MR BARTON'S ANSWERS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
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10 - 11	
12 - 13	
14 - 15	
16	
TOTAL	

AQA

Certificate in Further Mathematics
Level 2

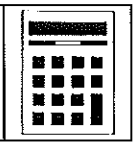
Further Mathematics 8360/2

Level 2

Specimen Paper 2

Calculator

For this paper you must have:
- a calculator
- mathematical instruments.



Time allowed
2 hours

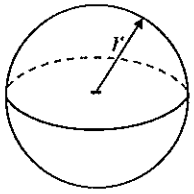
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Formulae Sheet

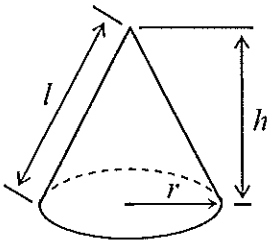
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



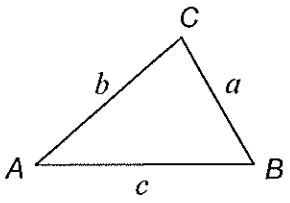
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ $\qquad \sin^2 \theta + \cos^2 \theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 a, b, c and d are consecutive integers.

Explain why $ab + cd$ is always even.

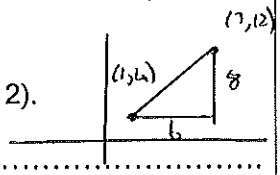
ab is even × odd or odd × even → even

cd is even × odd or odd × even → even

even + even = even

(3 marks)

2 Work out the distance between the point $A(1, 4)$ and the point $B(7, 12)$.



See sketch! → $AB = \sqrt{8^2 + 6^2}$

→ $AB = \sqrt{100}$

→ $AB = 10$

Answer 10 units *(2 marks)*

3 The nth term of a sequence is given by $\dfrac{3n + 1}{6n - 5}$

3 (a) Write down the first, tenth and hundredth terms of the sequence.

$n = 1$	$n = 10$	$n = 100$
$\frac{3(1)+1}{6(1)-5}$	$\frac{3(10)+1}{6(10)-5}$	$\frac{3(100)+1}{6(100)-5}$
$\frac{4}{1} = 4$	$\frac{31}{55}$	$\frac{301}{595}$

Answer 4, $\frac{31}{55}$, $\frac{301}{595}$ *(2 marks)*

3 (b) Show that the limiting value of $\dfrac{3n + 1}{6n - 5}$ is $\dfrac{1}{2}$ as $n \to \infty$

As $n \to \infty$ $3n + 1 \to 3n$, $6n - 5 \to 6n$

$\to \dfrac{3n}{6n} \to \dfrac{3}{6} \to \dfrac{1}{2}$

(2 marks)

9

4 The function f(x) is defined as $f(x) = x^2 + x$

4 (a) Write down the value of f(7)

$f(7) = 7^2 + 7 = 56$

Answer 56 *(1 mark)*

4 (b) Solve $f(x) = 0$

$x^2 + x = 0$

FACT!

$x(x+1) = 0$

Answer $x = 0$ and $x = -1$ *(2 marks)*

4 (c) Write an expression for $f(x+1) - f(x)$

Give your answer in its simplest form.

$f(x+1) = (x+1)^2 + (x+1)$

$= x^2 + 2x + 1 + x + 1$

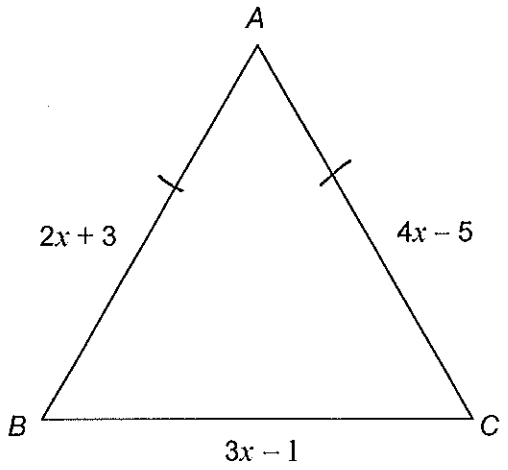
$= x^2 + 3x + 2$

$f(x+1) - f(x) = x^2 + 3x + 2 - (x^2 + x)$

Answer $2x + 2$ *(3 marks)*

Do not write outside the box

5 The diagram shows triangle ABC with $AB = AC$.



Not drawn accurately

Show that triangle ABC is equilateral.

$AB = AC \rightarrow 4x - 5 = 2x + 3$

$-2x \quad 2x - 5 = 3$

$+5 \quad 2x = 8$

$\div 2 \quad x = 4$

Use $x = 4$ to find side lengths

$AB \quad 2(4) + 3 = 11$

$AC \quad 4(4) - 5 = 11$

$BC \quad 3(4) - 1 = 11$

All sides are same length, ∴ triangle is equilateral

(5 marks)

6 x, y and z are three quantities such that

$x : y = 3 : 2$ and $y : z = 5 : 4$

get y to same amount.

Express the ratio $x : z$ in its simplest form.

$x : y = 3 : 2 = 15 : 10$

$y : z = 5 : 4 = 10 : 8$

$\therefore x : z = 15 : 8$

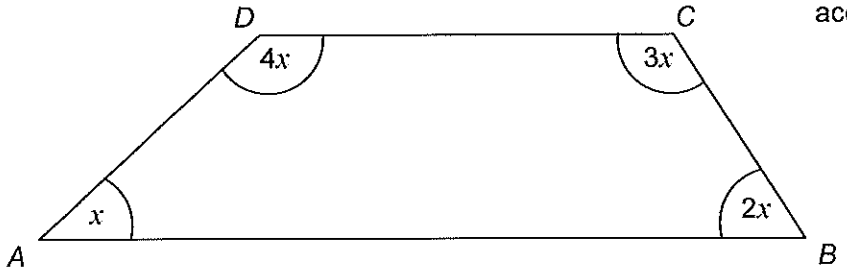
Answer 15 : 8 *(3 marks)*

14

Turn over ▶

7 *ABCD* is a quadrilateral.

Not drawn accurately



Prove that *AB* is parallel to *DC*.

Total angles add up to 360

$\rightarrow x + 4x + 3x + 2x = 10x = 360 \rightarrow x = 36$

Angle at D = $4 \times 36 = 144$

Angle at A = 36

$36 + 144 = 180^\circ$, so AB must be parallel to DC because of allied / interior angles add to 180°

(5 marks)

Do not write outside the box

8 The function f(x) is defined as $f(x) = \dfrac{1}{x^2 - 3x - 10}$

f(x) has domain all x except $x = a$ and $x = b$

Work out a and b. Denominator $\neq 0$

$f(x) = \dfrac{1}{(x-5)(x+2)}$ Values of x that cause it to equal 0 are $x = 5$ & $x = -2$

Answer $x = 5$, $x = -2$ *(3 marks)*

9 (a) Expand and simplify $(x - 5)(x^2 + 4x - 2)$

$x^3 + 4x^2 - 2x - 5x^2 - 20x + 10$

Answer $x^3 - x^2 - 22x + 10$ *(4 marks)*

9 (b) Factorise fully $(x^2 - 16) - (x - 4)(3x + 5)$

↑ Difference of 2 squares!

~~$(x^2 - 16) - [3x$~~

$(x-4)(x+4) - (x-4)(3x+5)$

$= (x-4)[(x+4) - (3x+5)]$

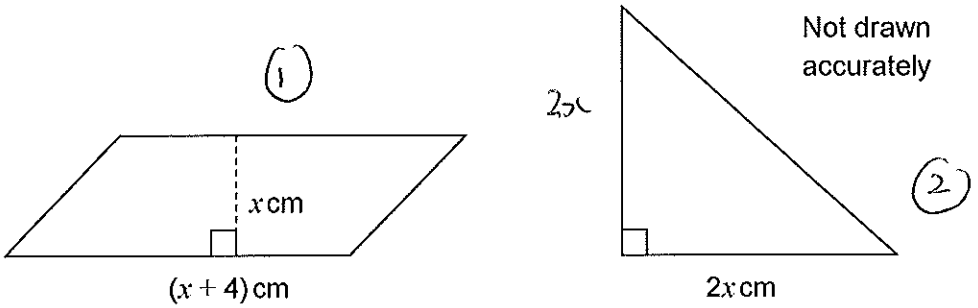
$= (x-4)(-2x-1)$

Answer $(x-4)(-2x-1)$ *(4 marks)*

16

Do not write outside the box

10 Here are a parallelogram and an isosceles triangle.



10 (a) The area of the triangle is greater than the area of the parallelogram.

Show that $x^2 - 4x > 0$

(1) $x(x+4)$ < (2) $(2x)(2x) \div 2$

$x^2 + 4x$ < $2x^2$

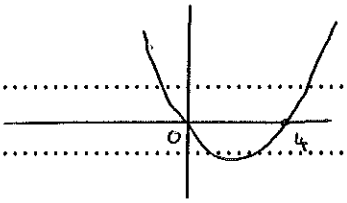
$-4x$ $0 < x^2 - 4x \rightarrow x^2 - 4x > 0$

(3 marks)

10 (b) Work out the least integer value for x.

$x^2 - 4x > 0$

$x(x-4) > 0$



$x = 0$
$x = 4$ (see sketch!)

Smallest value where graph above x-axis = 5

Answer $x = 5$ *(3 marks)*

11 Write $\dfrac{a^{\frac{1}{2}} \times a^{\frac{3}{2}}}{(a^3)^4}$ as a single power of a.

$= a^{\frac{4}{2}} = a^2$

$a^{12} \quad a^{12}$

Answer a^{-10} *(3 marks)*

Do not write outside the box

12 n is an integer.

Prove that $(n-2)^2 + n(8-n)$ is always a multiple of 4.

Expand: $n^2 - 4n + 4 + 8n - n^2$

$= 4n + 4$

$= 4(n+1)$

$\therefore$ must be a multiple of 4

(3 marks)

13 Solve the simultaneous equations $y^2 = x + 3$ and $y = 2x$

Do **not** use trial and improvement.

sub in!

$y^2 = x + 3$ and $y = 2x$

$\rightarrow (2x)^2 = x + 3$

$4x^2 = x + 3$

$-x$ -3 $\quad 4x^2 - x - 3 = 0$

$(4x + 3)(x - 1) = 0$

$4x + 3 = 0$ $\qquad x = 1$

$4x = -3$ $\qquad y = 2(1) = 2$

$x = -3/4$

$y = 2(-3/4)$

$= -1.5$

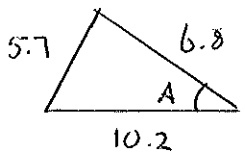
Answer $x = 1$, $y = 2$ and $x = -3/4$, $y = -1.5$ *(5 marks)*

17

Do not write outside the box

15 A triangle has sides 10.2 cm, 6.8 cm and 5.7 cm.

Work out the area of the triangle.



Need angle, so use Cosine Rule:

$\cos A = \frac{10.2^2 + 6.8^2 - 5.7^2}{2 \times 10.2 \times 6.8}$

$\Rightarrow \cos A = 0.84912...$

$\Rightarrow A = \cos^{-1}(0.84912) = 31.883...^{\circ}$

Area $= \frac{1}{2} ab \sin(c)$

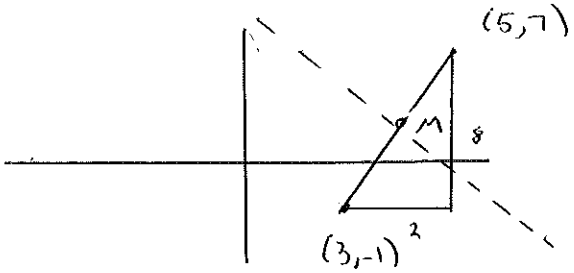
$= \frac{1}{2} \times 10.2 \times 6.8 \times \sin(31.883...)$

$= 18.3179...$

Answer 18.3 (1dp) cm^2 *(5 marks)*

16 Work out the equation of the perpendicular bisector of $P(3, -1)$ and $Q(5, 7)$.

Give your answer in the form $y = ax + b$



Mid Point = $\left[\frac{3+5}{2}, \frac{-1+7}{2}\right] = (4, 3)$

Gradient = $8/2 = 4$

Gradient of perpendicular = $-1/4$

$x_1 = 4$, $y_1 = 3$, $m = -1/4$

$y - y_1 = m(x - x_1)$

$y - 3 = -1/4(x - 4)$

$y - 3 = -1/4x + 1$

$y = -1/4x + 4$

Answer $y = -1/4x + 4$ *(5 marks)*

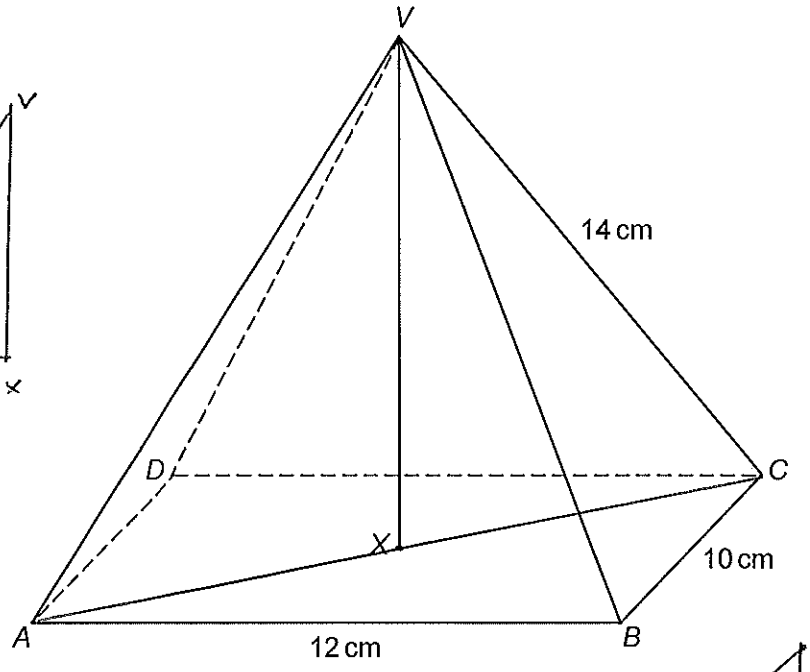
15

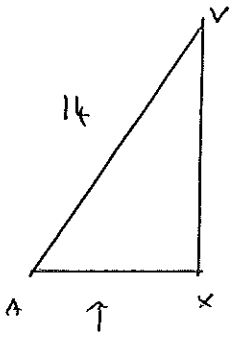
Do not write outside the box

17 *VABCD* is a rectangular based pyramid.

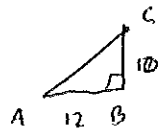
$AB = 12$ cm, $BC = 10$ cm and $VC = 14$ cm

The base *ABCD* is horizontal and the vertex *V* is directly above *X*, the centre of the base.





$\frac{\sqrt{244}}{2}$



17 (a) Work out the height of the pyramid, *VX*.

$AC = \sqrt{12^2 + 10^2}$

$= \sqrt{244}$

$AX = \frac{\sqrt{244}}{2}$

$VX^2 = 14^2 - \left(\frac{\sqrt{244}}{2}\right)^2$

$VX^2 = 135$

$VX = \sqrt{135}$

$= 11.6185...$

Answer 11.6 (1dp) cm *(4 marks)*

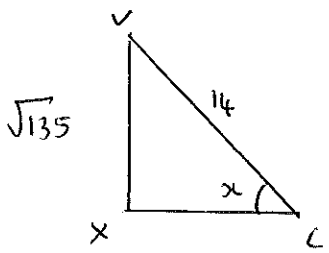
17 (b) Calculate the angle between *VC* and the plane *ABCD*.

$\sin(x) = \sqrt{135}/14$

$x = \sin^{-1}\left(\sqrt{135}/14\right)$

$= 56.091...$

Answer 56.1° (1dp) degrees *(2 marks)*



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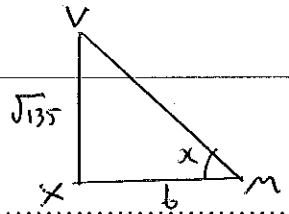
17 (c) Calculate the angle between the planes *VBC* and *ABCD*.



M = midpoint of BC

$\tan(x) = \sqrt{135}/6$

$x = \tan^{-1}(\sqrt{135}/6)$

$= 62.6882...$

Answer 62.7° (1dp) degrees *(3 marks)*

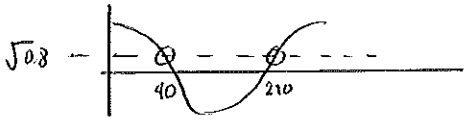
18 Solve the equation $\cos^2 x = 0.8$ for $0° \leqslant x \leqslant 360°$

$\sqrt{}$ { $\cos x = \pm\sqrt{0.8}$

$\cos(x) = \sqrt{0.8}$

$x = \cos^{-1}(\sqrt{0.8})$

$= 26.6°$



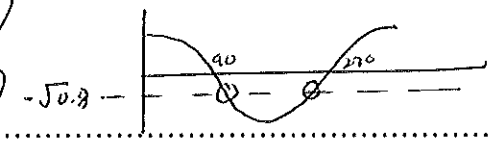
or $x = 360 - 26.6$

$= 333.4°$

$\cos x = -\sqrt{0.8}$

$x = \cos^{-1}(-\sqrt{0.8})$

$= 153.4°$



or $x = 360 - 153.4$

$= 206.6°$

Answer 26.6, 153.4, 206.6, 333.4 *(3 marks)*

19 $y = x^4(2x + 5)$

dy/dx

Work out the rate of change of y with respect to x when $x = 2$

d

$y = 2x^5 + 5x^4$

$dy/dx = 10x^4 + 20x^3$

when $x = 2$, $dy/dx = 10(2)^4 + 20(2)^3$

$= 320$

Answer 320 *(5 marks)*

17

20 (a) Matrix **A** = $\begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}$

$\begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Work out the image of point $P(2, -1)$ using transformation matrix **A**.

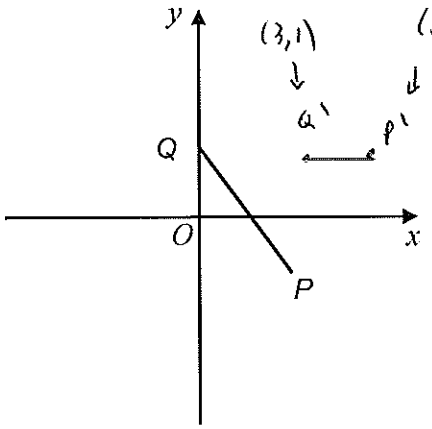
.....

.....

Answer (5 , 1) *(2 marks)*

20 (b) Point Q is (0, 1)

Line PQ is transformed to line $P'Q'$ using matrix **A**.



(3,1) (5,1)

Q' P'

Find Q'

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

Work out the length of $P'Q'$.

5 - 3 = 2

.....

.....

.....

Answer 2 units *(2 marks)*

USE FACTOR THEOREM!

21 Factorise fully $x^3 - 4x^2 - 11x + 30$ $= f(x)$

Try $f(2) = 2^3 - 4(2)^2 - 11(2) + 30 = 0$

$\therefore (x-2)$ is a factor

Try $f(-3) = (-3)^3 - 4(-3)^2 - 11(-3) + 30 = 0$

$\therefore (x+3)$ is a factor

$(x-2)(x+3)(\square) = x^3 - 4x^2 - 11x + 30$

must be -5 as $-2 \times 3 \times -5 = 30$

$(x-2)(x+3)(x-5)$

Answer ..... *(6 marks)*

Turn over for the next question

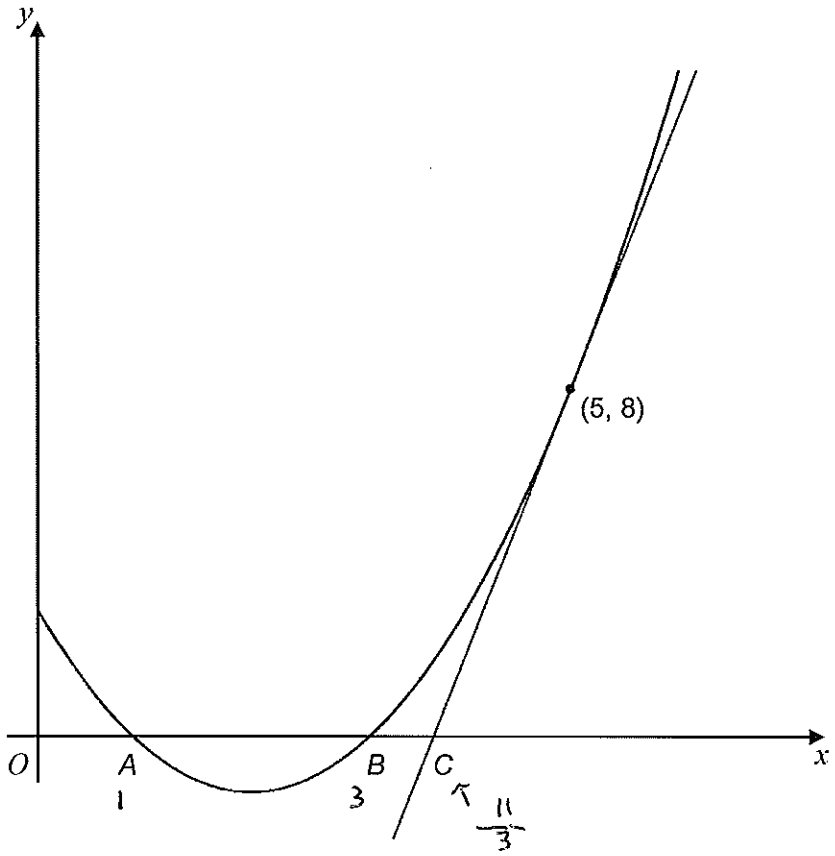
10

Do not write outside the box

22 The diagram shows the graph of $y = x^2 - 4x + 3$

The curve cuts the x-axis at the points A and B.

The tangent to the curve at the point (5, 8) cuts the x-axis at the point C.



Show that $AB = 3BC$

$y = x^2 - 4x + 3$

$\Rightarrow y = (x-1)(x-3)$

$x = 1$ (A) $x = 3$ (B)

Need equation of tangent!

$y = x^2 - 4x + 3$

$dy/dx = 2x - 4$

when $x = 5$, $dy/dx = 2(5) - 4 = 6$

$x_1 = 5$, $y_1 = 8$, $m = 5$

$y - y_1 = m(x - x_1)$

$y - 8 = 6(x - 5)$

$y - 8 = 6x - 30$

$y = 6x - 22$

At C, $y = 0 \Rightarrow 0 = 6x - 22$

$\Rightarrow x = 22/6 = 11/3$

(7 marks)

END OF QUESTIONS

$AB = 2$

$BC = \frac{11}{3} - 3 = \frac{2}{3}$

$3 \times (2/3) = 2$

7