MR BARTON'S SOLUTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Pages	Mark
3	
4 - 5	
6 - 7	
8 - 9	
10 - 11	
12 - 13	
14 - 15	
16 - 17	
TOTAL	

AQA Level 2 Certificate in Further Mathematics

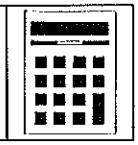
Further Mathematics 8360/2
Level 2

Practice Paper Set 1

Paper 2

Calculator

For this paper you must have:
- a calculator
- mathematical instruments.



Time allowed
2 hours

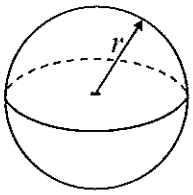
Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the space provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer booklet.

Formulae Sheet

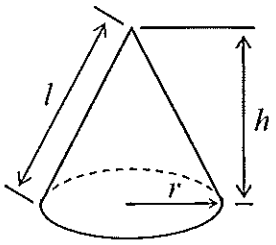
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



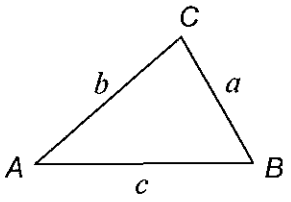
Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*

Area of triangle $= \frac{1}{2}ab \sin C$



Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ $\qquad \sin^2 \theta + \cos^2 \theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) The ratio of males to females at a party is 3 : 5
There are 12 more females than males.

How many people are at the party?

3 : 5
6 : 10
12 : 20
18 : 30 ✓

18 + 30 = 48

Answer 48 *(3 marks)*

1 (b) Show that $a\%$ of $b = b\%$ of a

$a\%$ of $b = \frac{a}{100} \times b = \frac{ab}{100}$

$b\%$ of $a = \frac{b}{100} \times a = \frac{ab}{100}$

(2 marks)

1 (c) A runner increases the distance she runs by 10% each week.
In week 1 she runs 16 miles.

In which week will she first run over 20 miles?
You **must** show your working.

week number (-1)

16 × 1.1

Week 1 = 16, Week 2 = 17.6 Week 3 = 19.36 week 4 = 21.296

Answer Week 4 *(2 marks)*

Turn over for the next question

7

2 (a) Expand and simplify $4(2x + 3) + 2(x - 7)$

$8x + 12 + 2x - 14$

Answer $10x - 2$ *(2 marks)*

2 (b) Expand $m^3(m + 2)$

Answer $m^4 + 2m^3$ *(2 marks)*

2 (c) Solve $\frac{9 - 2d}{4} = 1 - d$

$\times 4$ $9 - 2d = 4 - 4d$

$+4d$ $9 + 2d = 4$

-9 $2d = -5$

$\div 2$ $d = -2.5$ Answer $d =$ -2.5 *(3 marks)*

3 (a) The nth term of a sequence is $4n - 10$.

3 (a) (i) Show that the $(n + 1)$th term can be written as $4n - 6$.

$(n+1)$th term $= 4(n+1) - 10$

$= 4n + 4 - 10$

$= 4n - 6$ *(2 marks)*

3 (a) (ii) Prove that the sum of any two consecutive terms of the sequence is divisible by 8.

consecutive terms: n & $n + 1$

nth term $= 4n - 10$ $(n+1)$th term $= 4n - 6$

sum $= 4n - 10 + 4n - 6 = 8n - 16$

$= 8(n - 2)$

$\therefore$ divisible by 8 *(2 marks)*

3 (b) The nth term of a different sequence is $\frac{3n}{n+5}$

3 (b) (i) Explain why 1 is **not** a term in this sequence.

Try to solve: $\frac{3n}{n+5} = 1$

$\times (n+5)$: $3n = n + 5$ n needs to be

$-n$: $2n = 5$ an integer

$\div 2$: $n = 2.5$ *(3 marks)*

3 (b) (ii) Work out the limiting value of the sequence as $n \to \infty$

As $n \to \infty$, sequence $\to 3n/n = 3$

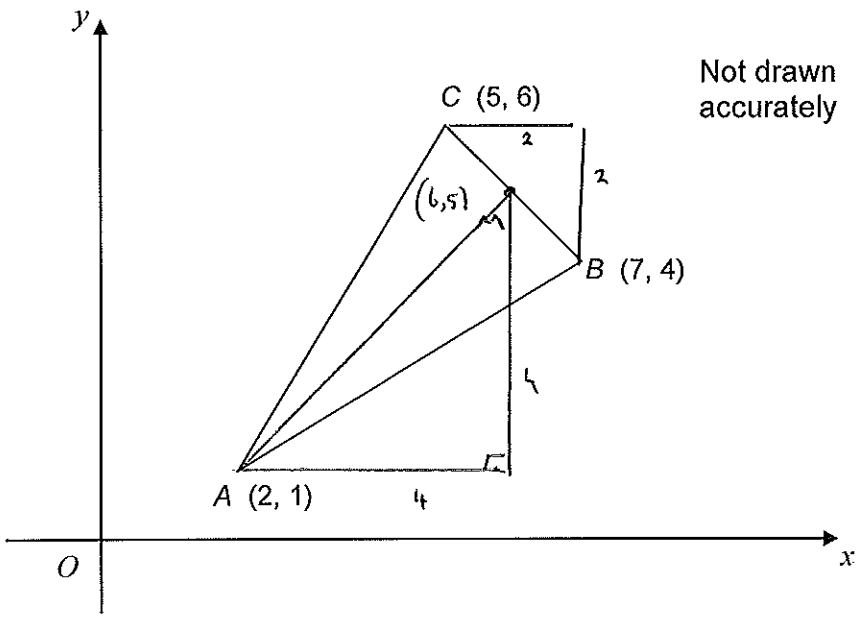
Answer 3 *(2 marks)*

Turn over for the next question

16

Do not write outside the box

4 The diagram shows an isosceles triangle ABC, with $AB = AC$.



Not drawn accurately

Work out the area of the triangle.

Length of BASE : BC $= \sqrt{2^2 + 2^2} = \sqrt{8}$

Midpoint of BC : $\left(\frac{5+7}{2}, \frac{6+4}{2}\right) = (6, 5)$

Length of AM (Height) $= \sqrt{4^2 + 4^2} = \sqrt{32}$

Area : $\frac{1}{2} b \times h = \frac{1}{2} \times \sqrt{8} \times \sqrt{32}$

$= \frac{1}{2} \times \sqrt{256} = \frac{1}{2} \times 16 = 8$

Answer 8 units^2 *(5 marks)*

Do not write outside the box

5 (a) Solve $x^2 - 11x + 28 = 0$

$(x-4)(x-7) = 0$

Answer $x = 4$ or $x = 7$ *(3 marks)*

5 (b) Use your answer to part **(a)** to solve $x - 11\sqrt{x} + 28 = 0$

Let $y = \sqrt{x}$

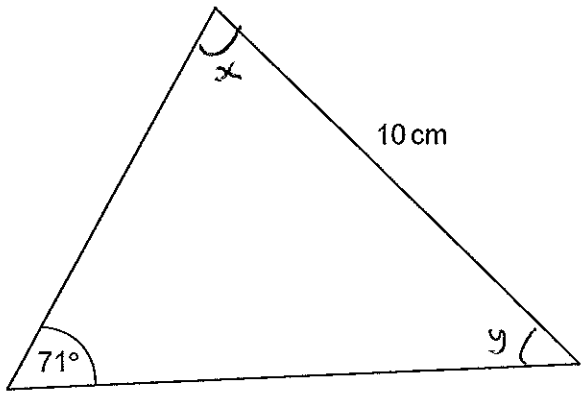
$\rightarrow y^2 - 11y + 28 = 0$

Answer $(y-4)(y-7) = 0$ *(2 marks)*

$y = 4$ $\quad$ $y = 7$

$\rightarrow x = 16$ or $x = 49$

6 Here is a triangle.



Not drawn accurately

SINE RULE

Work out the size of the smallest angle in the triangle.

$\frac{\sin x}{9} = \frac{\sin(71)}{10} \rightarrow \sin(x) = 9 \times \frac{\sin(71)}{10}$

$= 0.8509...$

$\rightarrow x = \sin^{-1}(0.8509...)$

$= 58.316^\circ$

Answer ..... degrees *(4 marks)*

So, other angle (y) $= 180 - 71 - 58.36...$

$= 50.683...^\circ$

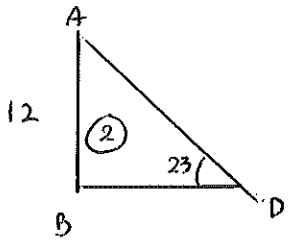
↑ smallest

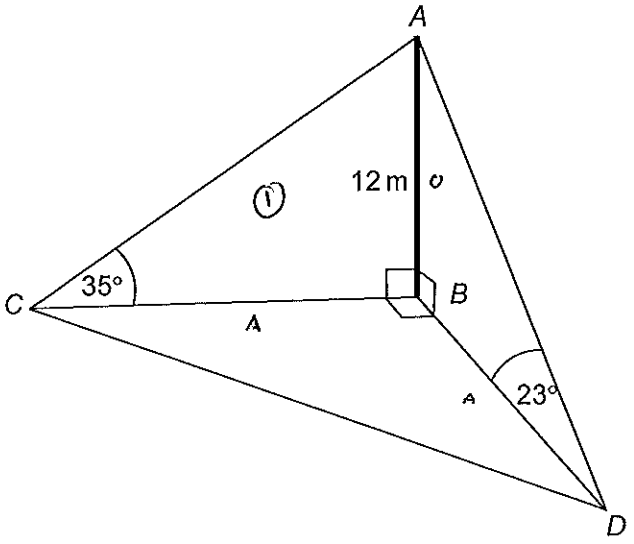
14

Do not write outside the box

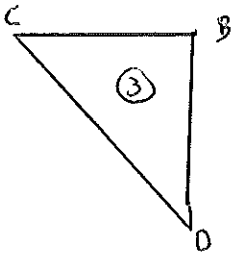
7 The diagram shows a vertical mast, *AB*, 12 metres high.
Points *B*, *C* and *D* are on a horizontal plane.

Point *C* is due West of *B*.
The angle of elevation of *A* from *C* is 35°.
Point *D* is due South of *B*.
The angle of elevation of *A* from *D* is 23°.





Not drawn accurately



7 (a) Calculate the distance *CD*.

① GET BC ... $\tan(35) = \frac{12}{BC} \rightarrow BC = \frac{12}{\tan(35)} = 17.137...$

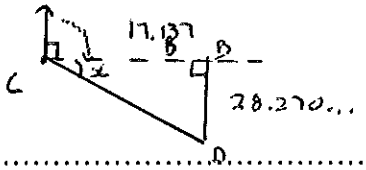
② GET BD ... $\tan(23) = \frac{12}{BD} \rightarrow BD = \frac{12}{\tan(23)} = 28.270...$

③ Now use Pythag: $CD^2 = \sqrt{17.137...^2 + 28.270...^2}$

$= 33.05...$ m

Answer 33.1 m (1dp) metres *(6 marks)*

7 (b) Calculate the bearing of *D* from *C*.
Give your answer to the nearest degree.



$\tan x = \frac{28.270...}{17.137}$

$\rightarrow x = \tan^{-1}\left(\frac{28.270...}{17.137}\right) = 58.77...$

must add 90° (see diagram)

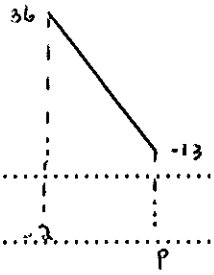
Answer $= 148.7 = 149°$ ° *(3 marks)*

8 (a) The function f(x) is defined as

$$f(x) = 22 - 7x \qquad -2 \leqslant x \leqslant p$$

The range of f(x) is $-13 \leqslant f(x) \leqslant 36$

Work out the value of p.



Solve: $22 - 7x = -13$

$+7x$: $22 = -13 + 7x$

$+13$: $35 = 7x$

$\div 7$: $5 = x$

Answer $p = 5$ *(2 marks)*

8 (b) The function g(x) is defined as

$$g(x) = x^2 - 4x + 5 \qquad \text{for all } x$$

8 (b) (i) Express g(x) in the form $(x-a)^2 + b$ complete the square

$(x-2)^2 + 1$

Answer $(x-2)^2 + 1$ *(2 marks)*

8 (b) (ii) Write down the range of g(x).

where $f(x) = x^2$

$= f(x-2) + 1$

Answer ∴ Range: $\geqslant 1$ *(1 mark)*

9 The equation of line *A* is $y = 5 - 2x$

Line *B* is parallel to line *A*.

Line *B* passes through the point (–3, 7).

Work out the coordinates of the point where line *B* intersects the x-axis.

Eq. of B: $y - y_1 = m(x - x_1)$

$m = -2$ $y - 7 = -2(x + 3)$

$x_1 = -3$ → $y - 7 = -2x - 6$

$y_1 = 7$ Intersects x axis when $y = 0$

→ $-7 = -2x - 6$

$+2x$: $2x - 7 = -6$

$+7$: $2x = 1$

$\div 2$: $x = 0.5$

Answer (0.5, 0) *(4 marks)*

18

Do not write outside the box

10 (a) Factorise fully $n^3 - n$

diff of 2 squares

$n(n^2 - 1) \rightarrow n(n-1)(n+1)$

Answer ..... *(2 marks)*

10 (b) n is an integer greater than 1.

Explain why $n^3 - n$ is divisible by 6.

$(n-1)$, n & $(n+1)$ are 3 consecutive integers. Any 3 consecutive integers must include a multiple of 2 & a multiple of 3, which gives a multiple of 6.

(2 marks)

11 You are given that $x = 5^m$ and $y = 5^n$

11 (a) Write 5^{m+2} in terms of x.

Give your answer in its simplest form.

$5^{m+2} = 5^m \times 5^2$

$= 5^m \times 25$

Answer $25x$ *(2 marks)*

11 (b) Write 5^{m-n} in terms of x and y.

$= 5^m \div 5^y$

Answer $\frac{x}{y}$ *(1 mark)*

11 (c) Write 5^{3n} in terms of y.

$= (5^n)^3$

Answer y^3 *(1 mark)*

11 (d) Write $5^{\frac{m+n}{2}}$ in terms of x and y.

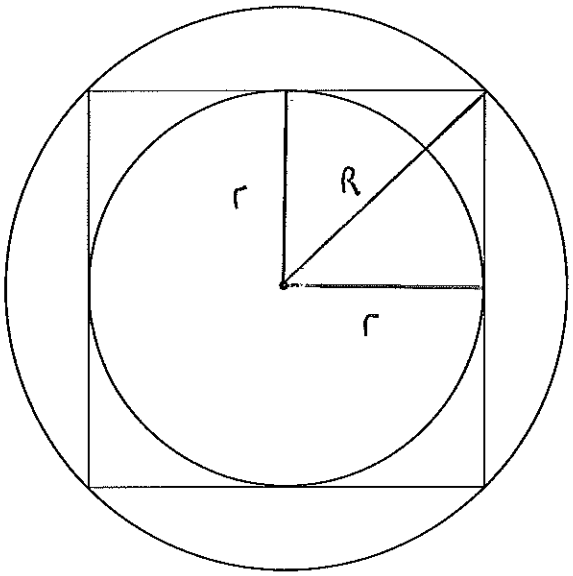
$= 5^{\frac{m}{2}} \times 5^{\frac{n}{2}} = (5^m)^{1/2} \times (5^n)^{1/2}$

Answer $\sqrt{x} \times \sqrt{y}$ *(2 marks)*

or $\sqrt{xy}$

Do not write outside the box

12 The diagram shows a square and two circles.
The smaller circle has radius r and touches the sides of the square.
The larger circle has radius R and passes through the vertices of the square.



See Diagram

Show that $R = r\sqrt{2}$

By Pythagoras: $R^2 = r^2 + r^2$

$\Rightarrow R^2 = 2r^2$

$\Rightarrow R = \sqrt{2r^2}$

$\Rightarrow R = r\sqrt{2}$

(3 marks)

Turn over for the next question

13

13 (a) Solve $5y - 4 < 2y + 6$

$-2y$ $3y - 4 < 6$

$+4$ $3y < 10$

$\div 3$ $y < 10/3$

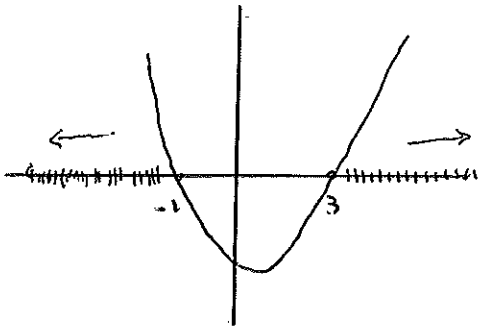
Answer ..... *(2 marks)*

13 (b) Solve $x^2 - 2x - 3 \geq 0$

$(x - 3)(x + 1) \geq 0$

(see graph below)

Answer $x \geq 3$ or $x \leq -1$ *(4 marks)*



where is graph ≥ 0 ?

Do not write outside the box

14 (a) Work out the stationary points on the curve $y = x^3 - 12x$

$dy/dx = 3x^2 - 12$

At stationary points, $dy/dx = 0$

$\rightarrow 3x^2 - 12 = 0$

$(\div 3) \quad x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = -2, \quad y = (-2)^3 - 12(-2) = 16$

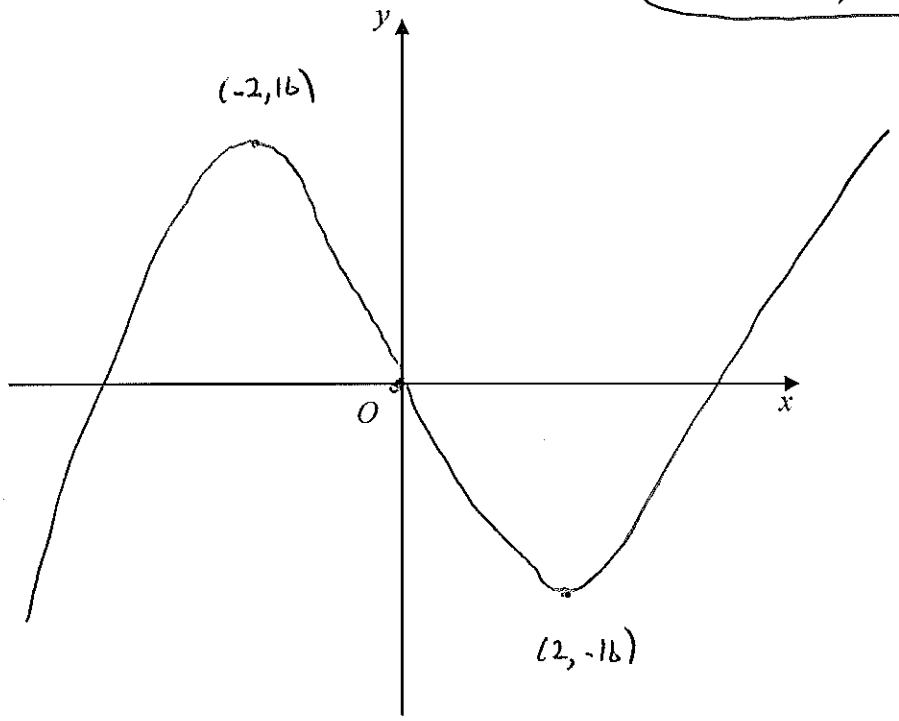
$x = 2$

$y = (2)^3 - 12(2) = -16$

Answer $(2, -16) \quad (-2, 16)$ *(4 marks)*

14 (b) Sketch the curve $y = x^3 - 12x$

When $x = 0$, $y = 0$



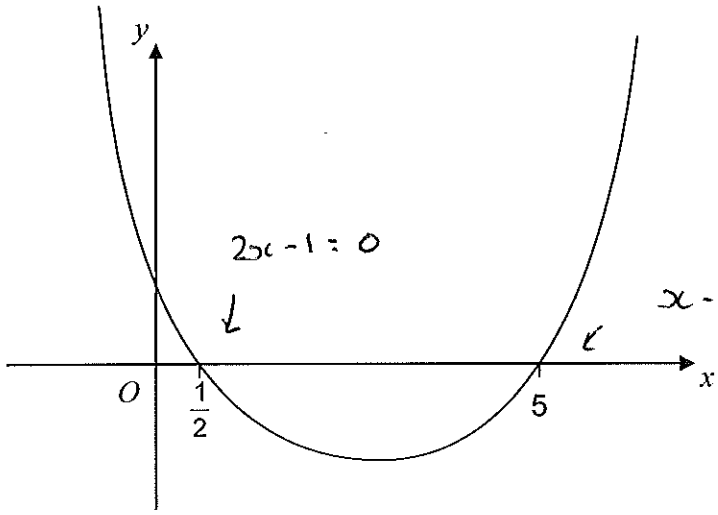
(3 marks)

Turn over for the next question

13

Do not write outside the box

15 The diagram shows a quadratic graph that intersects the x-axis when $x = \frac{1}{2}$ and $x = 5$.



Not drawn accurately

Work out the equation of the quadratic graph.

Give your answer in the form $y = ax^2 + bx + c$ where a, b and c are integers.

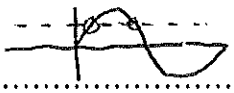
FACTORS must be $(2x-1)(x-5)$

$\rightarrow y = 2x^2 - 10x - x + 5$

$\rightarrow y = 2x^2 - 11x + 5$

Answer ..... *(3 marks)*

16 (a) Solve $\sin x = 0.8$ for $0° \leqslant x \leqslant 180°$



$\sin^{-1}(0.8) = 53.13...$

See diagram

Also: $180 - 53.13... = 126.86...$

Answer 53.13, 126.87 *(2 marks)*

16 (b) Solve $2 \sin x = -3 \cos x$ for $0° \leqslant x \leqslant 360°$

$\div \cos(x)$ $\frac{2\sin(x)}{\cos(x)} = -3$

$\frac{\sin(x)}{\cos(x)} = \tan(x)$

$\rightarrow 2\tan(x) = -3$

$\rightarrow \tan(x) = -3/2$

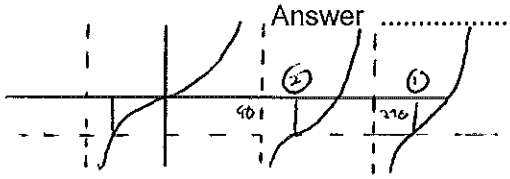
$\tan^{-1}(-3/2) = -56.309...$

(see diagram below)

① $x = 360 - 56.309 = 303.7$

② $x = 180 - 56.309 = 123.7$

Answer 123.7, 303.7 *(4 marks)*



17 Work out the equation of the normal to the curve $y = 2x^3 - x^2 + 1$ at the point (1, 2).

Give your answer in the form $y = mx + c$

Need gradient + point!

$dy/dx = 6x^2 - 2x$

when $x = 1$, $dy/dx = 6(1)^2 - 2(1) = 4$ (grad of tangent)

$\therefore$ gradient of normal $= -1/4$

$m = -1/4$ $\quad y - y_1 = m(x - x_1)$

$x_1 = 1$ $\quad y - 2 = -1/4(x - 1)$

$y_1 = 2$ $\quad y - 2 = -1/4x + 1/4$

(+2) $\quad y = -1/4x + 9/4$

Answer ..... *(5 marks)*

Turn over for the next question

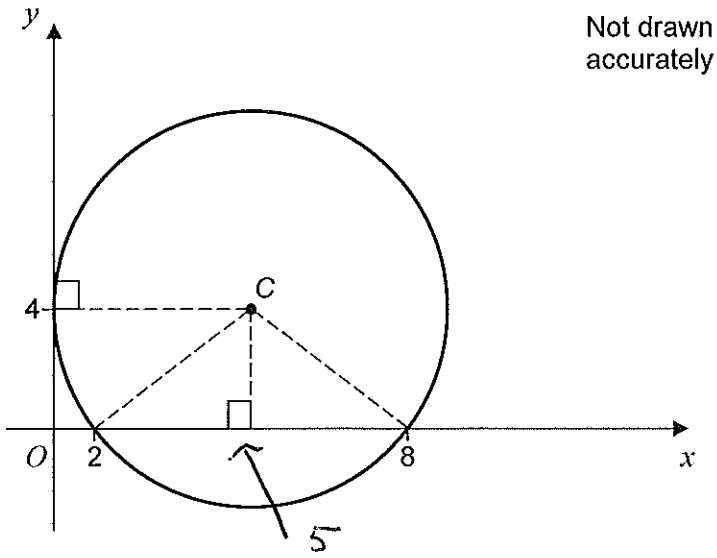
14

Do not write outside the box

18 The diagram shows a circle, centre C.

The circle touches the y-axis at (0, 4).

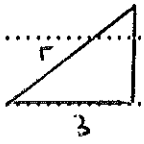
The circle intersects the x-axis at (2, 0) and (8, 0).



Work out the equation of the circle.

Centre of circle = (5, 4)

Need radius:



$r = \sqrt{3^2 + 4^2} = 5$

∴ Equation: $(x - 5)^2 + (y - 4)^2 = 5^2$

or $(x - 5)^2 + (y - 4)^2 = 25$

Answer ..... *(5 marks)*

Do not write outside the box

19 The equation $x^3 - x^2 + ax + b = 0$ has three integer solutions.
Two of these solutions are $x = 1$ and $x = 2$.

Work out the third solution to the equation.

Let $f(x) = x^3 - x^2 + ax + b$

by factor theorem, $f(1) = f(2) = 0$

$f(1) = 1 - 1 + a + b = 0 \rightarrow a + b = 0$

$f(2) = 8 - 4 + 2a + b = 0 \rightarrow 2a + b = -4$

Solve simultaneous equations: $a = -4$

$\rightarrow b = 4$

$\therefore x^3 - x^2 - 4x + 4 = 0$

By Factor Theorm $(x-1)(x-2)[\quad] = 0$

must be $x + 2$ to get $(+4)$ at the end

$x + 2 = 0 \rightarrow x = -2$

Answer $x =$ $x = -2$ *(5 marks)*

END OF QUESTIONS

10