Centre Number		Candidate Number	
Surname			
Other Names	MR BARTON'S		
Candidate Signature	WORKED SOLUTIONS.		

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MS2B

Unit Statistics 2B

Friday 12 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 In a survey of the tideline along a beach, plastic bottles were found at a constant average rate of 280 per kilometre, and drinks cans were found at a constant average rate of 220 per kilometre. It may be assumed that these objects were distributed randomly and independently.

Calculate the probability that:

(a) a 10 m length of tideline along this beach contains no more than 5 plastic bottles;

[2 marks]

(b) a 20 m length of tideline along this beach contains exactly 2 drinks cans;

[3 marks]

(c) a 30 m length of tideline along this beach contains a **total** of at least 12 but fewer than 18 of these two types of object.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

a) 280 per km = 2.8 per 10m

$\rightarrow \quad PB \sim Po(2.8)$

$P(PB \leq 5) = 0.9349$ (from tables)

b) 220 per km = 4.4 per 20m

$\rightarrow \quad DC \sim Po(4.4)$

$P(DC = 2) = e^{-4.4} \times \left(\frac{4.4^2}{2!}\right)$

$= 0.118844...$

c) Total = 500 per km = 15 per 30m

$\rightarrow \quad T \sim Po(15)$

$P(12 \leq T < 18)$

Need 12, 13, 14, 15, 16, 17

$\rightarrow \quad P(T \leq 17) - P(T \leq 11)$

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QUESTION PART REFERENCE	Answer space for question 1
	$= 0.7489 - 0.1848$
	$= 0.5641$

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2 The continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} \dfrac{1}{k} & a \leqslant x \leqslant b \\ 0 & \text{otherwise} \end{cases}$$

(a) Write down, in terms of a and b, the value of k.

[1 mark]

(b) (i) Given that $\mathrm{E}(X) = 1$ and $\mathrm{Var}(X) = 3$, find the values of a and b.

[4 marks]

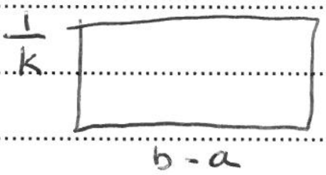
(ii) Four independent values of X are taken. Find the probability that exactly one of these four values is negative.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

a) Area $= 1$



$\rightarrow \quad k = (b - a)$

b) i) ① $E(x) = \frac{1}{2}(a+b) = 1$

② $Var(x) = \frac{1}{12}(b-a)^2 = 3$

② $\rightarrow \quad (b-a)^2 = 36$

$b - a = 6$ (cannot be -6 as $b > a$)

① $\rightarrow \quad a + b = 2$

$$\begin{aligned} & b - a = 6 \\ + \quad & b + a = 2 \\ \hline & 2b = 8 \rightarrow b = 4 \end{aligned}$$

$a = -2$

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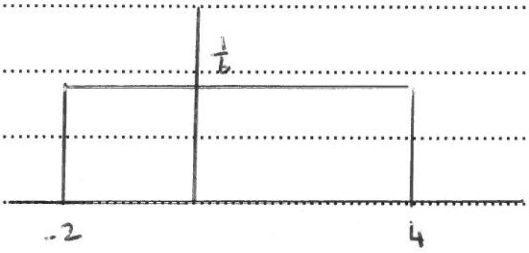
QUESTION PART REFERENCE	Answer space for question 2

ii)



$P(X < 0) = 2 \times \frac{1}{6} = \frac{1}{3}$

use binomial, with $P(\text{success}) = \frac{1}{3}$

$\rightarrow P(S = 1) = {}^4C_1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{2}{3}\right)^3$

$= 32/81$ or $0.39506\ldots$

Do not write outside the box

3 A machine fills bags with frozen peas. Measurements taken over several weeks have shown that the standard deviation of the weights of the filled bags of peas has been 2.2 grams.

Following maintenance on the machine, a quality control inspector selected 8 bags of peas. The weights, in grams, of the bags were

910.4 908.7 907.2 913.2 905.6 911.1 909.5 907.9

It may be assumed that the bags constitute a random sample from a normal distribution.

(a) Giving the limits to **four** significant figures, calculate a 95% confidence interval for the mean weight of a bag of frozen peas filled by the machine following the maintenance:

(i) assuming that the standard deviation of the weights of the bags of peas is known to be 2.2 grams;

[4 marks]

(ii) assuming that the standard deviation of the weights of the bags of peas may no longer be 2.2 grams.

[4 marks]

(b) The weight printed on the bags of peas is 907 grams. One of the inspector's concerns is that bags should not be underweight.

Make **two** comments about this concern with regard to the data and your calculated confidence intervals.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

a) i) From calc: $\bar{x} = 909.2$

From question: $\sigma = 2.2$, $n = 8$

we know σ, so use Z

95% CI multiplier $= \times 1.96$

$\rightarrow 909.2 \pm 1.96 \times \frac{2.2}{\sqrt{8}}$

$= (907.7, 910.7)$

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QUESTION PART REFERENCE	Answer space for question 3

ii) We no longer know σ, so need "s" from calculator

$\rightarrow s = 2.39165$

As we don't know σ and $n = 8$, we need to use t

$v = 8 - 1 = 7$

t_7 95% multiplier $= 2.365$

$\rightarrow 909.2 \pm 2.365 \times \frac{2.39165...}{\sqrt{8}}$

$= (907.2, 911.2)$

b) $\boxed{1}$ Both confidence intervals are above 907, so the claim about the AVERAGE weight is probably valid

$\boxed{2}$ One of the weights in the sample (905.6) is underweight, so no guarantee all weights are greater than 907

4 Wellgrove village has a main road running through it that has a 40 mph speed limit. The villagers were concerned that many vehicles travelled too fast through the village, and so they set up a device for measuring the speed of vehicles on this main road. This device indicated that the mean speed of vehicles travelling through Wellgrove was 44.1 mph. Ho

In an attempt to reduce the mean speed of vehicles travelling through Wellgrove, life-size photographs of a police officer were erected next to the road on the approaches to the village. The speed, X mph, of a sample of 100 vehicles was then measured and the following data obtained.

$$\sum x = 4327.0 \qquad \sum(x-\bar{x})^2 = 925.71$$

(a) State an assumption that must be made about the sample in order to carry out a hypothesis test to investigate whether the desired reduction in mean speed had occurred.

[1 mark]

(b) Given that the assumption that you stated in part **(a)** is valid, carry out such a test, using the 1% level of significance.

[8 marks]

(c) Explain, in the context of this question, the meaning of:

(i) a Type I error;

(ii) a Type II error.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 4**
a)	The sample of vehicles chosen must be RANDOM
b)	$H_0: \mu = 44.1$
	$H_1: \mu < 44.1$ (1TT)
	$\bar{x} = \dfrac{4327}{100} = 43.27$
	$s^2 = \dfrac{925.71}{99} = 9.3506...$
	$\rightarrow s = 3.0578...$

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QUESTION PART REFERENCE	Answer space for question 4

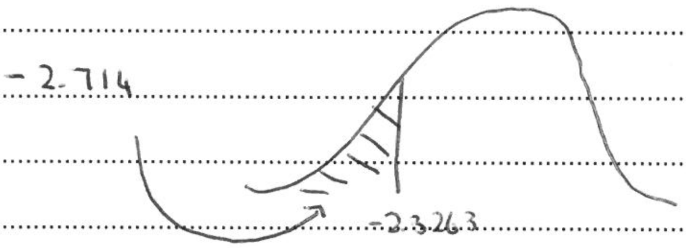
$n > 30 \rightarrow$ use Z because of Central Limit Theorem

$\boxed{\text{TEST STATISTIC}}$

$$Z = \frac{43.27 - 44.1}{3.0578/\sqrt{100}} = -2.714...$$

$\boxed{\text{CRITICAL VALUE}}$

1%, 1 tail ~~tada~~ test, Z $\rightarrow$ -2.3263



$-2.714 < -2.3263$

$\rightarrow$ Reject H_0

There is evidence at 1% significant level that the mean speed has reduced

c) i) $\boxed{\text{TYPE 1}}$ Concluding the mean speed has reduced when it has not (Reject H_0)

$\boxed{\text{TYPE 2}}$ Accepting mean speed is still 44.1 ~~km~~ mph when it has in fact reduced (accept H_0)

Turn over ▶

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5 In a particular town, a survey was conducted on a sample of 200 residents aged 41 years to 50 years. The survey questioned these residents to discover the age at which they had left full-time education and the greatest rate of income tax that they were paying at the time of the survey.

The summarised data obtained from the survey are shown in the table.

Greatest rate of income tax paid	Age when leaving education (years)			
	16 or less	17 or 18	19 or more	Total
Zero	32	3	4	39
Basic	102	12	17	131
Higher	17	5	8	30
Total	151	20	29	200

(a) Use a χ^2-test, at the 5% level of significance, to investigate whether there is an association between age when leaving education and greatest rate of income tax paid.

[9 marks]

(b) It is believed that residents of this town who had left education at a later age were more likely to be paying the higher rate of income tax. Comment on this belief.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 5

a) H_0: No association between age at which they left education and the rate of income tax they pay (Independent)

H_1: There is an association (Non-Independent)

Expected

	<16	17 or 18	≥19
Z	29.445	3.9	5.655
B	98.905	13.1	18.995
H	22.65	3	4.35

QUESTION PART REFERENCE	Answer space for question 5

3 values < 5 $\rightarrow$ combine '17 or 18' and ≥ 19 into "≥ 17"

New Expected

	<16	≥ 17
Z	29.445	9.555
B	98.905	32.095
H	22.65	7.35

CHI - SQUARED

$$\frac{\sum (O-E)^2}{E}$$

	<16	≥ 17
Z	0.221	0.6832
B	0.0968	0.2984
H	1.4093	4.3431

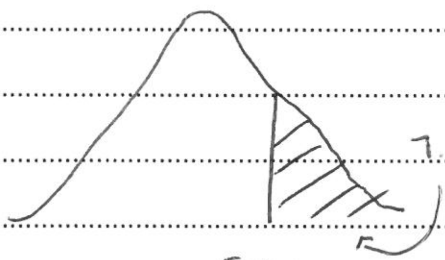
$\rightarrow$ Test Statistic $= 7.05$

$\boxed{\text{CRITICAL VALUE}}$

$v = (3-1)(2-1) = 2$

5%

$\rightarrow$ 5.991



$7.05 > 5.991$

$\rightarrow$ Reject H_0

7.05

Evidence at 5% level that there IS AN ASSOCIATION between age at leaving education & tax paid

5.991

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QUESTION PART REFERENCE

Answer space for question 5

b) Need to refer to data:

From Observed: 8 students aged 19 or more paid higher tax

From Expected: that number was 4.35

$\therefore$ The belief is supported

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6 The continuous random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x - \frac{1}{16}x^2 & 0 \leqslant x \leqslant 4 \\ 1 & x > 4 \end{cases}$$

(a) Find the probability that X lies between 0.4 and 0.8 .

[2 marks]

(b) Show that the probability density function, $f(x)$, of X is given by

$$f(x) = \begin{cases} \frac{1}{2} - \frac{1}{8}x & 0 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

[1 mark]

(c) (i) Find the value of $E(X)$.

[3 marks]

(ii) Show that $\mathrm{Var}(X) = \frac{8}{9}$.

[4 marks]

(d) The continuous random variable Y is defined by

$$Y = 3X - 2$$

Find the values of $E(Y)$ and $\mathrm{Var}(Y)$.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 6**
6	a) $P(0.4 < X < 0.8)$
	$= F(0.8) - F(0.4)$
	$= \left[\frac{0.8}{2} - \frac{0.8^2}{16}\right] - \left[\frac{0.4}{2} - \frac{0.4^2}{16}\right]$
	$= 0.36 - 0.19 = 0.17$
	b) $\frac{dF(x)}{dx} = f(x) \rightarrow \frac{1}{2} - \frac{2}{16}x$
	$= \frac{1}{2} - \frac{1}{8}x$

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QUESTION PART REFERENCE

Answer space for question 6

c) i) $E(X) = \int_0^4 x f(x)\, dx$

$= \int_0^4 \frac{1}{2}x - \frac{1}{8}x^2\, dx$

$= \left[\frac{1}{4}x^2 - \frac{1}{24}x^3\right]_0^4$

$= \frac{1}{4}(4^2) - \frac{1}{24}(4^3) - 0$

$= 4 - \frac{8}{3} = \frac{4}{3}$

ii) $Var(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \int_0^4 x^2 f(x)\, dx$

$= \int_0^4 \frac{1}{2}x^2 - \frac{1}{8}x^3\, dx$

$= \left[\frac{1}{6}x^3 - \frac{1}{32}x^4\right]_0^4$

$= \frac{1}{6}(4^3) - \frac{1}{32}(4^4) - 0$

$= 32/3 - 8 = 8/3$

$\rightarrow Var(X) = 8/3 - \left[\frac{4}{3}\right]^2 = \frac{8}{9}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

d) $Y = 3X - 2$

$E(Y) = 3E(X) - 2$

$= 3\left(\frac{4}{3}\right) - 2 = 2$

$Var(Y) = 3^2 Var(X)$

$= 9\left(8/9\right) = 8$

Do not write outside the box

7 Each week, a newsagent stocks 5 copies of the magazine *Statistics Weekly*. A regular customer always buys **one** copy. The demand for **additional** copies may be modelled by a Poisson distribution with mean 2.

The number of copies sold in a week, X, has the probability distribution shown in the table, where probabilities are stated correct to three decimal places.

x	1	2	3	4	5
$\mathrm{P}(X=x)$	0.135	0.271	0.271	a	b

(a) Show that, correct to three decimal places, the values of a and b are 0.180 and 0.143 respectively.

[3 marks]

(b) Find the values of $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$, showing the calculations needed to obtain these values, and hence calculate the standard deviation of X.

[5 marks]

(c) The newsagent makes a profit of £1 on each copy of *Statistics Weekly* that is sold and loses 50 p on each copy that is not sold. Find the mean weekly profit for the newsagent from sales of this magazine.

[2 marks]

(d) Assuming that the weekly demand remains the same, show that the mean weekly profit from sales of *Statistics Weekly* will be greater if the newsagent stocks only 4 copies.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7

7 a) $\cancel{X}$ $A \sim Po(2)$ A = Additional copies

So, for $x = 4$, we need $A = 3$

$P(A = 3) = \dfrac{e^{-2} \times 2^3}{3!} = 0.1804...$

$= 0.180$ (3dp)

~~For x = 5, we need A = 4~~

$\cancel{P(A = 4) = \dfrac{e^{-2} \times 2^4}{4!}}$

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QUESTION PART REFERENCE	Answer space for question 7

For $x = 5$, need $1 - (0.135 + 0.271 + 0.271 + 0.1804)$
$= 0.143$

b) $E(x) = (1 \times 0.135) + (2 \times 0.271) + (3 \times 0.271)$
$+ (4 \times 0.180) + (5 \times 0.143)$
$= 2.925$

$E(x^2) = (1^2 \times 0.135) + (2^2 \times 0.271) + (3^2 \times 0.271)$
$+ (4^2 \times 0.180) + (5^2 \times 0.143)$
$= 10.113$

$VAR(x) = 10.113 - 2.925^2$

$\Rightarrow SD(x) = \sqrt{10.113 - 2.925^2}$
$= 1.25$

c)

x	1	2	3	4	5
PROFIT	-1	0.5	2	3.5	5
PROB	0.135	0.271	0.271	0.180	0.143

Mean profit $= E(\text{profit}) = (-1 \times 0.135) + (0.5 \times 0.271)$
$+ (2 \times 0.271) + (3.5 \times 0.180)$
$+ (5 \times 0.143) = £1.89$

Turn over ▶

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QUESTION PART REFERENCE

Answer space for question 7

d) New distribution:

x	1	2	3	4
Profit	-0.5	1	2.5	4
Prob	0.135	0.271	0.271	0.323

$E(\text{Profit}) = (-0.5 \times 0.135) + (1 \times 0.271)$
$+ (3 \times 0.271) + (4 \times 0.323)$
$= £2.17$

$£2.17 > £1.89$, so profit is greater.