Centre Number					Candidate Number				
Surname									
Other Names									
Candidate Signature									

AQA

Level 2 Certificate in Further Mathematics

Further Mathematics
Level 2

8360/2

Practice Paper Set 3

Paper 2

Calculator

For this paper you must have:	
• a calculator	
• mathematical instruments.	

Time allowed
2 hours

Instructions
* Use black ink or black ball-point pen. Draw diagrams in pencil.
* Fill in the boxes at the top of this page.
* Answer **all** questions.
* You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
* Do all rough work in this book. Cross through any work that you do not want to be marked.
* In all calculations, show clearly how you work out your answer.

Information
* The marks for questions are shown in brackets.
* The maximum mark for this paper is 105.
* You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.
* The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.
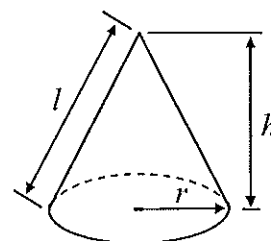
8360/2

Formulae Sheet

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$
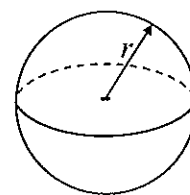
Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$

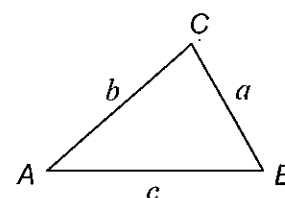**In any triangle** ABC

Area of triangle $= \dfrac{1}{2}ab \sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \dfrac{\sin \theta}{\cos \theta}$ $\qquad$ $\sin^2 \theta + \cos^2 \theta \equiv 1$
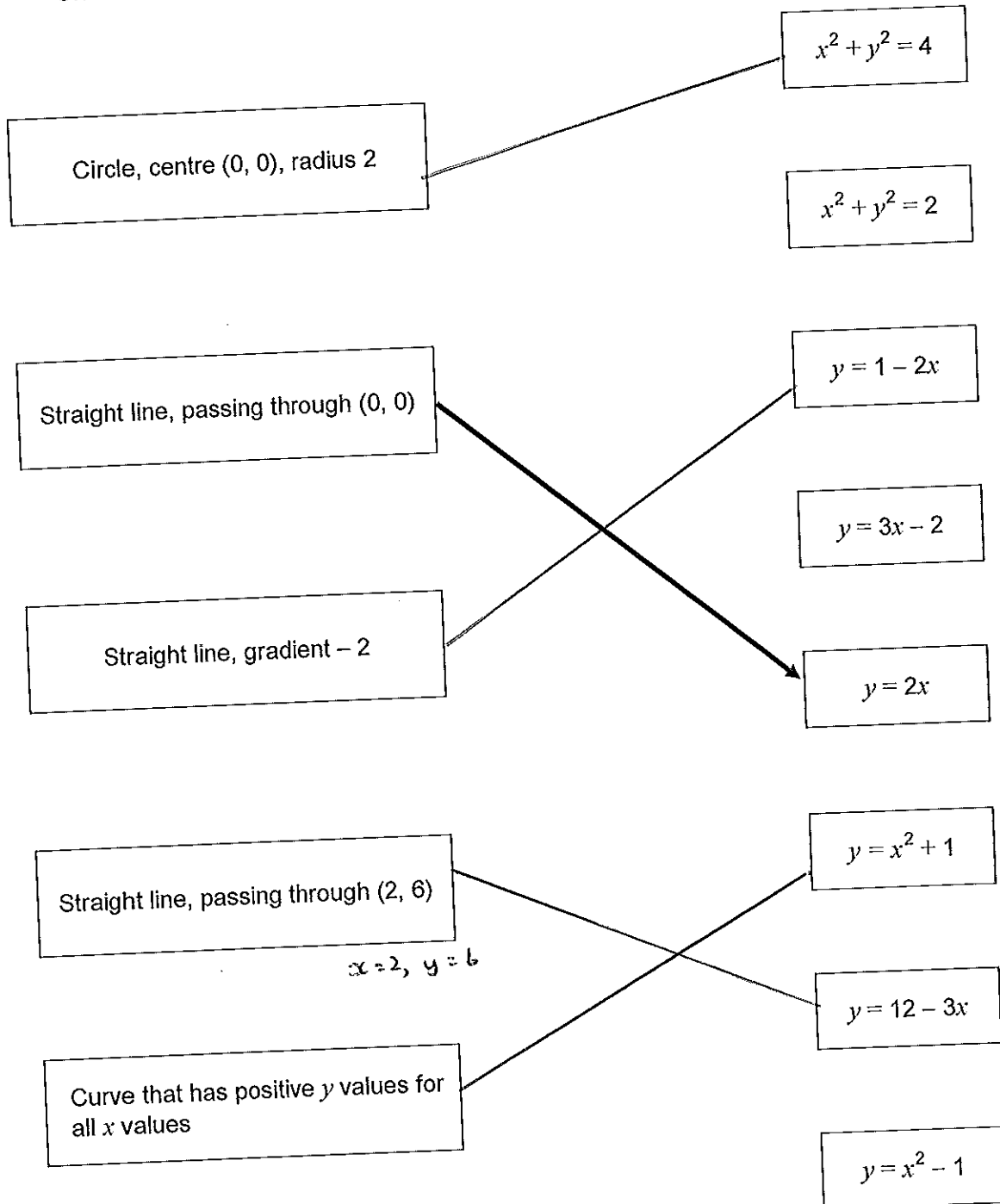
Answer **all** questions in the spaces provided.

1   Match each graph with the correct equation.

You will **not** use all of the equations.

One has been done for you.

$x^2 + y^2 = 4$

Circle, centre (0, 0), radius 2

$x^2 + y^2 = 2$

Straight line, passing through (0, 0)

$y = 1 - 2x$

$y = 3x - 2$

Straight line, gradient – 2

$y = 2x$

Straight line, passing through (2, 6)

$x = 2, \ y = 6$

$y = x^2 + 1$

$y = 12 - 3x$

Curve that has positive y values for all x values

$y = x^2 - 1$

(4 marks)

4

Turn over ▶

8360/2

2 h is 60% of m.

$h = \dfrac{60}{100} m$

2 (a) Write an equation connecting h and m.

Answer .... $h = 0.6\,m$ .... (1 mark)

2 (b) Also, r is 75% of m.

$r = 0.75m$

Work out h as a percentage of r.

$$\dfrac{0.6m}{0.75m} = 0.8$$

Answer .... 80 .... % (2 marks)

3 A curve has gradient function $\dfrac{dy}{dx} = 2x^2 - 7$

3 (a) Work out the gradient of the curve when $x = -3$

$$\dfrac{dy}{dx} = 2(-3)^2 - 7$$

$$= 2 \times 9 - 7 = 11$$

Answer .... 11 .... (2 marks)

$\dfrac{dy}{dx}$

3 (b) Work out the values of x for which the rate of change of y with respect to x is 1.

$$1 = 2x^2 - 7$$

$$0 = 2x^2 - 8$$

$$(\div 2) \quad 0 = x^2 - 4 \qquad \left\{ \begin{array}{l} x^2 - 4 = 0 \\ \Rightarrow x^2 = 4 \end{array} \right.$$

$$\Rightarrow x = \pm\sqrt{4} = 2 \text{ or } -2$$

Answer .... (3 marks)

4   The straight line shows a sketch of $y = f(x)$ for the full domain of the function.
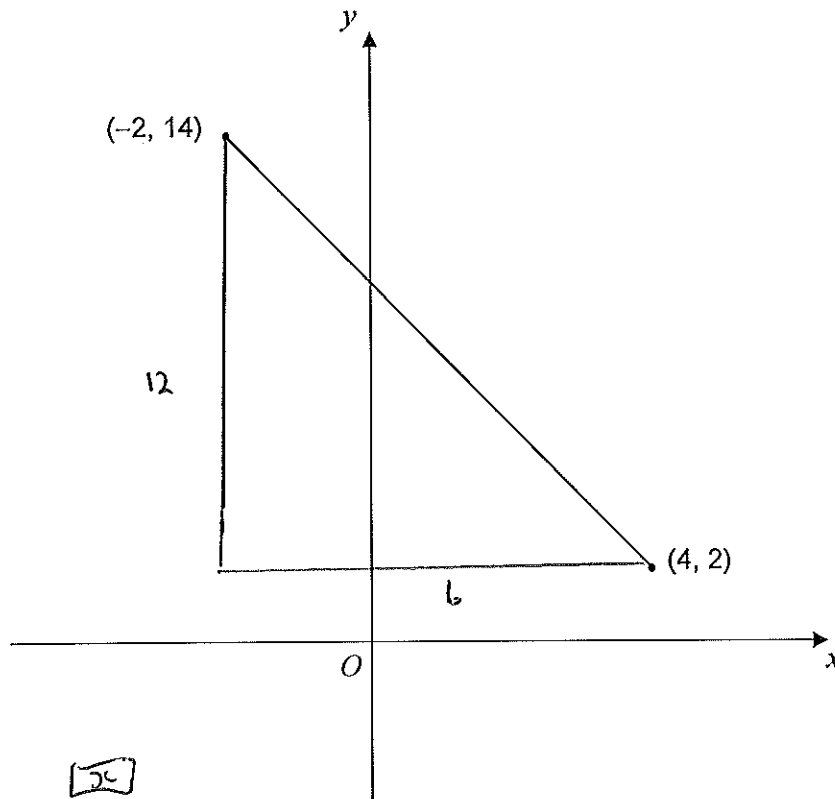
4 (a)   State the domain of the function.

Answer ....-2........ $\leq x \leq$ ....4........ *(1 mark)*

4 (b)   Work out the equation of the line.

Gradient $= -\frac{12}{6} = -2$

$x_1 = 4$ $y - y_1 = m(x - x_1)$

$y_1 = 2$ $y - 2 = -2(x - 4)$

$\rightarrow y - 2 = -2x + 8$

Answer ....or $y = -2x + 16$........ *(3 marks)*

Turn over ▶

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5

$$\begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} -3 & -4 \\ -5 & t \end{pmatrix} = I \quad \swarrow \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{Identity Matrix!}$$

Work out the value of t.

$$\begin{pmatrix} -3 & -4 \\ -5 & t \end{pmatrix}$$

$$\begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\rightarrow \quad (-7) \times (-4) + 4t = 0$

$\rightarrow \quad 28 + 4t = 0$

$\rightarrow \quad 4t = -28$

$\rightarrow \quad t = -7$

$t = $ .... -7 .... (3 marks)

6   Rearrange $c = 5a^2 - b$ to make a the subject.

....

.... See below ....

....

....

Answer .... $a = \sqrt{\dfrac{c + b}{5}}$ .... (3 marks)

$c = 5a^2 - b$

$+b \quad \begin{cases} c + b = 5a^2 \\[2mm] \div 5 \quad \dfrac{c+b}{5} = a^2 \\[2mm] \sqrt{} \quad \sqrt{\dfrac{c+b}{5}} = a \end{cases}$

7 Triangle *ABC* is isosceles with *AC = BC*

Triangle *CDE* is isosceles with *CD = CE*

ACD and *DEF* are straight lines.

Not drawn
accurately
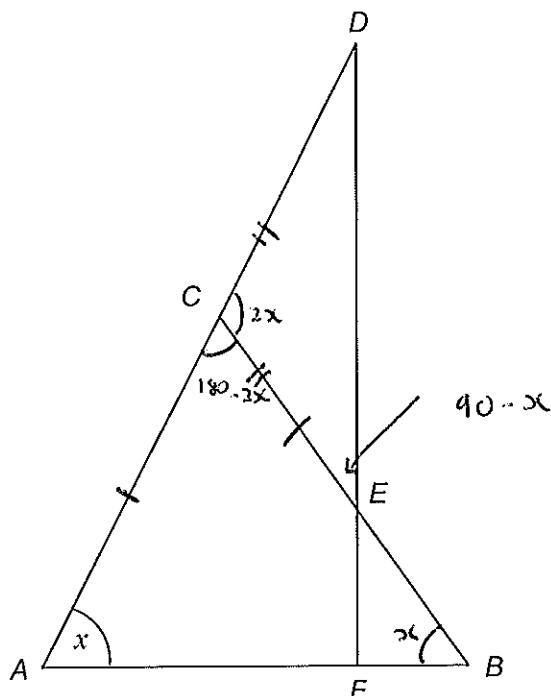
7 (a) Prove that angle $DCE = 2x$

CBA = x (isosceles)

ACB = 180 - 2x (angles in $\triangle$) $x = 180$

DCE = 180 - (180 - 2x) = 2x (angles on straight line)

(2 marks)

7 (b) Prove that *DF* is perpendicular to *AB*.

CED is an

CED = $\frac{180 - 2x}{2}$ = 90 - x (isosceles triangle)

BEF = 90 - x (vertically opposite angles are
equal)

EFB = 180 - x - (90 - x) = 90° (angles in a $\triangle$) = 180

∴ DF is perpendicular to AB

(3 marks)

11

8 (a) $0 < w < 1$

Are the following statements true or false?

Tick a box for each statement.

	True	False
$1 - w > 0$	✓	
$w^3 > w^2$		✓
$\dfrac{1}{w} > w$	✓	

$eg \ 1 - 0.5$

$eg \ w \ \frac{1}{2}^3 : \frac{1}{8}$

$\frac{1}{2}^2 : \frac{1}{4}$

$eg \ \frac{1}{0.5} : 2$

(3 marks)

8 (b) $x < -1$

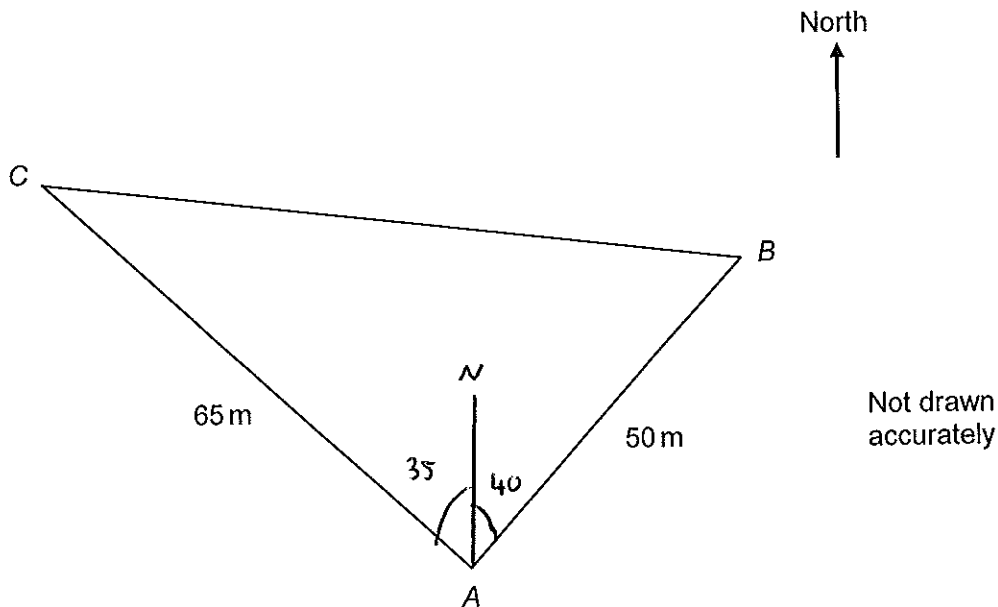Write the following expressions in order of size.
Start with the smallest.

$eg \ -2$

x	x^2	x^3	x^4
-2	4	-8	16

....

....

Answer .... x^3 ..... x ..... x^2 .... x^4 .... *(2 marks)*

9   *B* is 50 metres from *A* on a bearing of 040°.

C is 65 metres from *A* on a bearing of 325°.

North

C

B

N

65 m

Not drawn accurately

50 m

35   40

A

9 (a)   Explain why angle *CAB* is 75°.

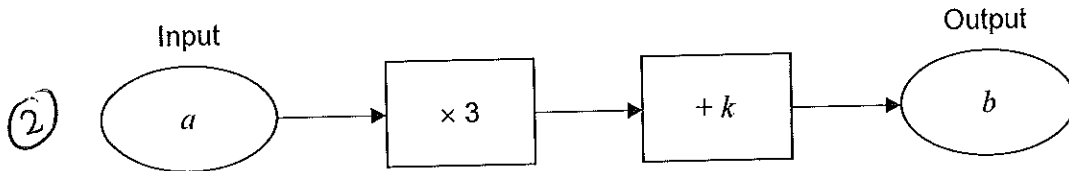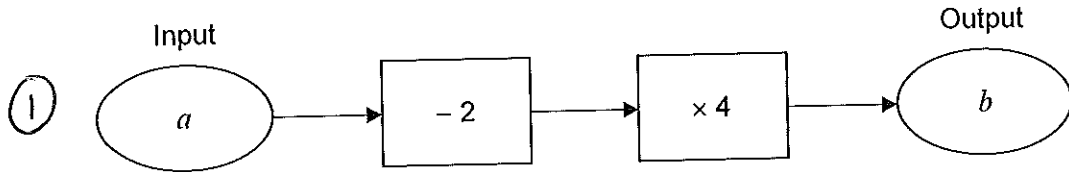$CAN = 360 - 325 = 35$

$35 + 40 = 75$

(2 marks)

9 (b)   Work out the distance *BC*.   Cosine rule!

$(BC)^2 = 50^2 + 65^2 - 2 \times 50 \times 65 \times \cos(75)$

$(BC)^2 = 5042.676$

✓ $\Rightarrow BC = 71.012$

$BC = 71.012$ m *(3 marks)*

10

Turn over ▶

10 Here are two number machines.

Input · Output

①

a → -2 → $\times 4$ → b

Input · Output

②

a → $\times 3$ → $+k$ → b

Work out a in terms of k.

① $4(a-2) = b$

② $3a + k = b$

① also must $=$ ② → $4(a-2) = 3a + k$

→ $4a - 8 = 3a + k$
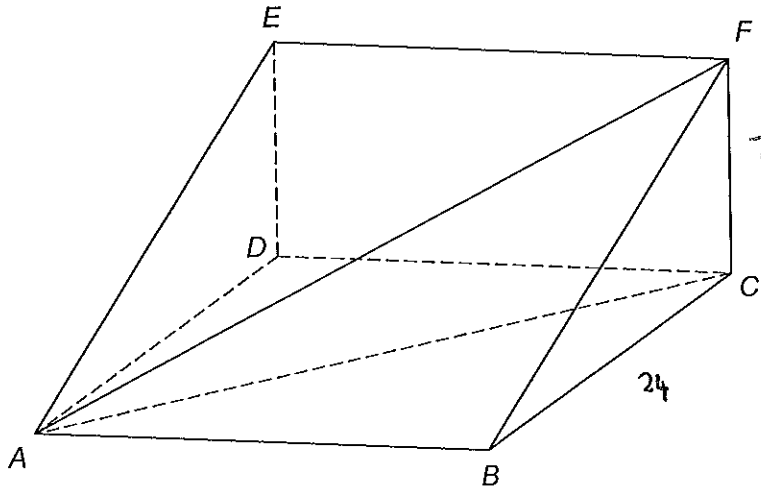
→ $a = k + 8$

$a =$ .... $k + 8$ .... (4 marks)

11   The diagram shows part of a skate ramp, modelled as a triangular prism.

ABCD represents horizontal ground.
The vertical rise of the ramp, CF, is 7 feet.
The distance BC = 24 feet.

Not drawn
accurately

You are given that   gradient = $\dfrac{\text{vertical rise}}{\text{horizontal distance}}$

11 (a)   The gradient of BF is **twice** the gradient of AF.

Write down the distance AC.

.... must = 2 × BC = 2 × 24 ....
....

AC = ... 48 .... feet (1 mark)

11 (b)   Greg skates down the ramp along FB.

How much further would he travel if he had skated along FA?   Pythagoras!

$(BF)^2 = 7^2 + 24^2 = 625$
⟹ BF = $\sqrt{625}$ = 25 feet
$(AF)^2 = 48^2 + 7^2 = 2353$
⟹ AF = $\sqrt{2353}$ = 48.507 feet
Difference = 48.507 - 25

Answer = 23.51 .... feet (4 marks)

9

Turn over ▶

8360/2

12 $f(x) = x^2 + px + q$ for all values of x.

p and q are integers.

    f(0) is an odd number.

    f(1) is an odd number.

Show that p is an odd number.

$f(0) = 0 + 0 + q = q = odd\ number$

$f(1) = 1 + p + q = odd\ number$

$1 + q = odd + odd = even$

$A.s. (1 + q) + p\ must\ be\ odd,\ p\ must\ be\ odd$

(3 marks)

$as\ odd\ even + odd = odd$

13 $y = x^{\frac{1}{2}}(x^{\frac{7}{2}} - x^{\frac{1}{2}})$ $Add\ powers!$

Work out $\dfrac{dy}{dx}$.

$y = x^{\frac{8}{2}} - x^{\frac{2}{2}}$

$\rightarrow y = x^4 - x^1$

$\rightarrow \dfrac{dy}{dx} = 4x^3 - 1$

$\dfrac{dy}{dx} = $ ....$4x^3 - 1$.......... *(4 marks)*

8360/2

14 Points *A*, *B* and *C* lie on a straight line.

BC is 50% longer than *AB*.

↓

≈ ratio 2 : 3

Not drawn accurately

C (11, 18)

3

15

B

2

A (1, 3)

10
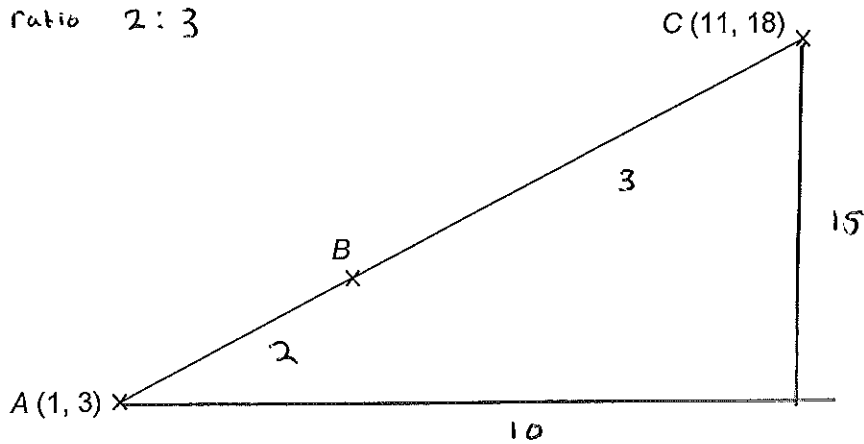
Work out the coordinates of *B*.

[x] 10 in ratio 2:3 = 4:6

→ x coordinate = 1 + 4 = 5

[y] 15 in ratio 2:3 = to 6 : 9

→ y coordinate = 3 + 6 = 9

Answer (......5......, ...9......) *(4 marks)*

Turn over for the next question

11

15 (a) Factorise $2x^2 - 3x - 14$

$$(2x - 7)(x + 2)$$

Answer $(2x - 7)(x + 2)$ .... *(2 marks)*

15 (b) Hence, or otherwise, solve $2(y - 5)^2 - 3(y - 5) - 14 = 0$

Let $(y - 5) = x$

$\rightarrow 2x^2 - 3x - 14 = 0$

$\rightarrow (2x - 7)(x + 2) = 0$

$2x - 7 = 0 \qquad\qquad x = -2$

$\rightarrow x = \frac{1}{2} \qquad\qquad \rightarrow y - 5 = -2$

$\rightarrow y - 5 = \frac{1}{2} \qquad\qquad \rightarrow y = 3$

Answer .... *(3 marks)*

$\rightarrow y = 5 + \frac{1}{2}$

$= 8.5$

$\therefore y = 8.5 \quad$ and $\quad 3$

16 (a) Write $(x^2)^2$ as a single power of x.   *multiply powers*

....

Answer ....x^4........................ *(1 mark)*

16 (b) Factorise fully $x^4 - 1$ *Diff of 2 squares.*
                                              Twice!

....

............................$(x^2 - 1)(x^2 + 1)$..............................

....................$(x - 1)(x + 1)(x^2 + 1)$....................

Answer .... *(2 marks)*

Turn over for the next question

.

8

Turn over ▶

17 The unit square is shaded on the grid.
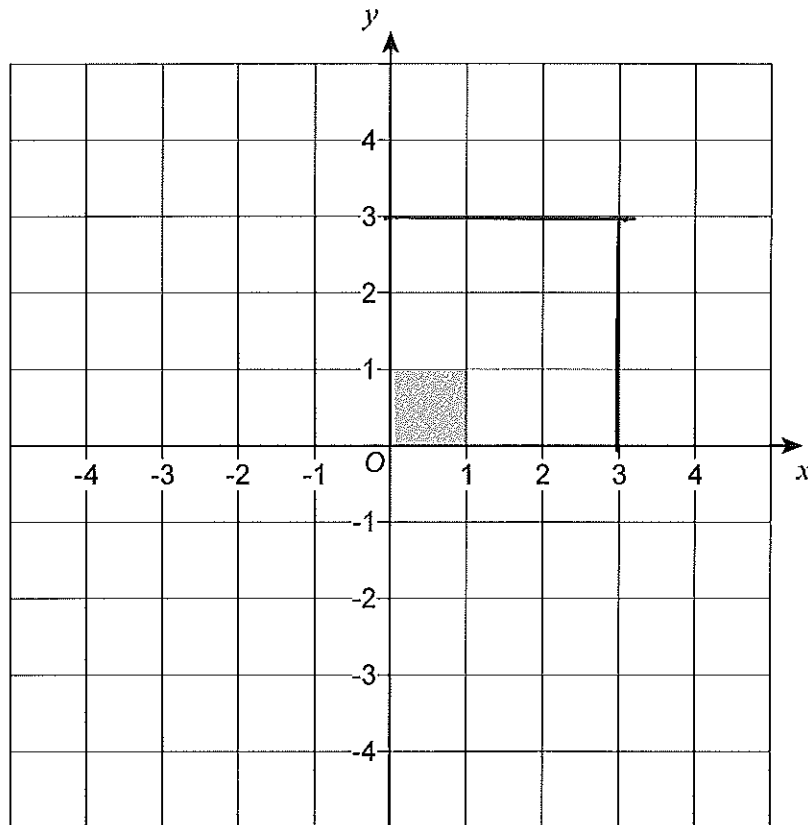
17 (a) On the grid, draw the image of the unit square after it is transformed using the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.
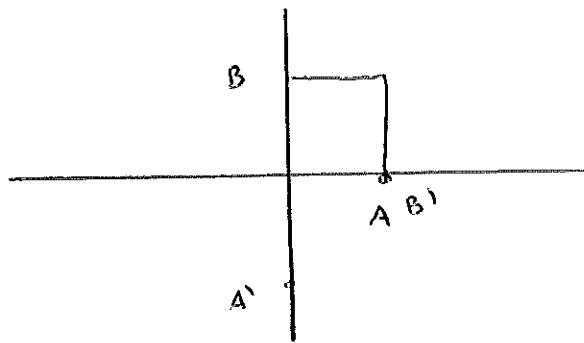
Enlargement

Scale Factor 3

Centre (0,0)

(2 marks)

17 (b)   Work out the matrix that transforms the unit square by a 270° rotation about O.

↗ Antn Cw

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Answer $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(2 marks)

Turn over for the next question

4

18

$C = \dfrac{3x + 7}{x + 1}$ and $D = \dfrac{4x - 11}{2x + 3}$

$\dfrac{3x + 7}{x + 1} + \dfrac{4x - 11}{2x + 3} = 5$

Work out the value of x when $C + D = 5$

$$\rightarrow \frac{(3x + 7)(2x + 3)}{(x + 1)(2x + 3)} + \frac{(4x - 11)(x + 1)}{(2x + 3)(x + 1)} = 5$$

$$\rightarrow \frac{6x^2 + 9x + 14x + 21 + 4x^2 + 4x - 11x - 11}{(x + 1)(2x + 3)} = 5$$

$$10x^2 + 16x + 10 = 5(x + 1)(2x + 3)$$

$$\rightarrow 10x^2 + 16x + 10 = 5[2x^2 + 3x + 2x + 3]$$

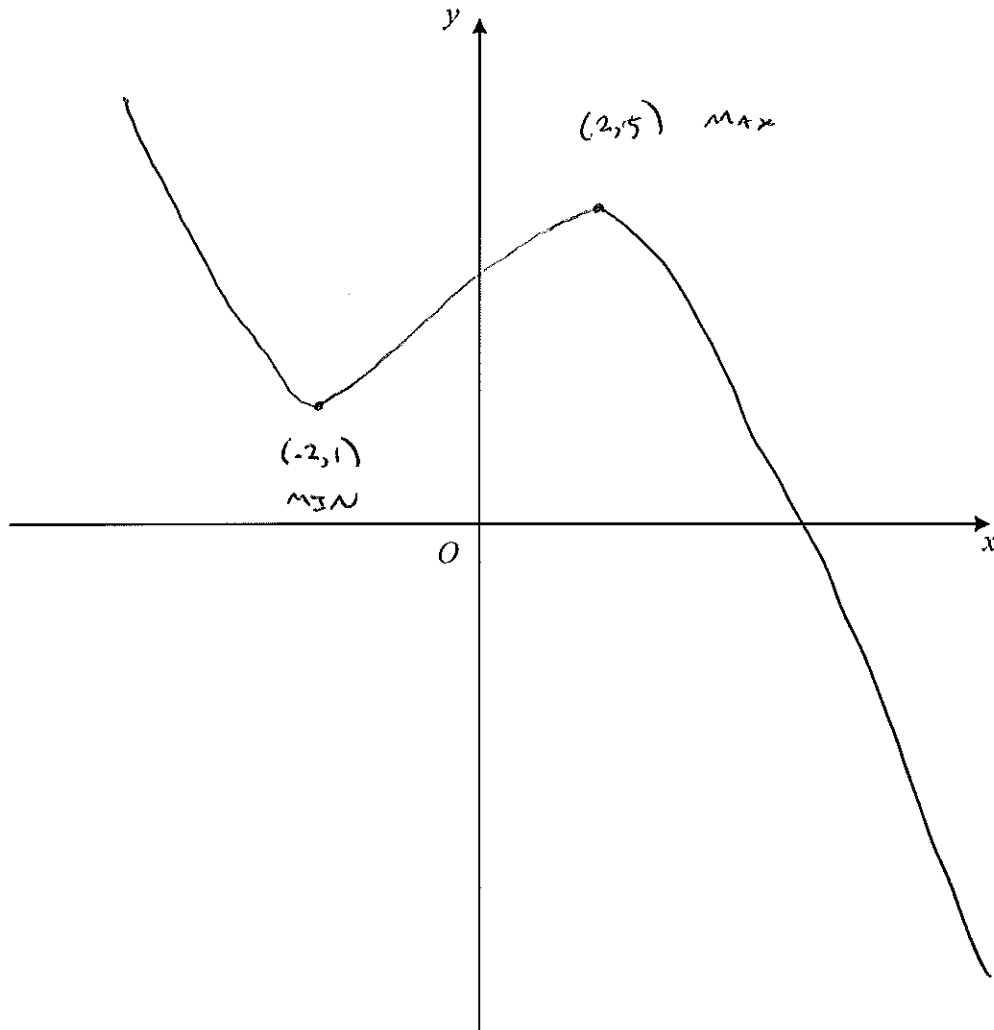$$\rightarrow 10x^2 + 16x + 10 = 10x^2 + 25x + 15$$

$x = \text{...}$ (5 marks)

$$\rightarrow -10x^2 \left\{ \begin{array}{l} 16x + 10 = 25x + 15 \\[1em] 10 = 9x + 15 \\[1em] -5 = 9x \\[1em] x = \boxed{\dfrac{-5}{9}} \end{array} \right.$$

$-16x$

-15

$\div 5$

19   A cubic function f(x) has domain all values of x.

The curve $y = $ f(x) has two stationary points.

There is a minimum point at (−2, 1).

There is a maximum point at (2, 5).
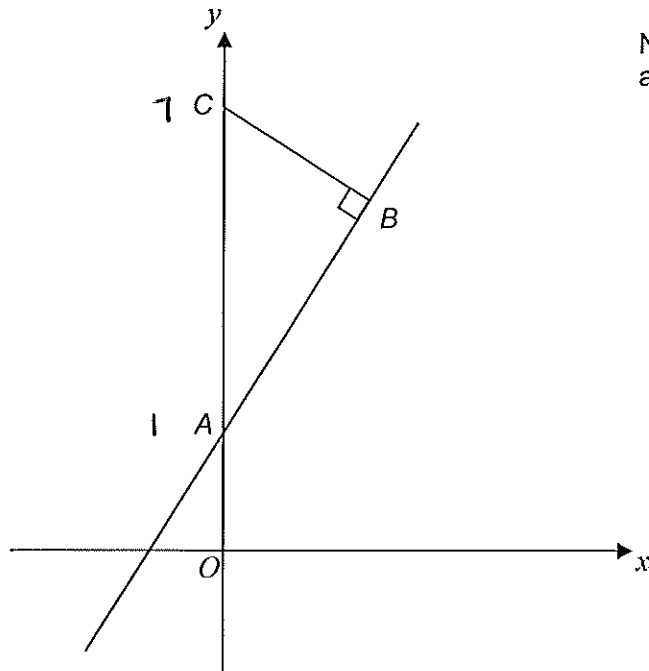
Sketch the graph of $y = $ f(x) on these axes.

(3 marks)

Turn over for the next question

21 On the diagram

- *A* and *C* are on the *y*-axis
- *C* is (0, 7)
- the equation of the line through *A* and *B* is $y = 2x + 1$ → $A = (0, 1)$
- *BC* is perpendicular to *AB*.

Not drawn accurately

Work out the area of triangle *ABC*.

Gradient of $AB = 2$

∴ Gradient of $CB = -\frac{1}{2}$

Need equation of CB

$m = -\frac{1}{2}$ $y - y_1 = m(x - x_1)$

$x_1 = 0$ $y - 7 = -\frac{1}{2}x$

$y_1 = 7$ → $y = -\frac{1}{2}x + 7$

Find B by using intersection of lines: $y = -\frac{1}{2}x + 7$ & $y = 2x + 1$

→ $2x + 1 = -\frac{1}{2}x + 7$

→ $2\frac{1}{2}x = 6$ Area of $\triangle = \frac{1}{2}b \times h$

→ $x = 2.4$ $= \frac{1}{2} \times 6 \times 2.4$

                 $= 7.2$

Answer .... 7.2 units^2 .... *(6 marks)*

Turn over ▶

22   Expand and simplify $xy(2x + 3y)(5x - 2y)$

$\left(2x^2y + 3xy^2\right)\left(5x - 2y\right)$

$10x^3y - 4x^2y^2 + 15x^2y^2 - 6xy^3$

Answer $10x^3y + 11x^2y^2 - 6xy^3$ .... *(4 marks)*

23 $g(x) = x^3 + 3x^2$ for all values of x.   replace $(3x)$ with (x)

Show that $g(3x) = kx^2(x + 1)$ where k is an integer.

$\rightarrow (3x)^3 + 3(3x)^2$

$\rightarrow 27x^3 + 27x^2$

$= 27x^2(x + 1)$

(3 marks)

24 (a) Factorise $5s^2 - 2s$

Answer .... $s(5s - 2)$ .... *(1 mark)*

use part a)!

24 (b) Solve $5\sin^2 x - 2\sin x = 0$ for $0° \leqslant x \leqslant 360°$

$\sin(x)\left[5\sin(x) - 2\right] = 0$

$\sin(x) = 0$ $5\sin(x) - 2 = 0$

$\rightarrow \sin(x) = \frac{2}{5}$

$x = \sin^{-1}(\frac{2}{5})$

$= 23.6°$

0.4

$23.6°$
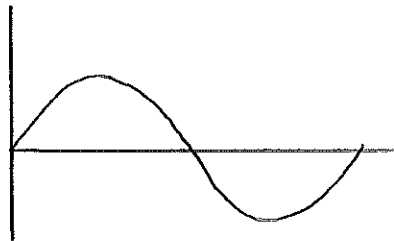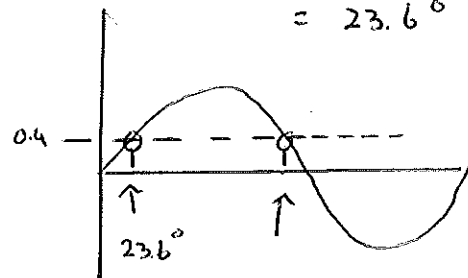
$180 - 23.6$

$= 156.4°$

$x = 0°, \quad 180°, \quad 360°$

$x = 0, \quad 180°, \quad 360°, \quad 23.6°, \quad 156.4°$

Answer .... *(4 marks)*

Turn over for the next question

12

25 $x^3 + ax^2 + bx + 150$ factorises to $(x + c)^2(x + d)$

a, b, c and d are positive integers and $c \neq 1$

Work out the values of a, b, c and d.

$c \times c \times d$ must $= 150$

$\Rightarrow$ must be $5 \times 5 \times 6$

$\Rightarrow (x + 5)(x + 5)(x + 6)$

$= (x^2 + 10x + 25)(x + 6)$

$\Rightarrow x^3 + 6x^2 + 10x^2 + 60x + 25x + 150$

$\Rightarrow x^3 + 16x^2 + 85x + 150$

Answer $a =$ 16

$b =$ 85

$c =$ 5

$d =$ .... 6 .... *(6 marks)*

6

END OF QUESTIONS