MR BARTON'S SOLUTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
TOTAL	

AQA

Certificate in Further Mathematics
Level 2

Further Mathematics 8360/1

Level 2

Specimen Paper 1

Non-Calculator

For this paper you must have:
- mathematical instruments.

You may **not** use a calculator.

Time allowed
1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

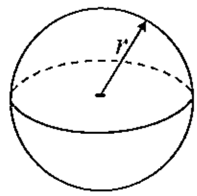
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

8360/1

Formulae Sheet

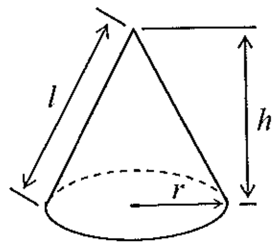
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



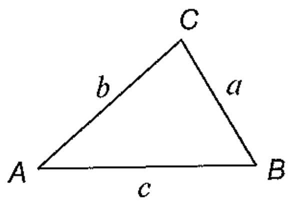
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle ABC

Area of triangle $= \frac{1}{2}ab \sin C$



Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) Solve $7(3x - 1) + 2(x + 7) = 3(6x - 1)$

$21x - 7 + 2x + 14 = 18x - 3$

$23x + 7 = 18x - 3$

$-18x$ $5x + 7 = -3$

-7 $5x = -10$

$\div 5$ $x = -2$

Answer $x =$ -2 *(4 marks)*

1 (b) Solve $\sqrt{3x + 10} = 4$

^{2} $3x + 10 = 16$

-10 $3x = 6$

$\div 3$ $x = 2$

Answer $x =$ 2 *(2 marks)*

Turn over for the next question

6

Do not write outside the box

2 (a) The nth terms of two sequences are $4n + 13$ and $6n - 21$

Which term has the same value in each sequence?

Need when $4n + 13 = 6n - 21$

$-4n$ $13 = 2n - 21$

$+21$ $34 = 2n$

Answer $n = 17$ (17th term) *(3 marks)*

2 (b) The first five terms of a quadratic sequence are 4 10 18 28 40

Work out an expression for the nth term.

1st diff. 6 8 10 12

2nd diff. 2 2 2 → $\boxed{n^2}$

Sequence 4 10 18 28 40

n^2 1 4 9 16 25

Difference 3 6 9 12 15 → $\boxed{3n}$

Answer $n^2 + 3n$ *(5 marks)*

Do not write outside the box

3 (a) On the axes below sketch the graph of $y = x^2 - 9$

Label clearly any points of intersection with the x-axis.

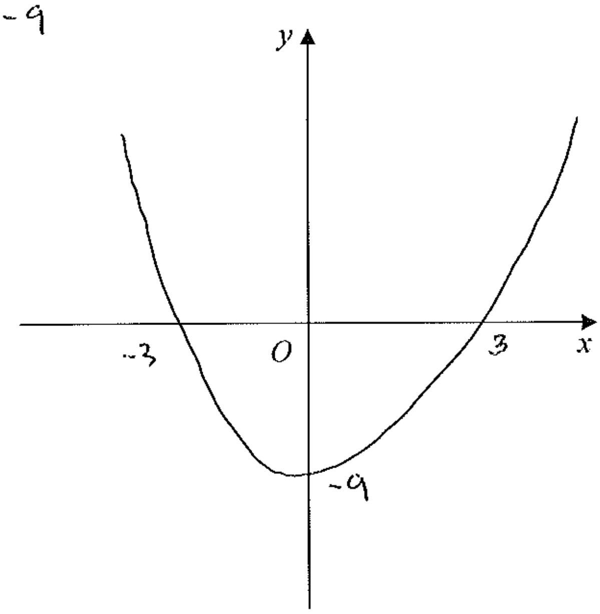
$\boxed{x\text{-axis}}$

Factorise $\rightarrow (x+3)(x-3)=0$

$x=-3 \quad x=3$

$x=0 \rightarrow y=-9$

$\boxed{y\text{-axis}}$



(2 marks)

3 (b) Write down all the integer solutions to $x^2 - 9 < 0$ ← graph below axes

Answer -2, -1, 0, 1, 2 *(2 marks)*

Turn over for the next question

12

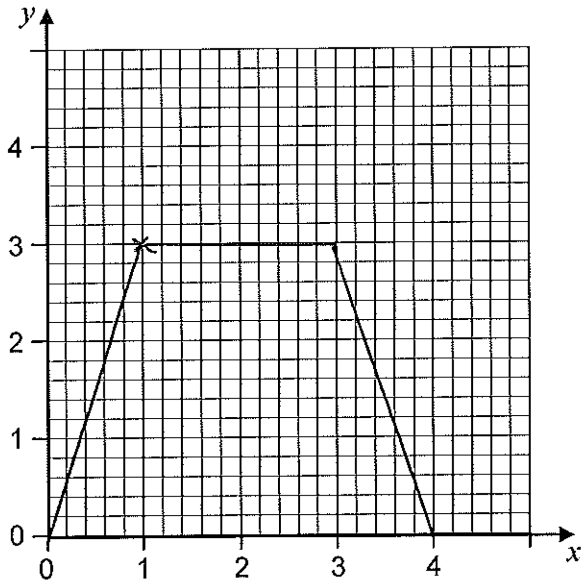
Turn over ▶

Do not write outside the box

4 A function f(x) is defined as

$$\begin{aligned} f(x) &= 3x && 0 \leqslant x < 1 \qquad y = 3x \\ &= 3 && 1 \leqslant x < 3 \qquad y = 3 \\ &= 12 - 3x && 3 \leqslant x \leqslant 4 \qquad y = 12 - 3x \end{aligned}$$

Calculate the area enclosed by the graph of $y = f(x)$ and the x-axis.



= Trapezium! Area = $\frac{1}{2}(4+2) \times 3$

= 3×3

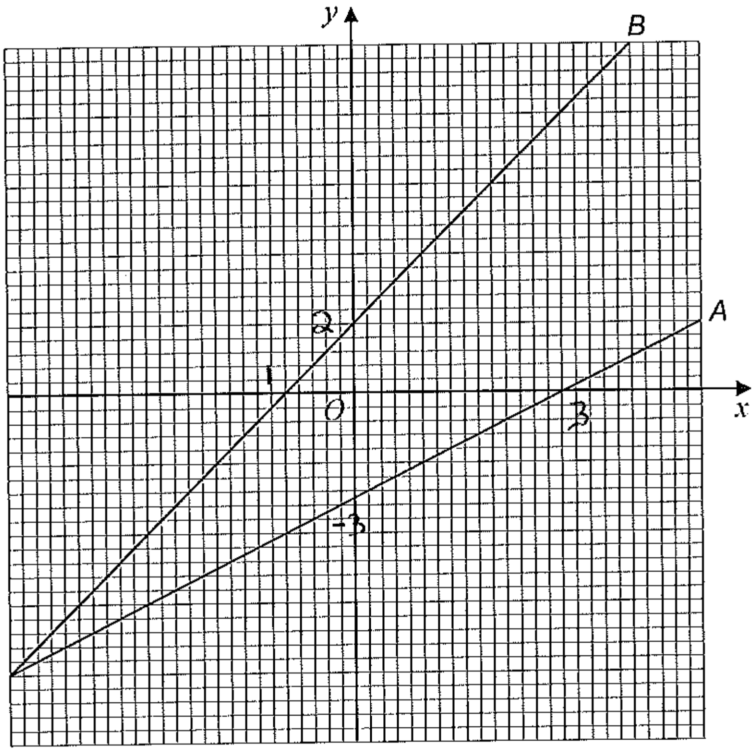
Answer ......9...... units^2 *(5 marks)*

8360/1

Do not write outside the box

5 The graph shows two lines A and B.

The equation of line B is $y = 2x + 2$



Work out the equation of line A.

$\boxed{B}$ $y = 2x + 2$ y-axis crossing $= 2$

x-axis crossing, $y = 0 \Rightarrow x = -1$

$\therefore$ $\boxed{A}$ y axis $= -3$, x-axis $= 3$

$\therefore$ gradient $= 1$, y-intercept $= -3$

Answer $y = x - 3$ *(4 marks)*

9

6 Work out $2\frac{2}{3} - 1\frac{3}{4} \div 1\frac{1}{8}$ Make top heavy!

Give your answer as a fraction in its simplest form.

$8/3 - 7/4 \div 9/8$ ← BIDMAS! must do ÷ first!

$\rightarrow 8/3 - [7/4 \times 8/9]$

$\rightarrow 8/3 - [14/9]$

$\rightarrow 24/9 - 14/9 = 10/9$

Answer $10/9$ *(5 marks)*

7 (a) Solve $x^{\frac{2}{3}} = 9$

$\sqrt{\ }$ $x^{1/3} = 3$

3 $x = 27$

Answer $x =$ 27 *(2 marks)*

7 (b) The reciprocal of $y^{\frac{1}{2}}$ is 5

Work out the value of y.

$\frac{1}{\sqrt{y}} = 5$

$\times\sqrt{y}$ $1 = 5\sqrt{y}$

$\div 5$ $\frac{1}{5} = \sqrt{y}$

Answer $y = 1/25$ *(2 marks)*

$\sqrt{\ }$ $\frac{1}{25} = y$

Do not write outside the box

8 Make d the subject of $c = \dfrac{8(c-d)}{d}$

$\times d$ $\quad cd = 8(c-d)$

Expand $\quad cd = 8c - 8d$

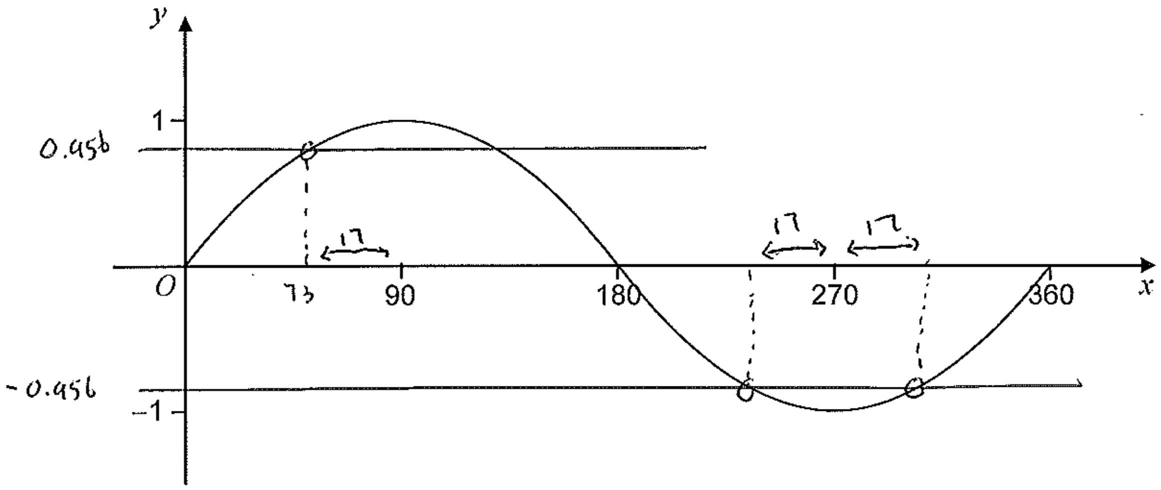
$+8d$ $\quad cd + 8d = 8c$

Fact! $\quad d(c+8) = 8c$

$\div(c+8)$ $\quad d = 8c/c+8$

Answer $d = \dfrac{8c}{c+8}$ *(4 marks)*

9 The sketch shows $y = \sin x$ for $0° \leqslant x \leqslant 360°$



The value of sin 73° = 0.956 to 3 significant figures.

Use the sketch to find **two** angles between 0° and 360° for which $\sin x = -0.956$

270 ± 17

Answer 253 and 287 *(2 marks)*

15

Turn over ▶

Do not write outside the box

10 (a) Write $\sqrt{75} + \sqrt{12}$ in the form $a\sqrt{b}$ where a and b are integers.

$\sqrt{25} \times \sqrt{3} + \sqrt{4} \times \sqrt{3}$

$= 5\sqrt{3} + 2\sqrt{3}$

Answer $7\sqrt{3}$ *(2 marks)*

10 (b) Rationalise and simplify $\dfrac{(2\sqrt{2}+1)}{(\sqrt{2}-3)} \times \dfrac{(\sqrt{2}+3)}{(\sqrt{2}+3)}$

$= \dfrac{4 + 6\sqrt{2} + \sqrt{2} + 3}{2 + 3\sqrt{2} - 3\sqrt{2} - 9} = \dfrac{7 + 7\sqrt{2}}{-7}$

Answer $-1 - \sqrt{2}$ *(5 marks)*

11 The points $A(-1, -7)$ and $B(24, 23)$ are on a straight line ACB.

$AC : CB = 2 : 3$

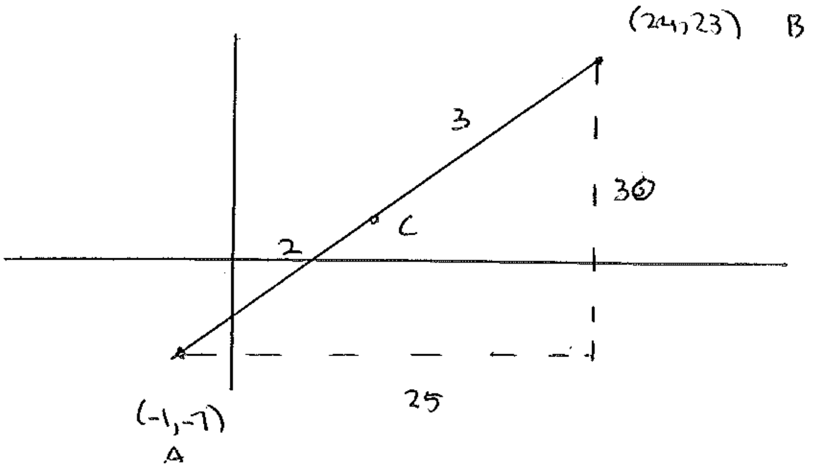
See sketch!

Work out the coordinates of C.

$\boxed{x}$ $(25/5) \times 2 = 10$ $-1 + 10 = 9$

$\boxed{y}$ $(30/5) \times 2 = 12$ $-7 + 12 = 5$

Answer (5 , 9) *(4 marks)*



12 Prove that $\tan^2 x - 1 \equiv \dfrac{1 - 2\cos^2 x}{\cos^2 x}$

$\tan^2(x) = \dfrac{\sin^2 x}{\cos^2(x)}$

$\rightarrow \dfrac{\sin^2(x)}{\cos^2(x)} - 1$

$= \dfrac{\sin^2(x)}{\cos^2(x)} - \dfrac{\cos^2(x)}{\cos^2(x)}$

$= \dfrac{\sin^2(x) - \cos^2(x)}{\cos^2(x)}$

$\sin^2(x) = 1 - \cos^2(x)$

$\rightarrow \dfrac{1 - \cos^2(x) - \cos^2(x)}{\cos^2(x)}$

$\rightarrow \dfrac{1 - 2\cos^2(x)}{\cos^2(x)}$

(3 marks)

13 (a) Work out the coordinates of the stationary point for the curve $y = x^2 + 3x + 4$

$dy/dx = 2x + 3$

At stationary points, $dy/dx = 0$

$\rightarrow 2x + 3 = 0 \rightarrow x = -3/2 = -1\tfrac{1}{2}$

$\boxed{\text{FIND Y}}$ $y = (-3/2)^2 + 3(-3/2) + 4$

$\rightarrow y = 9/4 - 9/2 + 4$

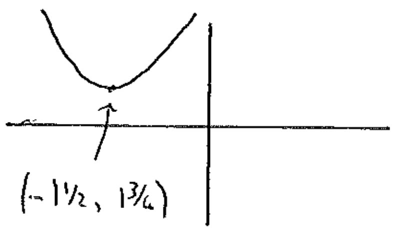
$\rightarrow y = 9/4 - 18/4 + 4 = 1\tfrac{3}{4}$

Answer ($-1\tfrac{1}{2}$, $1\tfrac{3}{4}$) *(4 marks)*

13 (b) Explain why the equation $x^2 + 3x + 4 = 0$ has no real solutions.

Sketch below shows minimum point is <u>above</u> x-axis, so no real solutions to $x^2 + 3x + 4 = 0$

(2 marks)



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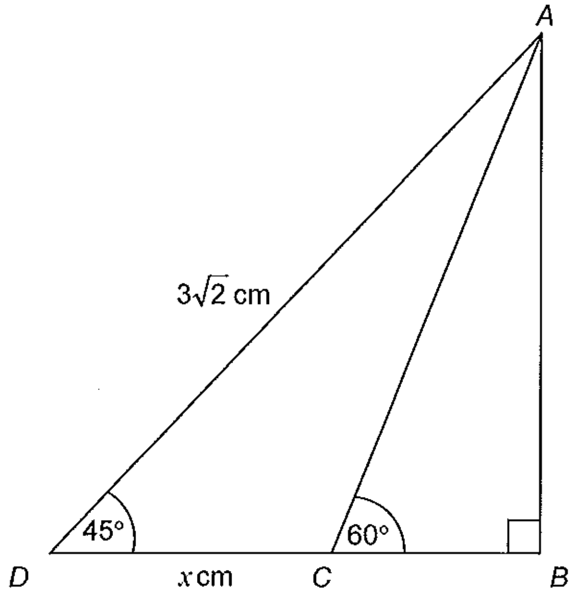
14 In the diagram, *DCB* is a straight line.



Special triangles

$\cos 45 = \frac{1}{\sqrt{2}}$

$\sin 45 = \frac{1}{\sqrt{2}}$

$\tan 60 = \sqrt{3}$

Work out the length of *DC*, marked x on the diagram.

Write your answer in the form $a - \sqrt{b}$

① $\cos(45) = {}^{DB}/_{3\sqrt{2}}$

$\Rightarrow DB = \cos(45) \times 3\sqrt{2}$

$\Rightarrow DB = {}^{1}/_{\sqrt{2}} \times 3\sqrt{2}$

$\Rightarrow DB = 3$

② $AB = 3$ as

isosceles triangle

③ $\tan(60) = \frac{3}{CB}$

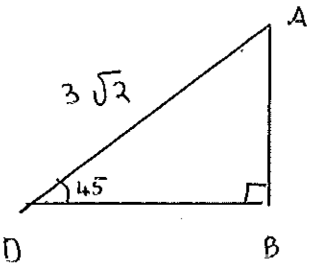
$\Rightarrow CB = {}^{3}/_{\sqrt{3}}$

RATIONALISE: $\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

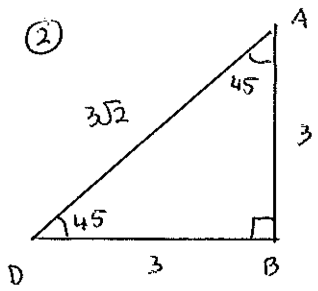
$= \frac{3\sqrt{3}}{3} = \sqrt{3}$

Answer $DC = DB - CB = 3 - \sqrt{3}$ cm *(4 marks)*

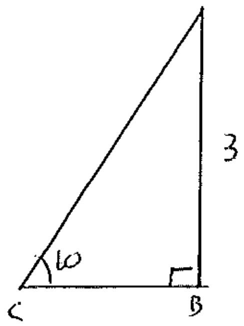
①



②



③



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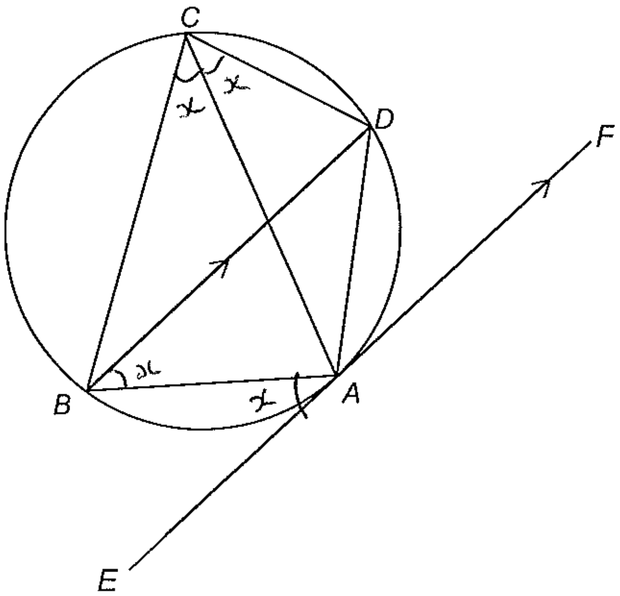
15 A, B, C and D are points on the circumference of a circle such that BD is parallel to the tangent to the circle at A.



Prove that AC bisects angle BCD.

Give reasons at each stage of your working.

BCA = BAE (alternate segment theorem) = x

DBA = BAE (alternate angles are equal) = x

DBA = DCA (angles in same segment are equal) = x

∴ BCA = DCA = x

∴ AC bisects BCD

(4 marks)

END OF QUESTIONS

8