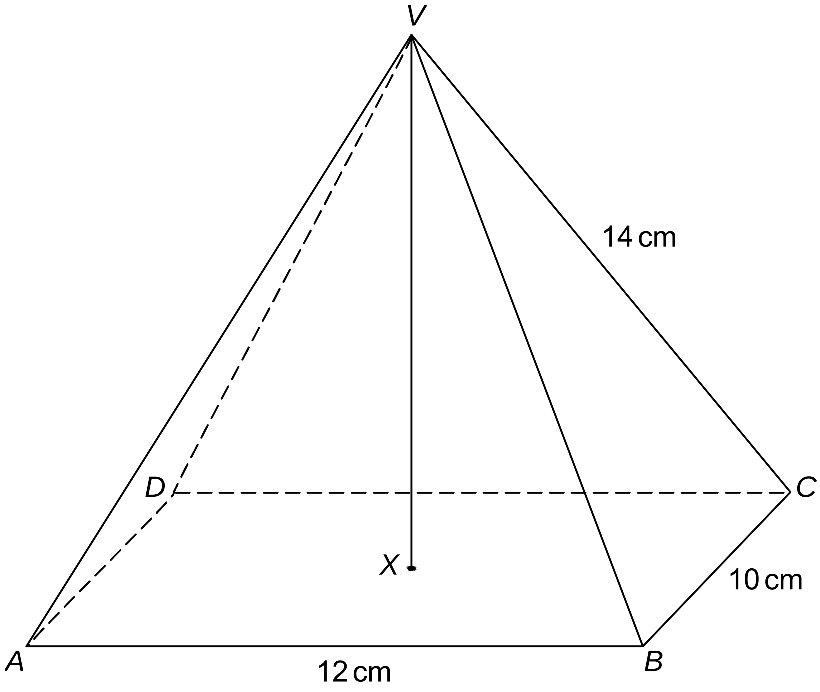
15

Do not write outside the box

17 *VABCD* is a rectangular based pyramid.

$AB = 12$ cm, $BC = 10$ cm and $VC = 14$ cm

The base *ABCD* is horizontal and the vertex *V* is directly above *X*, the centre of the base.



17 (a) Work out the height of the pyramid, *VX*.

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Answer .... cm *(4 marks)*

17 (b) Calculate the angle between *VC* and the plane *ABCD*.

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Answer .... degrees *(2 marks)*

Do not write outside the box

17 (c) Calculate the angle between the planes *VBC* and *ABCD*.

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Answer .... degrees *(3 marks)*

18 Solve the equation $\cos^2 x = 0.8$ for $0° \leqslant x \leqslant 360°$

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Answer .... *(3 marks)*

19 $y = x^4(2x + 5)$

Work out the rate of change of y with respect to x when $x = 2$

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Answer .... *(5 marks)*

17

20 (a) Matrix **A** = $\begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}$

Work out the image of point $P(2, -1)$ using transformation matrix **A**.

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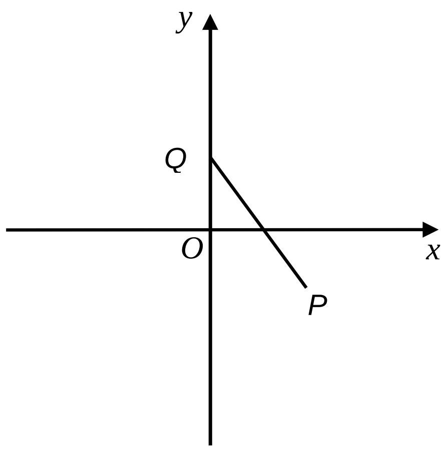
Answer (..............................., ...) *(2 marks)*

20 (b) Point Q is (0, 1)

Line PQ is transformed to line $P'Q'$ using matrix **A**.



Work out the length of $P'Q'$.

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Answer .... units *(2 marks)*

21 Factorise fully $x^3 - 4x^2 - 11x + 30$

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Answer .... *(6 marks)*

Turn over for the next question

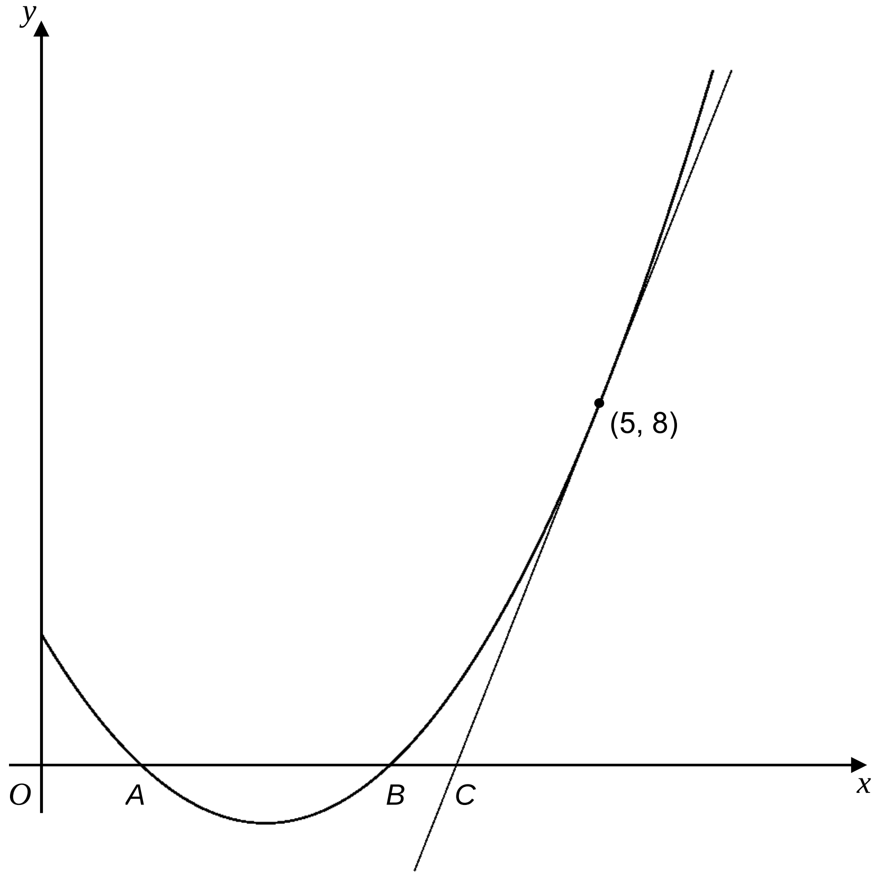
10

Do not write outside the box

22 The diagram shows the graph of $y = x^2 - 4x + 3$

The curve cuts the x-axis at the points A and B.

The tangent to the curve at the point (5, 8) cuts the x-axis at the point C.



Show that $AB = 3BC$

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(7 marks)

END OF QUESTIONS

7

Level 2 Certificate in Further Mathematics

Specimen Paper 2 8360/2

Mark Scheme

Mark Schemes

Principal Examiners have prepared these mark schemes for specimen papers. These mark schemes have not, therefore, been through the normal process of standardising that would take place for live papers.

It is not possible to indicate all the possible approaches to questions that would gain credit in a ‘live’ examination. The principles we work to are given in the glossary on page 3 of this mark scheme.

- Evidence of any method that would lead to a correct answer, if applied accurately, is generally worthy of credit.
- Accuracy marks are awarded for correct answers following on from a correct method. The correct method may be implied, but in this qualification there is a greater expectation that method will be appropriate and clearly shown.

Set and published by the Assessment and Qualifications Alliance.

Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Paper 2 - Calculator

Q	Answer	Mark	Comments
1	Any consecutive pair contains an even (and an odd)	M1	$n(n + 1) + (n + 2)(n + 3)$
	Even × odd = even	M1	$n^2 + n + n^2 + 3n + 2n + 6$ Allow 1 error
	Even + even = even	A1	$2(n^2 + 3n + 3)$ so even
2	$\sqrt{((7-1)^2 + (12-4)^2)}$	M1	
	10	A1	
3(a)	4 $\frac{31}{55}$ $\frac{301}{595}$	B2	B1 For two correct oe
3(b)	Reference to $3n + 1 \to 3n$ or $6n - 5 \to 6n$ when n is large	B1	oe Must include reference to n being large
	$\frac{3n}{6n}$ cancelled to $\frac{1}{2}$	B1	
Alt 3(b)	$\dfrac{\frac{3n}{n} + \frac{1}{n}}{\frac{6n}{n} - \frac{5}{n}}$	M1	
	$\frac{3}{6}$ since $\frac{1}{n}$ and $\frac{5}{n} \to 0$ as $n \to \infty$	A1	oe
4(a)	56	B1	
4(b)	$x(x + 1) = 0$	M1	
	0 and –1	A1	
4(c)	$(x + 1)^2 + x + 1 - x^2 - x$	M1	Allow 1 sign error
	$x^2 + x + x + 1 + x + 1 - x^2 - x$	A1	oe
	$2x + 2$ or $2(x + 1)$	A1	

Q	Answer	Mark	Comments
5	$4x - 5 = 2x + 3$	M1	
	$4x - 2x = 3 + 5$	M1	Allow one sign error
	$x = 4$	A1ft	
	Substitute their x into one of the equal sides	M1	eg, $4 \times 4 - 5$ or $2 \times 4 = 3$ (=11)
	Shows BC is $3 \times 4 - 1 = 11$ and 11 obtained for either AB or AC	A1	

Q	Answer	Mark	Comments
6	Attempt at common value for y in order to eliminate y eg, $3 \times 5 : 2 \times 5$ and $5 \times 2 : 4 \times 2$	M1	Attempt to find two equations in order to eliminate y eg, $y = \frac{2x}{3}$ and $y = \frac{5z}{4}$
	15 : 10 and 10 : 8	A1	oe eg, $\frac{2x}{3} = \frac{5z}{4}$ or $8x = 15z$
	15 (: 10) : 8	A1	15 : 8

Q	Answer	Mark	Comments
7	$x + 2x + 3x + 4x = 360$	M1	oe
	$10x = 360$ ($x = 36$)	M1	
	Their 36 × 2 and their 36 × 3 or their 36 × 4	M1	
	36 + 144 = 180 or 72 + 108 = 180	M1	oe
	Concludes that *AB* is parallel to *DC* because allied/interior angles add up to 180°	A1	

Q	Answer	Mark	Comments
Alt 7	$x + 4x + 3x + 2x = 360$	M1	oe
	$10x = 360$ ($x = 36$)	M1	
	$5x = 180$	M1	
	$x + 4x = 5x$, so angle *A* + angle *D* = 180° or $3x + 2x = 5x$, so angle *C* + angle *B* = 180°	M1	oe
	Concludes that *AB* is parallel to *DC* because allied/interior angles add up to 180°	A1	oe

Q	Answer	Mark	Comments
8	Sets denominator to zero or attempts to factorise in the form $(x \pm a)(x \pm b)$ where $ab = 10$	M1	$x^2 - 3x - 10 = 0$
	$(x + 2)(x - 5)$	A1	
	– 2 (and) 5	B1ft	ft From their factors

Q	Answer	Mark	Comments
9(a)	$x^3 + 4x^2 - 2x$	M1	Allow 1 error
	$-5x^2 - 20x + 10$	M1	Allow 1 error
	$x^3 + 4x^2 - 2x - 5x^2 - 20x + 10$	A1	
	$x^3 - x^2 - 22x + 10$	A1ft	
9(b)	$(x + 4)(x - 4) - (x - 4)(3x + 5)$	M1	
	$(x - 4)(\ldots\ldots\ldots\ldots\ldots\ldots\ldots..)$	M1	
	$(x - 4)(x + 4 - 3x - 5)$	A1	
	$(x - 4)(-2x - 1)$	A1ft	oe eg, $-(x - 4)(2x + 1)$
Alt 9(b)	$(-)(3x^2 - 12x + 5x - 20)$	M1	
	$-2x^2 + 7x + 4$	A1	
	$(x + a)(-2x + b)$ $ab = 4$	M1	
	$(x - 4)(-2x - 1)$	A1 ft	oe eg, $-(x - 4)(2x + 1)$

10(a)	Attempt to work out both areas	M1	ie, $\frac{1}{2}(2x \times 2x)$ and $x(x + 4)$ Allow one error
	Correct expression for both areas	A1	
	$2x^2 > x^2 + 4x$	A1	
10(b)	$x(x - 4) > 0$	M1	Attempts U-shaped sketch of $y = x^2 - 4x$ crossing x-axis at $x = 0$ and $x = 4$
	$(x < 0$ and$)$ $x > 4$	M1	
	5	A1	

Q	Answer	Mark	Comments
11	(numerator) a^2	B1	
	(denominator) a^{12}	B1	
	a^{-10}	B1ft	ft If numerator and denominator seen as powers of a
12	$n^2 - 4n + 4 + 8n - n^2$	M1	Allow one error or omission
	$4n + 4$	A1	
	$4(n + 1)$	A1	$(4n + 4) \div 4 = n + 1$
13	Attempt to eliminate one variable eg, $(2x)^2 = x + 3$	M1	$y^2 = \frac{y}{2} + 3$
	$4x^2 - x - 3 = 0$	A1	$2y^2 - y - 6 = 0$
	Attempt at solution eg, $(4x + 3)(x - 1)$	M1	$(2y + 3)(y - 2)$ Allow correct use of formula
	$x = -\frac{3}{4}$ (and) $x = 1$	A1	$y = 2$ (and) $y = -1\frac{1}{2}$
	$y = -1\frac{1}{2}$ (and) $y = 2$	A1	$x = 1$ (and) $x = -\frac{3}{4}$
14(a)	$x^2 + y^2 = 9$	B1	
14(b)	$2^2 + 2^2$ (= 8)	M1	
	Inside and valid justification	A1	eg, inside and $8 < 9$, inside and $2.8\ldots < 3$
14(c)	$(x - 2)^2 + (y - 4)^2 =$ their 9	B2 ft	$(x + 2)^2 + (y + 4)^2 =$ their 9 B1 ft Their part (a)

Q	Answer	Mark	Comments
15	$\cos A = \dfrac{6.8^2 + 5.7^2 - 10.2^2}{2 \times 6.8 \times 5.7}$	M1	$\cos B = \dfrac{10.2^2 + 5.7^2 - 6.8^2}{2 \times 10.2 \times 5.7}$ or $\cos C = \dfrac{10.2^2 + 6.8^2 - 5.7^2}{2 \times 10.2 \times 6.8}$
	–0.32649(…) or –0.3265	A1	0.77648(…) or 0.7765 or 0.8491(…)
	109(.05 …)° or 109.06° or 109.1°	A1	39(.05 …) or 39.1 or 31.88(…) or 31.9 or 32
	$\frac{1}{2}$ × 6.8 × 5.7 × sin their 109	M1	$\frac{1}{2}$ × 10.2 × 5.7 × sin their 39 or $\frac{1}{2}$ × 10.2 × 6.8 × sin their 32
	18.3	A1ft	

Q	Answer	Mark	Comments
16	Gradient = 4	B1	
	Gradient of perpendicular = $-\frac{1}{4}$	B1ft	
	Midpoint = (4, 3)	B1	
	$y - 3 = -\frac{1}{4}(x - 4)$	M1	oe
	$y = -\frac{1}{4}x + 4$	A1	oe

Q	Answer	Mark	Comments
17(a)	$(AC^2 =)\ 12^2 + 10^2$ or $(AX^2 =)\ 6^2 + 5^2$	M1	$(VM^2 =)\ 14^2 - 5^2$ or $(VN^2 =)\ 14^2 - 6^2$
	$(AX =)\ \frac{\sqrt{244}}{2}$ $(\sqrt{61})$	A1	oe $(VM =)\ \sqrt{171}$ or $(VN =)\ \sqrt{160}$
	$(VX^2 =)\ 14^2 -$ their AX^2	M1	$(VX^2 =)$ their $VM^2 - 6^2$ or $(VX^2 =)$ their $VN^2 - 5^2$
	11.6(2)	A1	
17(b)	$\sin VCX = \frac{\text{their } VX}{14}$	M1	$\cos VCX = \frac{\text{their } \sqrt{61}}{14}$ or $\tan VCX = \frac{\text{their } VX}{\text{their } \sqrt{61}}$
	56.1°	A1	
17(c)	Use of right-angled triangle VMX where M is the mid-point of BC	M1	So that MV and MX are both at right angles to BC, thus defining the angle
	$\tan VMX = \frac{\text{their } VX}{6}$	M1	$\cos VMX = \frac{6}{\sqrt{14^2 - 5^2}}$ or $\sin VMX = \frac{\text{their } VX}{\sqrt{14^2 - 5^2}}$
	62.7°	A1	

18	$\cos x = (\pm)\ 0.894427$	M1	
	26.6, 153.4, 206.6, 333.4	A2	A1 For 2 or 3 answers

19	$2x^5 + 5x^4$	M1	
	$10x^4 + 20x^3$	A2 ft	ft Their two terms differentiated
	$10(2)^4 + 20(2)^3$	M1	$x = 2$ in their terms from differentiating
	320	A1 ft	ft If M2 awarded

Q	Answer	Mark	Comments
20(a)	(5,1)	B2	B1 For (5, k) or (c, 1) or $\begin{pmatrix}5\\1\end{pmatrix}$
20(b)	(3,1) or $\begin{pmatrix}3\\1\end{pmatrix}$	B1	
	2	B1ft	ft Their two points
21	f(2) = 8 – 16 – 22 + 30	M1	
	x – 2 is a factor	A1	
	(x – 2)(x^2……– 15)	M1	
	(x – 2)(x^2 –2x – 15)	A1	
	(x – 2) (x + a) (x + b) ab = –15	M1	
	(x – 2)(x – 5) (x + 3)	A1	
22	(x – 1)(x – 3)	M1	
	A (1, 0) and B (3, 0) or AB = 2	A1	oe
	Attempts to differentiate, evidenced by at least one term correct	M1	$\frac{dy}{dx} = 2x - 4$
	Evidence of substituting x = 5 to find the gradient of the tangent	M1	When x = 5, $\frac{dy}{dx}$ = 2 × 5 – 4 (= 6)
	Attempt to work out equation of tangent	M1	Tangent is y – 8 = their 6(x – 5) oe eg, y = 6x – 22
	Substitutes y = 0 into their equation in an attempt to obtain $x = \frac{11}{3}$	M1	oe
	$3 \times (\frac{11}{3} - 3) = 2$	A1	oe

AQA Level 2 Certificate in Further Mathematics from 2011 onwards

Qualification Accreditation Number: 600/2123/8

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Centres should be aware that candidates who enter for more than one GCSE qualification with the same classification code will have only one grade counted for the purpose of the School and College Performance Tables. In the case of a candidate taking two qualifications with the same classification code that are of the same size and level, eg two full course GCSEs, the higher grade will count.

Centres may wish to advise candidates that, if they take two specifications with the same classification code, schools and colleges are very likely to take the view that they have achieved only one of the two GCSEs. The same view may be taken if candidates take two GCSE specifications that have different classification codes but have significant overlap of content. Candidates who have any doubts about their subject combinations should check with the institution to which they wish to progress before embarking on their programmes.

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