MR BARTON'S ANSWERS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
14 - 15	
16 - 17	
18 - 19	
20 - 21	
22	
TOTAL	

AQA Level 2 Certificate in Further Mathematics

Further Mathematics 8360/2
Level 2

Practice Paper Set 2

Paper 2

Calculator

For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
2 hours

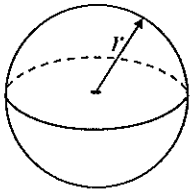
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet

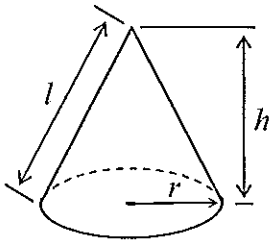
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



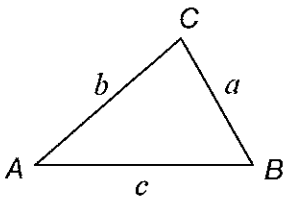
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*

Area of triangle $= \frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

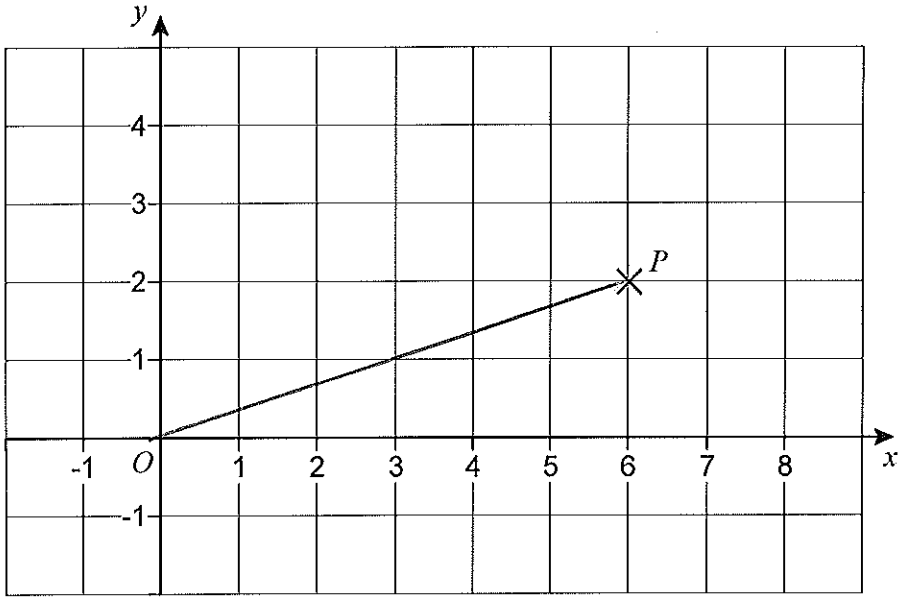
The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$ $\qquad \sin^2\theta + \cos^2\theta \equiv 1$

Answer **all** questions in the spaces provided.

1 Point P is marked on the grid.



1 (a) Work out the equation of the line that passes through O and P. Grad = $\frac{1}{3}$

.....

Answer ..... $y = \frac{1}{3}x$ ..... *(2 marks)*

1 (b) Work out the distance OP.

Give your answer to 2 significant figures.

By Pythag: $OP = \sqrt{6^2 + 2^2}$

$= \sqrt{36 + 4} = \sqrt{40}$

$= 6.32455...$

Answer ..... 6.3 (2sf) ..... *(4 marks)*

Turn over for the next question

6

Do not write outside the box

2 Write as single powers of m.

2 (a) $(m^2)^5$ (×)

Answer m^{10} *(1 mark)*

2 (b) $m^{12} \div m^{-4}$ (−)

Answer m^{16} *(1 mark)*

2 (c) $m^{\frac{1}{2}} \times m^{\frac{3}{2}}$ (+)

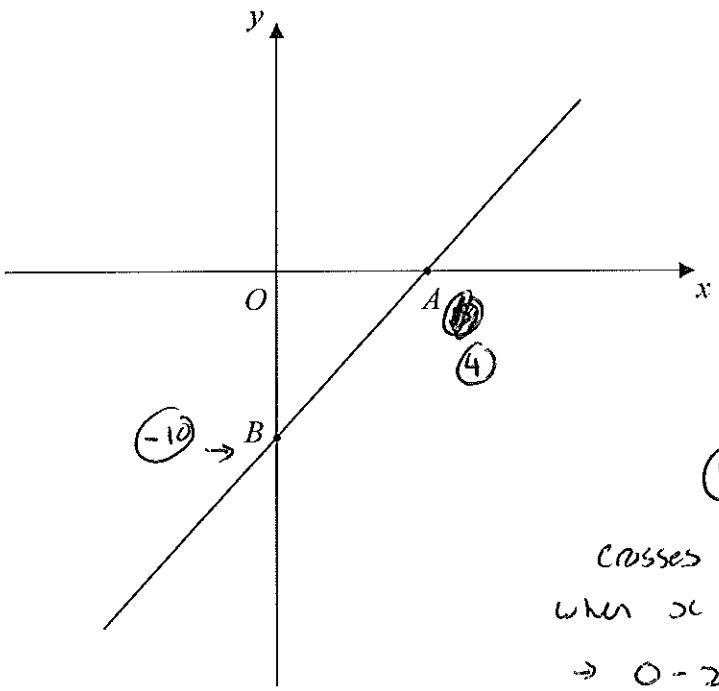
Answer $m^{\frac{4}{2}} = m^2$ *(1 mark)*

2 (d) $\sqrt{\frac{1}{m^6}} = \sqrt{m^{-6}} = (m^{-6})^{1/2}$ (×)

Answer m^{-3} *(2 marks)*

3 The line $5x - 2y = 20$ crosses the axes at the points A and B, as shown.

(A)
crosses x-axis
when $y = 0$
$\rightarrow 5x = 20$
$\rightarrow x = 4$



(B)
crosses y-axis
when $x = 0$
$\rightarrow 0 - 2y = 20$
$\rightarrow y = -10$

Work out the area of triangle OAB.

Area $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 10 \times 4$
$= 20$

Answer 20 units squared *(4 marks)*

4 Solve $\frac{x-4}{3} + \frac{x}{5} = 2$

$\times 15$

$\frac{5(x-4)}{15} + \frac{3(x)}{15} = 2$

$5x - 20 + 3x = 2$
15

$8x - 20 = 30$
$8x = 50$
$x = 50/8$

Answer $x =$ 6.25 *(4 marks)*

13

5 The function f(x) is defined as

$$f(x) = x^2 - 4 \qquad 0 \leqslant x < 3$$
$$= 14 - 3x \qquad 3 \leqslant x \leqslant 5$$

5 (a) Work out the value of f(1).

$(1)^2 - 4$

Answer -3 *(1 mark)*

5 (b) Work out the value of f(4). use $14 - 3x$

$14 - 3(4)$

Answer 2 *(1 mark)*

5 (c) Solve $f(x) = 0$

(1) Need: $x^2 - 4 = 0$

$\rightarrow x^2 = 4$

$\rightarrow x = 2$ or -2

But / cannot $= -2$ as $0 \leqslant x \leqslant 3$

(2) Need $14 - 3x = 0$

$\rightarrow 14 = 3x$

$\rightarrow x = 14/3$

Answer 2 or $14/3$ *(3 marks)*

5 (d) Work out the range of f(x).

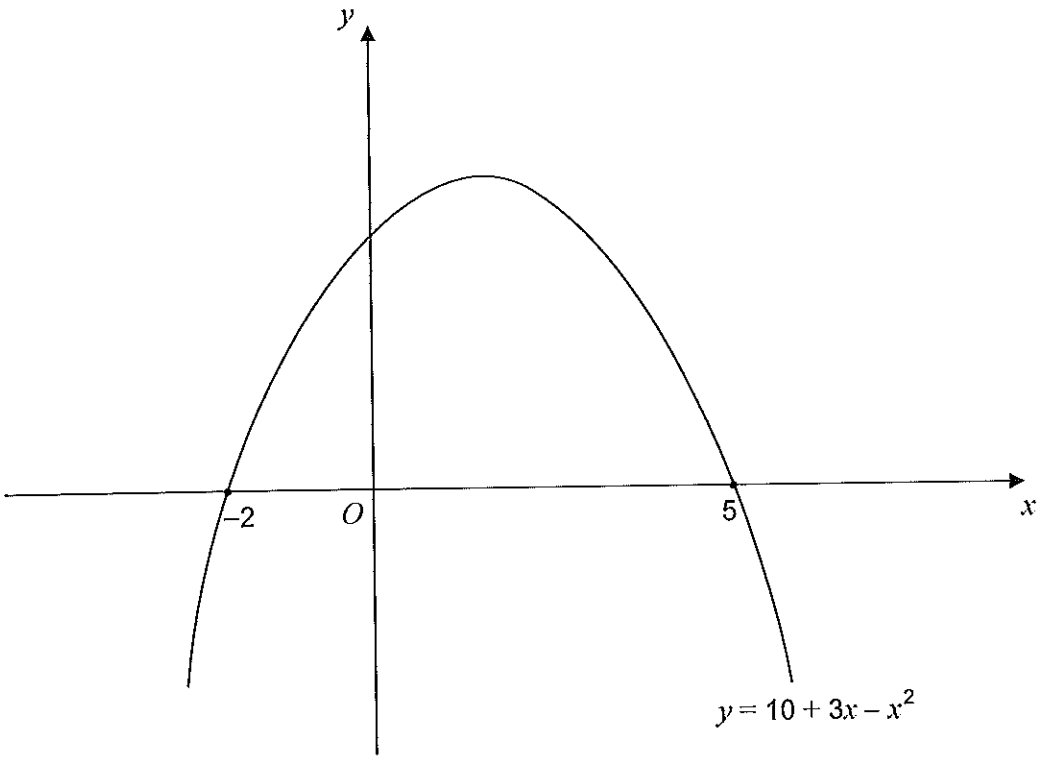
$f(0) = 0^2 - 4 = -4$ smallest $= -4$

$f(3) = 3^2 - 4 = 5$ biggest $= 5$

$f(5) = 14 - 3(5) = -1$ $\therefore$ range: $-4 \leqslant f(x) \leqslant 5$

Answer *(3 marks)*

6 Here is a sketch of $y = 10 + 3x - x^2$



6 (a) Write down the two solutions of $10 + 3x - x^2 = 0$

From graph

Answer $x =$ -2 and $x =$ 5 *(1 mark)*

6 (b) Write down the equation of the line of symmetry of $y = 10 + 3x - x^2$

$\frac{-2 + 5}{2}$

Answer $x = 1.5$ *(1 mark)*

6 (c) Write down the solution of $10 + 3x - x^2 \geqslant 0$

Graph above x-axis

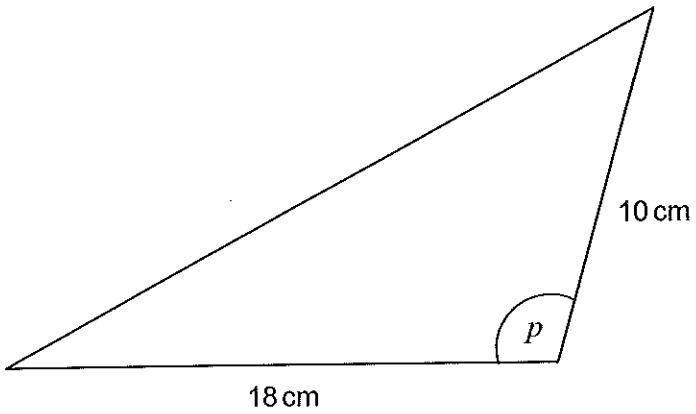
Answer $-2 \leq x \leq 5$ *(2 marks)*

Turn over for the next question

12

Do not write outside the box

7 The area of this triangle is 27 cm^2.



Not drawn accurately

$\frac{1}{2}$ ab sin(C) = Area

Work out the size of obtuse angle p.

$\frac{1}{2} \times 18 \times 10 \times \sin(p) = 27$

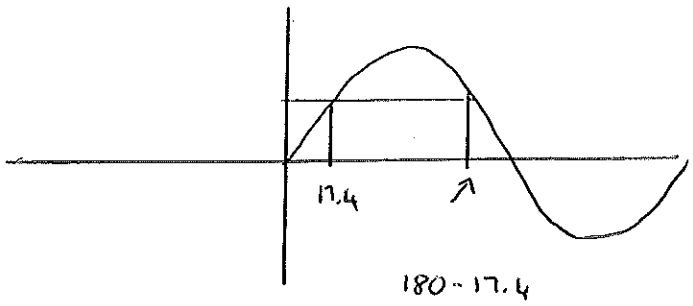
$90 \sin(p) = 27$

$\sin p = 27/90$

$p = \sin^{-1}(27/90) = 17.457...$

Answer 162.542... degrees *(3 marks)*

180 - 17.457...



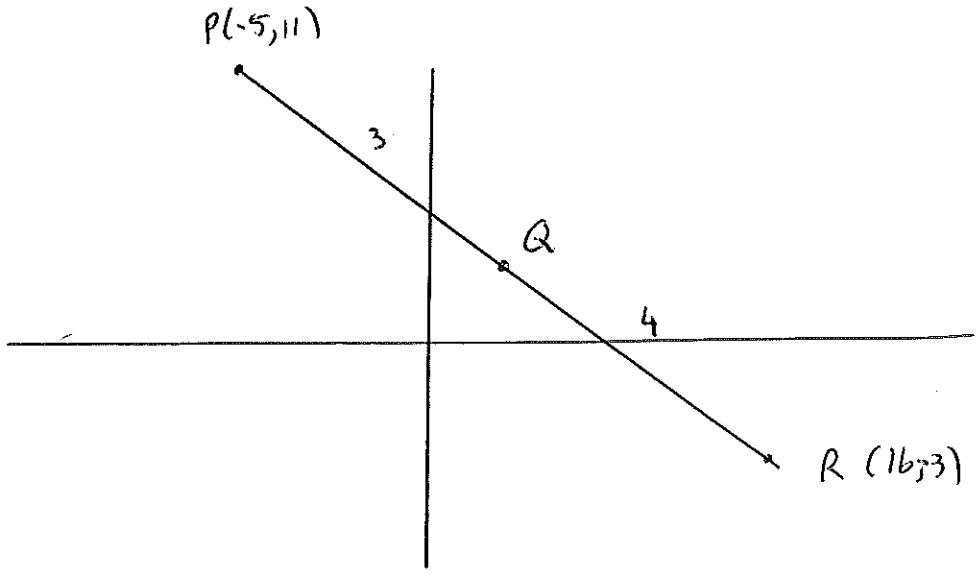
Do not write outside the box

8 PQR is a straight line.

P is (–5, 11) and R is (16, –3).

$PQ : QR = 3 : 4$

Work out the coordinates of Q.



[x] P → R = 16 - -5 = 21

7 parts → 1 part = 3

3 : 4 → 9 : 12

∴ x co-ordinate = -5 + 9 = 4

[y] P → R = -3 - 11 = -14

7 parts → 1 part = -2

3 : 4 → -6 : -8

∴ y- co-ordinate = 11 - 6 = 5

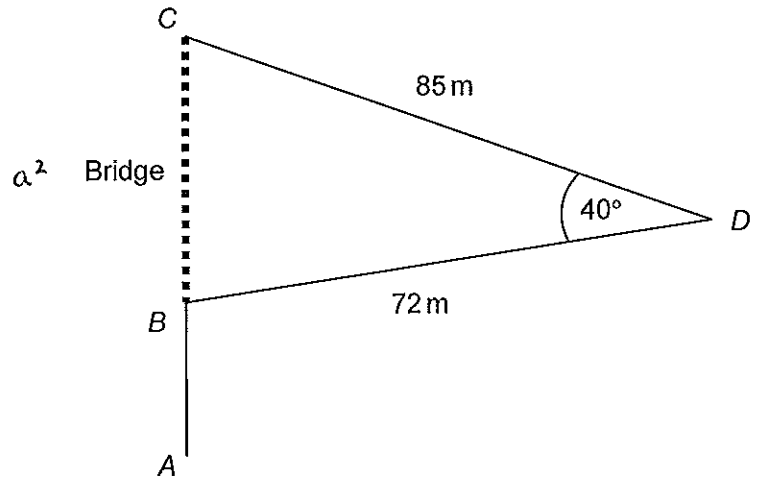
Answer (4, 5) *(4 marks)*

7

9 Sue is walking due North from *A* to *C*.

The bridge between *B* and *C* is closed.

She has to walk along paths *BD* and *DC* instead.



Not drawn accurately

Work out how much further Sue has to walk.

cosine rule:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$a^2 = 72^2 + 85^2 - 2 \times 72 \times 85 \times \cos(40)$

$a^2 = 3032.6...$

$\rightarrow a = \sqrt{3032.6...} = 55.0...$

$\therefore$ Extra distance $= 72 + 85 - 55.0$

$= 102$ m

Answer ..... m *(4 marks)*

10 Simplify fully $\dfrac{9x^3 - 16x}{6x + 8}$

diff of 2 squares

$= \dfrac{x(9x^2 - 16)}{2(3x + 4)}$

$= \dfrac{x(3x + 4)(3x - 4)}{2(3x + 4)}$

Answer $\dfrac{x(3x-4)}{2}$ *(4 marks)*

Do not write outside the box

11 The nth term of sequence X is $an + b$ (1)

The nth term of sequence Y is $bn + a$ (2)

11 (a) Show that the sequences have the same first term.

1st term; $n = 1$

(1) $a(1) + b = a + b$ (2) $b(1) + a = a + b$

(1 mark)

11 (b) The 2nd term of sequence X is equal to the 3rd term of sequence Y.

Show that $a = 2b$

[X] $n = 2 \rightarrow 2a + b$

[Y] $n = 3 \rightarrow 3b + a$

$\rightarrow 2a + b = 3b + a$

$-a$ $\quad a + b = 3b$

$-b$ $\quad a = 2b$

(2 marks)

11 (c) Prove that

$$\frac{n\text{th term of sequence } X}{n\text{th term of sequence } Y} = \frac{2n+1}{n+2}$$

$\frac{an + b}{bn + a}$ we know $a = 2b$

$\rightarrow \frac{2bn + b}{bn + 2b} = \frac{b(2n+1)}{b(n+2)}$

$= \frac{2n+1}{n+2}$

(3 marks)

Turn over for the next question

14

Do not write outside the box

12 A curve has equation $y = x^3 - 9x^2 + 24x - 16$

12 (a) Show that the curve passes through the point (1, 0).

$x = 1 \Rightarrow y = (1)^3 - 9(1)^2 + 24(1) - 16$

$= 1 - 9 + 24 - 16 = 0$

$\therefore (1, 0)$ on curve

(1 mark)

12 (b) State the coordinates of the point where the curve intersects the y-axis.

$x = 0$ on y-axis $\Rightarrow y = -16$

Answer (0, −16) *(1 mark)*

12 (c) Work out $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 3x^2 - 18x + 24$$

Answer ..... *(2 marks)*

Do not write outside the box

12 (d) Work out the coordinates of the **two** stationary points on the curve.

At st. points $dy/dx = 0$

$\rightarrow 3x^2 - 18x + 24 = 0$

$(\div 3) \rightarrow x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2$ $\qquad$ $x = 4$

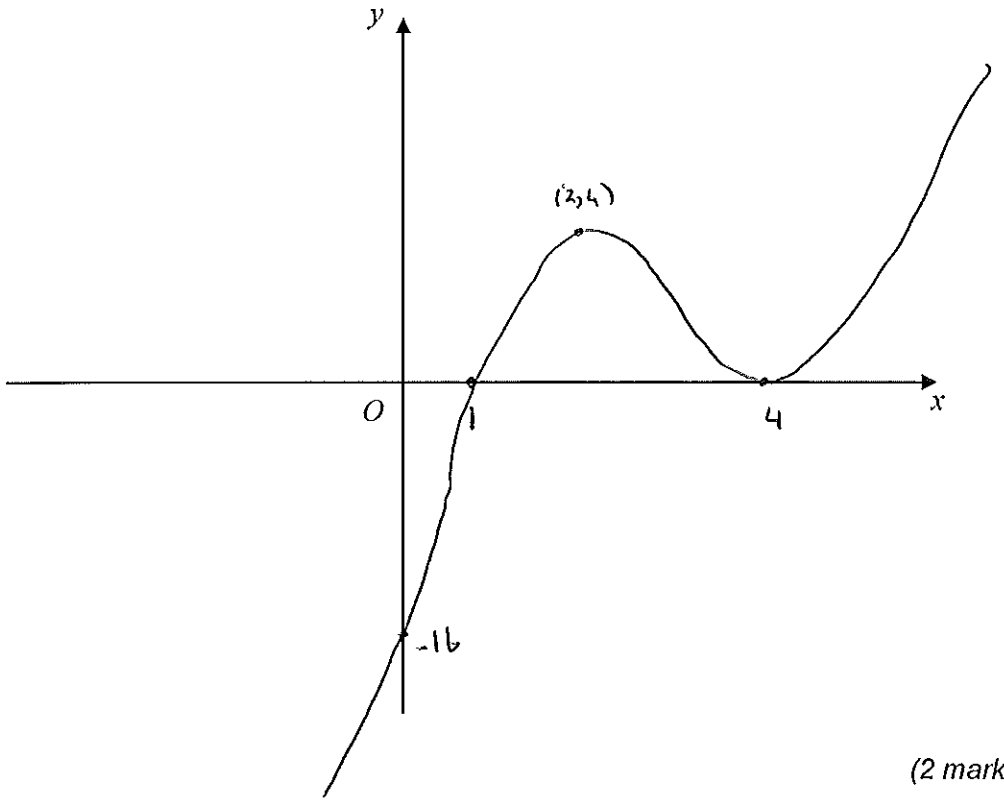
$\rightarrow y = 2^3 - 9(2)^2 + 24(2) - 16$ $\qquad$ $\rightarrow y = 4^3 - 9(4)^2 + 24(4) - 16$

$= 4$ $\qquad$ $= 0$

Answer (2, 4) and (4, 0) *(4 marks)*

12 (e) You are given that the curve has one maximum point and one minimum point.

Sketch the curve.



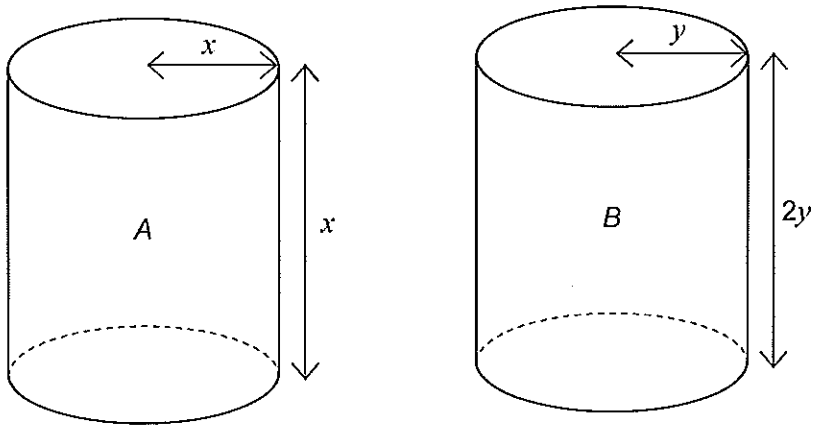
(2 marks)

10

Do not write outside the box

13 Cylinder A has radius x cm and height x cm.
Cylinder B has radius y cm and height $2y$ cm.

Not drawn accurately



Surface Area = $2\pi rh$ (curve) + $2\pi r^2$ (top + bottom)

You are given that

total surface area of cylinder A = **total** surface area of cylinder B

13 (a) Show that $x^2 = \frac{3}{2}y^2$

(A) Surface Area = $2\pi x^2 + 2\pi x^2 = 4\pi x^2$

(B) Surface Area = $2\pi y(2y) + 2\pi y^2 = 6\pi y^2$

$6\pi y^2 = 4\pi x^2$

$\div 4\pi$ { $\frac{3}{2} y^2 = x^2$

(4 marks)

13 (b) Which cylinder has the greater height?
You **must** show your working.

use calculator ✓

$x = \sqrt{\frac{3}{2} y^2} = \sqrt{\frac{3}{2}}\, y$

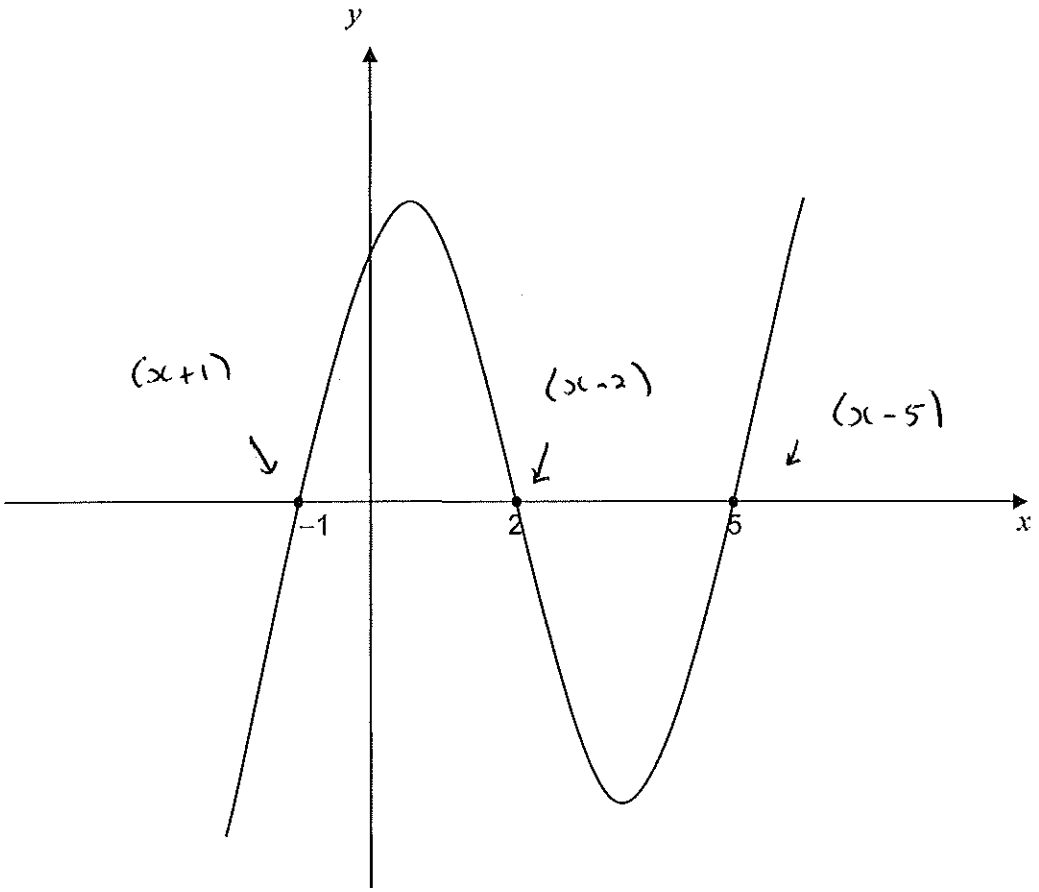
$\Rightarrow x = 1.224\ldots y$

(A) height = $1.224\ldots y$, (B) height = $2y$

Answer Cylinder B *(2 marks)*

Do not write outside the box

14 Here is a sketch of $y = x^3 + bx^2 + cx + d$ where b, c and d are constants.



Work out the values of b, c and d.

$y = (x+1)(x-2)(x-5)$

1st 2 brackets: $y = [x^2 - x - 2](x-5)$

$\rightarrow y = (x^3 - 5x^2 - x^2 + 5x - 2x + 10)$

$\rightarrow y = x^3 - 6x^2 + 3x + 10$

Answer $b =$ -6

$c =$ 3

$d =$ 10 *(4 marks)*

10

Do not write outside the box

15 (a) Work out the values of a and b such that

$(x-3)(x-3)$
$= x^2 - 6x + 9$

$$x^2 - 6x + 5 \equiv (x + a)^2 + b$$

$(x-3)^2 - 4$

Answer $a =$ -3

$b =$ -4 *(2 marks)*

15 (b) Rearrange the equation $m = 12 - (p - 1)^2$ to make p the subject.

$+(p-1)^2$ { $m + (p-1)^2 = 12$

$-m$ { $(p-1)^2 = 12 - m$

$\sqrt{\ }$ { $p - 1 = \pm\sqrt{12-m}$

$+1$ { $p = \pm\sqrt{12-m} + 1$

Answer ..... *(4 marks)*

Do not write outside the box

16 Matrix $\mathbf{P} = \begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix}$ Matrix $\mathbf{Q} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

You are given that $\mathbf{PQ} = \mathbf{QP}$

Work out the values of a and b.

PQ

$$\begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ a & a+b \end{pmatrix}$$

QP

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix} = \begin{pmatrix} 2+a & 3+b \\ a & b \end{pmatrix}$$

$\therefore \quad 5 = 3 + b \rightarrow b = 2$

$a + b = b \rightarrow a = 0$

Answer $a =$ 0

$b =$ 2 *(5 marks)*

Turn over for the next question

11

Do not write outside the box

18 (a) Expand and simplify $(2s + 1)(s - 1)$

$2s^2 - 2s + s - 1$

Answer $2s^2 - s - 1$ *(2 marks)*

18 (b) Hence, or otherwise, solve $2\sin^2\theta - \sin\theta - 1 = 0$ for $0° \leqslant \theta \leqslant 360°$

From a) → $(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0$ $\quad$ $\sin\theta - 1 = 0$

→ $\sin\theta = -1/2$ $\quad$ $\sin\theta = 1$

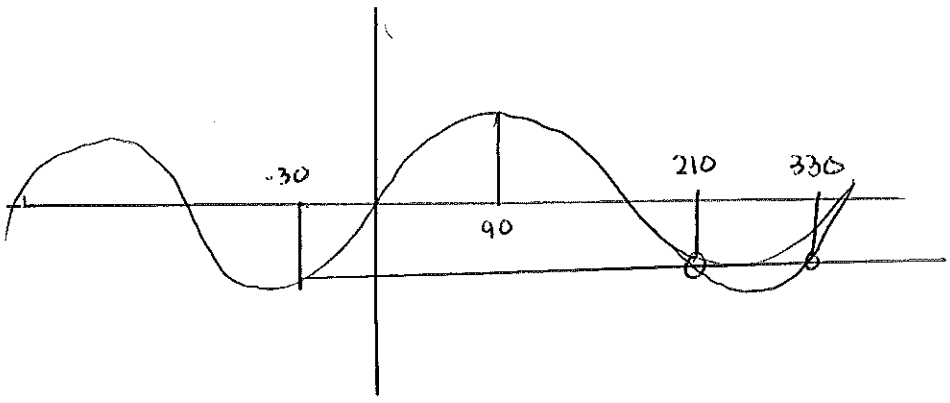
→ $\theta = \sin^{-1}(-1/2)$ $\quad$ → $\theta = \sin^{-1}(1)$ *(3 marks)*

$= -30°$ $\quad$ → $\theta = 90°$

(see graph)

$\theta = 90°, 210°, 330°$

Turn over for the next question

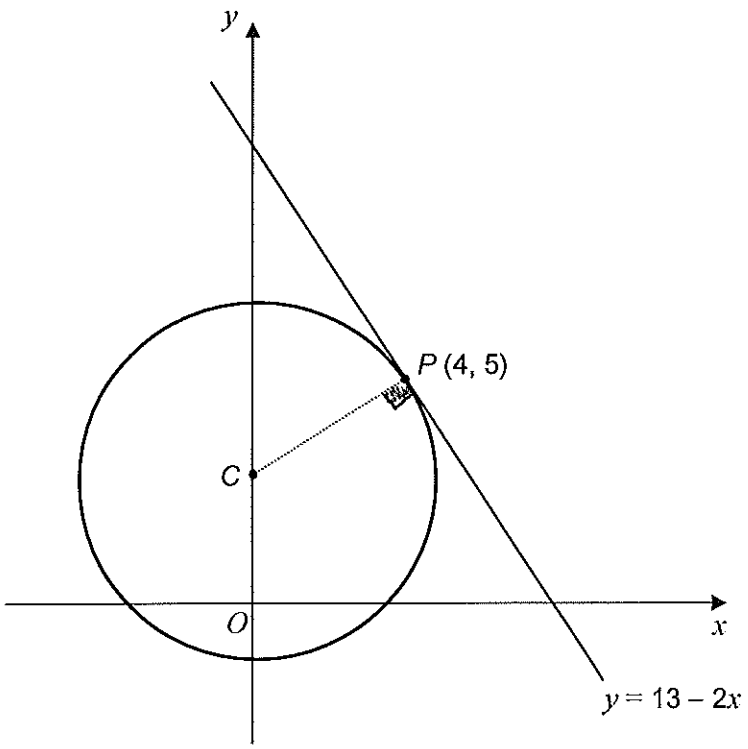


10

Do not write outside the box

19 The sketch shows point P on a circle, centre C.

The equation of the tangent at P is $y = 13 - 2x$



19 (a) Work out the gradient of PC.

Perpendicular → $^1/_2$

Answer $^1/_2$ *(1 mark)*

Do not write outside the box

19 (b) Work out the equation of the circle.

Need equation of radius

$m = \frac{1}{2}$ $\quad y - y_1 = m(x - x_1)$

$x_1 = 4$ $\quad \rightarrow y - 5 = \frac{1}{2}(x - 4)$

$y_1 = 5$ $\quad \rightarrow y - 5 = \frac{1}{2}x - 2$

$\rightarrow y = \frac{1}{2}x + 3$

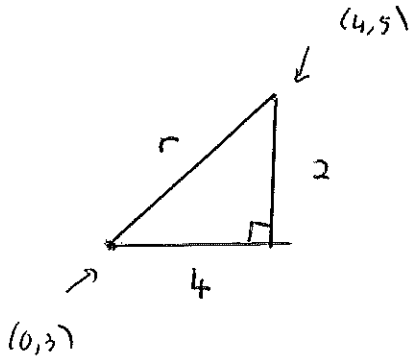
centre when $x = 0 \rightarrow y = 3 \rightarrow (0, 3)$

For radius, need distance from $(0,3)$ to $(4,5)$ (see below)

$\rightarrow r = \sqrt{20}$

Answer $(x^2) + (y-3)^2 = 20$ *(5 marks)*

Turn over for the next question



$r^2 = 4^2 + 2^2$

$\rightarrow r^2 = 20$

$\rightarrow r = \sqrt{20}$

6

Do not write outside the box

20 Factorise fully $x^3 + 4x^2 - 25x - 28$ $= f(x)$

Need a factor!

$f(-1) = (-1)^3 + 4(-1)^2 - 25(-1) - 28 = 0$

$\therefore (x+1)$ is a factor!

$(x+1)(\uparrow \square \uparrow) = x^3 + 4x^2 - 25x - 28$

$x^2 \qquad -28$

$\rightarrow (x+1)(x^2 + 3x - 28)$

$\rightarrow (x+1)(x-4)(x+7)$

Answer ..... *(6 marks)*

6

END OF QUESTIONS

$\square$ $(x+1)(x^2 + px - 28)$

Number of xs must $= -25$

$\rightarrow \quad px - 28x = -25x$

$\rightarrow \underline{\underline{p = 3}}$