

MR BARTON'S ANSWERS

Centre Number								Candidate Number						
Surname														
Other Names														
Candidate Signature														

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
TOTAL	



Level 2 Certificate in Further Mathematics

Further Mathematics

8360/1

Level 2

Practice Paper Set 2

Paper 1

Non-Calculator

<p>For this paper you must have:</p> <ul style="list-style-type: none">• mathematical instruments. <p>You may not use a calculator.</p>	
---	--

Time allowed
1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information

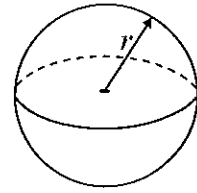
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 70.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

8360/1

Formulae Sheet

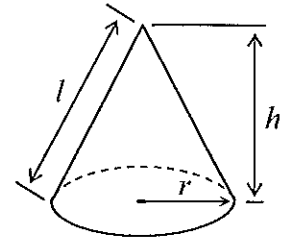
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

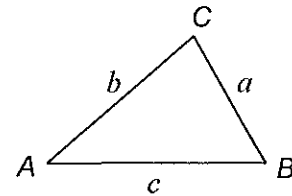
$$\text{Curved surface area of cone} = \pi r l$$



In any triangle ABC

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

$$\text{Sine rule} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



$$\text{Cosine rule} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$

Answer all questions in the spaces provided.

1

$$y = 2x^3 - 5x$$

Work out $\frac{dy}{dx}$.

$$3 \times 2x^2 - 5$$

Answer $6x^2 - 5$ (2 marks)

2

Here is a linear sequence.

	4	11	18	25
$(7n)$	7	14	21	28	

2 (a)

Work out an expression for the n th term.

.....
.....

Answer $7n - 3$ (2 marks)

2 (b)

How many terms are less than 150?

.....

$$\text{Solve } 7n - 3 < 150$$

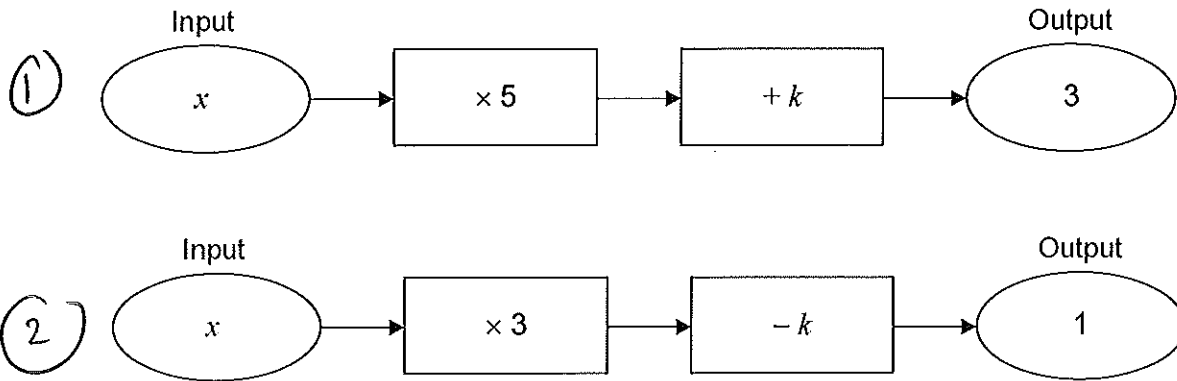
$$\rightarrow 7n < 153$$

$$\rightarrow n < 21 \frac{3}{7}$$
.....
.....

Answer 21 terms (2 marks)

Turn over for the next question

3 Here are two number machines with the same input, x .



Work out the value of x .

$$\begin{array}{r} \textcircled{1} \quad 5x + k = 3 \\ \textcircled{2} \quad \textcircled{+} \quad 3x - k = 1 \\ \hline 8x = 4 \\ \rightarrow x = 4/8 = 1/2 \end{array}$$

Answer $x = \dots\dots\dots 0.5 \dots\dots\dots$ (4 marks)

4 The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3, 4)$ onto the point $(2, b)$.

Work out the values of a and b .

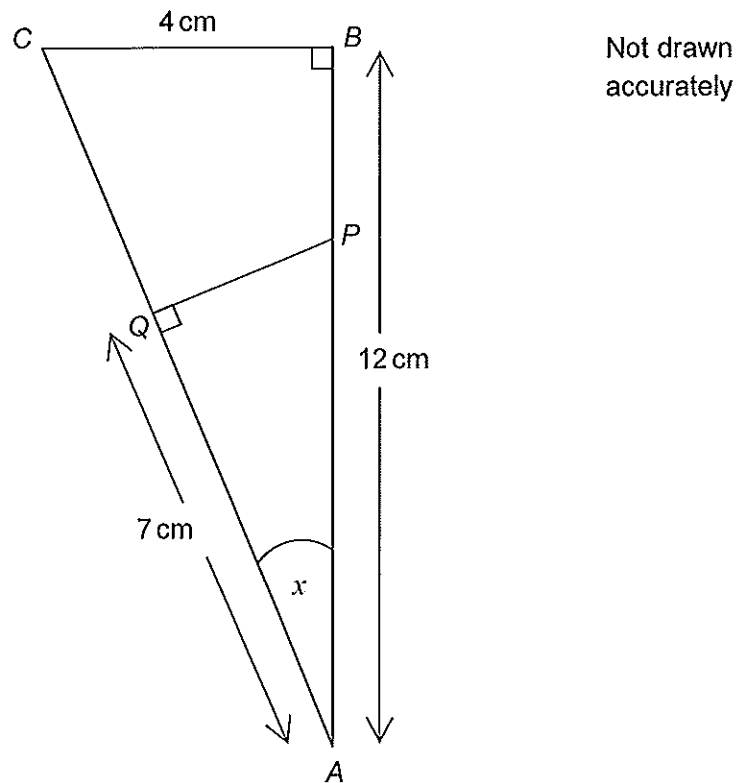
$$\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ b \end{pmatrix}$$

$$\begin{aligned} \therefore 3a + 2 \times 4 &= 2 \\ \rightarrow 3a + 8 &= 2 \\ \rightarrow 3a &= -6 \\ \rightarrow a &= -2 \end{aligned}$$

$$\begin{aligned} -3 + 4 &= b \\ \rightarrow b &= 1 \end{aligned}$$

Answer $a = \dots\dots\dots -2 \dots\dots\dots, b = \dots\dots\dots 1 \dots\dots\dots$ (3 marks)

- 5 The diagram shows two right-angled triangles ABC and APQ .



- 5 (a) Using triangle ABC , write down the value of $\tan x$. = $\frac{\text{Opp}}{\text{Adj}}$

Answer $\tan x = \dots \frac{4}{12} \text{ or } \frac{1}{3} \dots$ (1 mark)

- 5 (b) Work out the length of PQ .

$$\dots \tan(x) = \frac{PQ}{7} \dots$$

$$\dots \rightarrow \frac{1}{3} = \frac{PQ}{7} \dots$$

$$\dots \rightarrow 7 \times \frac{1}{3} = PQ \dots$$

Answer $\dots \frac{7}{3} \text{ or } 2\frac{1}{3} \dots$ cm (2 marks)

Turn over for the next question

6 Expand and simplify $(5x - 2y)(4x + 3y)$

$$20x^2 + 15xy - 8xy - 6y^2$$

Answer $20x^2 + 7xy - 6y^2$ (3 marks)

7 $p \Delta t$ is defined as $3p^2 + p - t^2 - t$

$$\begin{aligned} \text{For example } 7 \Delta 2 &= 3(7^2) + 7 - 2^2 - 2 \\ &= 147 + 7 - 4 - 2 \\ &= 148 \end{aligned}$$

7 (a) Work out $4 \Delta -3$

$$3(4^2) + 4 - (-3)^2 - (-3)$$

$$48 + 4 - 9 + 3$$

Answer 46 (2 marks)

7 (b) Work out all the solutions of the equation $x \Delta 5 = 0$

$$3x^2 + x - 5^2 - 5 = 0$$

$$3x^2 + x - 30 = 0$$

$$(3x + 10)(x - 3) = 0$$

$$3x + 10 = 0$$

$$x - 3 = 0$$

$$x = -10/3$$

$$x = 3$$

Answer (4 marks)

8 The equation of line L is $2x + y = 3$

8 (a) Line M is parallel to line L and passes through $(1, -6)$.

Work out the equation of line M .

Give your answer in the form $y = ax + b$

$$\text{Gradient: } 2x + y = 3 \rightarrow y = -2x + 3$$

$$\therefore \text{Gradient} = -2$$

$$m = -2 \quad y - y_1 = m(x - x_1)$$

$$x_1 = 1 \quad y + b = -2(x - 1)$$

$$y_1 = -6 \rightarrow y + b = -2x + 2$$

$$y = -2x - 4$$

Answer (3 marks)

8 (b) Line N is perpendicular to line L and passes through $(-5, 4)$.

Work out the point of intersection of line N and the x -axis.

$$\text{If perpendicular, gradient} = \frac{1}{2} \quad (-2 \times \frac{1}{2} = -1)$$

$$m = \frac{1}{2} \quad y - y_1 = m(x - x_1)$$

$$x_1 = -5 \quad y - 4 = \frac{1}{2}(x + 5)$$

$$y_1 = 4 \quad y - 4 = \frac{1}{2}x + \frac{5}{2}$$

$$y = \frac{1}{2}x + \frac{13}{2}$$

$$\boxed{x - ax + b} \quad \text{when } y = 0 \rightarrow 0 = \frac{1}{2}x + \frac{13}{2}$$

$$\frac{1}{2}x = -\frac{13}{2} \rightarrow x = -13$$

Answer (..... -13....., 0.....) (4 marks)

Turn over for the next question

9 Simplify fully $\frac{3x}{(x-3)(x+6)} - \frac{2}{(x+6)}$

$$\frac{3x}{(x-3)(x+6)} - \frac{2(x-3)}{(x-3)(x+6)}$$

$$= \frac{3x - 2(x-3)}{(x-3)(x+6)}$$

$$= \frac{3x - 2x + 6}{(x-3)(x+6)} = \frac{x+6}{(x-3)(x+6)} = \frac{1}{(x-3)}$$

Answer (4 marks)

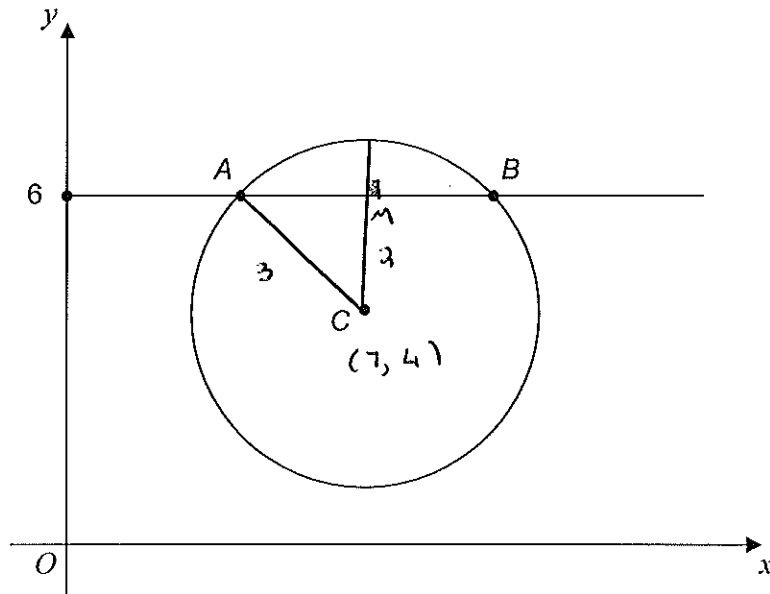
10 Make y the subject of the formula $x = \sqrt{\frac{y+1}{y-2}}$

$$\begin{aligned} x^2 &= \frac{y+1}{y-2} \\ x^2(y-2) &= y+1 \\ x^2y - 2x^2 &= y+1 \\ x^2y &= 2x^2 + y + 1 \\ x^2y - y &= 2x^2 + 1 \\ y(x^2 - 1) &= 2x^2 + 1 \\ y &= \frac{2x^2 + 1}{x^2 - 1} \end{aligned}$$

Answer (5 marks)

- 12 The diagram shows a sketch of the circle $(x - 7)^2 + (y - 4)^2 = 9$ with centre C .
The line $y = 6$ intersects the circle at A and B .

Show that $AB = 2\sqrt{5}$



Centre of circle = $(7, 4)$, Radius = $\sqrt{9} = 3$
 $CM = 2$ ($6 - 4$)

$$\rightarrow AC^2 = AM^2 + CM^2$$

$$3^2 = AM^2 + 2^2$$

$$9 - 4 = AM^2 \rightarrow AM^2 = 5 \rightarrow AM = \sqrt{5}$$

$$\therefore AB = 2 \times \sqrt{5} = 2\sqrt{5}$$

(5 marks)

- 13 Work out the value of x if $\frac{\sqrt{x} \times \sqrt{8}}{\sqrt{3}} = 4\sqrt{5}$

$$\begin{aligned} & \times \sqrt{3} \quad \left\{ \begin{array}{l} \sqrt{x} \times \sqrt{8} = 4\sqrt{15} \\ \div \sqrt{8} \quad \left\{ \begin{array}{l} \sqrt{x} = \frac{4\sqrt{15}}{\sqrt{8}} \\ \phantom{\sqrt{x}} \end{array} \right. \\ \phantom{\sqrt{x}} \quad \left\{ \begin{array}{l} x = \frac{16 \times 15}{8} = 30 \end{array} \right. \end{array} \right. \end{aligned}$$

Answer $x = \dots\dots\dots 30 \dots\dots\dots$ (4 marks)

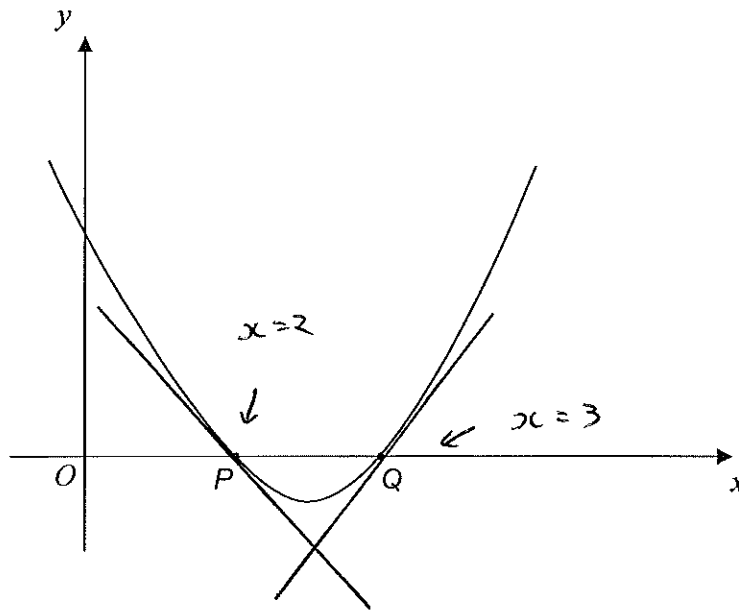
- 14 $(x - 5)$ is a factor of $x^3 - 6x^2 + ax - 20 = f(x)$

Work out the value of a .

$$\begin{aligned} & \text{By Factor Theorem: } f(5) = 0 \\ & \rightarrow 5^3 - 6(5)^2 + 5a - 20 = 0 \\ & \rightarrow 125 - 150 + 5a - 20 = 0 \\ & 5a - 45 = 0 \\ & 5a = 45 \\ & a = 9 \end{aligned}$$

Answer $a = \dots\dots\dots$ (3 marks)

15

The graph shows a sketch of $y = (x-2)(x-3)$ The curve intersects the x -axis at P and Q .Show that the tangents at P and Q are perpendicular.

$$\dots\dots\dots y = x^2 - 3x - 2x + 6 \dots\dots\dots \rightarrow \dots\dots\dots y = x^2 - 5x + 6 \dots\dots\dots$$

Find Gradient of Tangents

$$\dots\dots\dots \frac{dy}{dx} = 2x - 5 \dots\dots\dots$$

$$\boxed{P} \quad x=2, \quad \frac{dy}{dx} = 2(2) - 5 = -1$$

$$\boxed{Q} \quad x=3, \quad \frac{dy}{dx} = 2(3) - 5 = 1$$

$$\dots\dots\dots -1 \times 1 = -1 \dots\dots\dots$$

\therefore product of gradients = -1

\therefore tangents are perpendicular

(5 marks)

16

 n is a positive integer.Prove that $(n+2)^2 + (n+1)^2 - 1$ is always a multiple of 4.

$$\dots\dots\dots n^2 + 4n + 4 \dots\dots + n^2 + 2n + 1 \dots\dots - 1 \dots\dots$$

$$\rightarrow \dots\dots 2n^2 + 6n + 4 \dots\dots$$

$$= \dots\dots 2(n^2 + 3n + 2) \dots\dots$$

$$= \dots\dots 2(n+1)(n+2) \dots\dots$$

$(n+1)$ & $(n+2)$ are consecutive numbers, so one must be even. Then the answer is multiplied (6 marks) by 2. Any 2 even numbers multiplied together are divisible by 4.

11

END OF QUESTIONS