

Stats 2 - June 2014

① a) From calc: $\sum x = 1904$

$$\bar{x} = 380.8 \quad s = 4.3817 \dots \quad n = 5$$

Don't know σ and $n < 30$, so need t

$$v = 5 - 1 = 4$$

$$90\% \text{ } t \text{ value, } v = 4 \text{ (2 tailed)} = 2.132$$

$$\mu = \bar{x} \pm t \times s/\sqrt{n}$$

$$\mu = 380.8 \pm 2.132 \times 4.3817/\sqrt{5}$$

$$\mu = 380.8 \pm 4.182 \dots$$

$$\mu = (377, 385)$$

b) 90% \rightarrow 10% outside $\rightarrow 3/30 = 3$

② a) Observed

	Eng	Scot	Wales	N.I.	
Male	57	44	27	17	145
Female	39	43	19	4	105
	96	87	46	21	250

b) Expected

	Eng	Scot	W	N.I.
Male	55.68	50.46	26.68	12.18
Female	40.32	36.54	19.32	8.82

χ^2 VALUES

	Eng	Scot	W	N.I.
Male	0.03124	0.82102	0.00383	1.90742
Female	0.04321	1.14207	0.00530	2.63405

Test Statistic

$$= \sum \chi^2$$

$$= 6.59$$

Critical value, $v = (4-1)(2-1) = 3$, 90% level

$$= 6.251$$

H_0 : No association between country & gender

H_1 : Association between country & gender



$$6.59 > 6.251$$

\therefore Reject H_0

There is significant evidence at 10% level of an association between country and gender of recruits.

c) More female recruits than expected recruited from Scotland (43 compared to 36.54)

③ a) $P(X \leq 4) = 0.2 + 0.1 = 0.3$

$\therefore P(\text{both } X \leq 4) = 0.3^2 = 0.09$

b) i) $0.1 + 0.2 + a + 0.3 + b = 1$

$\rightarrow a + b + 0.6 = 1$

$\rightarrow a + b = 0.4$ ①

Using $E(X)$: $3(0.1) + 4(0.2) + 5a + 6(0.3) + 7b = 5.1$

$0.3 + 0.8 + 5a + 1.8 + 7b = 5.1$

$\rightarrow 5a + 7b = 2.2$ ②

① $\times 5 \rightarrow 5a + 5b = 2$

② $\rightarrow 5a + 7b = 2.2$

② - ① $\rightarrow 2b = 0.2 \rightarrow \boxed{b = 0.1}$

① $5a + 5(0.1) = 2$

$\rightarrow 5a = 1.5 \rightarrow \boxed{a = 0.3}$

ii) $E(X^2) = 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.3 + 6^2 \times 0.3 + 7^2 \times 0.1$
 $= 27.3$

$\rightarrow \text{Var}(X) = 27.3 - (5.1)^2$

$= 1.29$ QED $\ddot{\smile}$

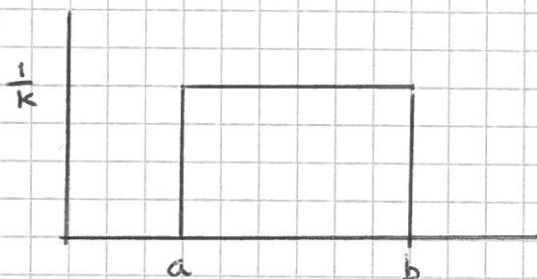
$$\text{iii) } N = 2X - 5$$

$$\begin{aligned} E(N) &= 2(E(X)) - 5 \\ &= 2 \times 5.1 - 5 = 5.2 \end{aligned}$$

$$\begin{aligned} \text{Var}(N) &= 2^2 \text{Var}(X) \\ &= 4 \times 1.29 = 5.16 \end{aligned}$$

$$\rightarrow \text{sd}(N) = \sqrt{5.16} = 2.271\dots$$

④ a) i)



$$\text{Area} = 1$$

$$\rightarrow \text{abw. (V.A.) norm}$$

$$(b-a)(1/K) = 1$$

$$\rightarrow b-a = K$$

$$\text{ii) } E(X) = \frac{1}{2}(a+b)$$

$$\begin{aligned} \text{iii) } E(X^2) &= \int_a^b x^2 \varphi(x) dx \\ &= \int_a^b x^2 / K dx = \frac{1}{K} \int_a^b x^2 dx \\ &= \frac{1}{K} \left[\frac{x^3}{3} \right]_a^b \\ &= \frac{1}{K} \left[\frac{b^3}{3} - \frac{a^3}{3} \right] \end{aligned}$$

$$\text{we know } K = (b-a)$$

$$\begin{aligned} \rightarrow & \left(\frac{1}{b-a} \right) \left(\frac{b^3}{3} - \frac{a^3}{3} \right) = \left(\frac{1}{b-a} \right) \left(\frac{b^3 - a^3}{3} \right) \\ &= \frac{b^3 - a^3}{3(b-a)} = \frac{(b-a)(b^2 + ab + a^2)}{3(b-a)} \\ &= \frac{1}{3}(b^2 + ab + a^2) \end{aligned}$$

$$\begin{aligned} \text{iv) } \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= \frac{1}{3}(b^2 + ab + a^2) - \left[\frac{1}{2}(a+b) \right]^2 \\ &= \frac{1}{3}b^2 + \frac{1}{3}ab + \frac{1}{3}a^2 - \frac{1}{4}(a+b)^2 \\ &= \frac{4}{12}b^2 + \frac{4}{12}ab + \frac{4}{12}a^2 - \frac{3}{12}a^2 - \frac{6}{12}ab - \frac{3}{12}b^2 \\ &= \frac{1}{12}b^2 - \frac{2}{12}ab + \frac{1}{12}a^2 \end{aligned}$$

$$= \frac{1}{2}(b^2 - 2ab + a^2)$$

$$= \frac{1}{2}(b-a)^2$$

$$b) \quad \frac{1}{2}(b-4)^2 = 3$$

$$(b-4)^2 = 36$$

$$b-4 = \pm 6$$

$$\rightarrow b = 10 \text{ or } -2$$

b must be 10 as $b > 0$

$$\rightarrow E(X) = \frac{1}{2}(4+10) = 7$$

$$\textcircled{5} \text{ a) } \mu = \frac{\sum x_i}{n} = \frac{128}{40} = 3.2 \text{ as required per } \lambda$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{126.4}{39} = 3.2410$$

this is close to λ

\therefore Evidence supports Peter's belief

$$b) \quad X \sim \text{Po}(3.2)$$

$$\text{i) } P(X > 5) = 1 - P(X \leq 5) \quad (\text{tables})$$

$$= 1 - 0.8946 = 0.1054$$

$$\text{ii) } P(3 \leq X < 8)$$

can be: 3, 4, 5, 6, 7

$$\rightarrow P(X \leq 7) - P(X \leq 2)$$

$$= 0.9832 - 0.3799 = 0.6033$$

$$\text{iii) } P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - 0.0408 = 0.9592$$

$$P(\text{both have } \emptyset) = 0.0408^2$$

$$P(1 \text{ has } \emptyset \text{ and other has mail}) = 2 \times 0.0408 \times 0.9592$$

$$\text{TOTAL} = 0.0408^2 + 2 \times 0.0408 \times 0.9592$$

$$= 0.0799$$

$$c) T \sim P_0(5 + 3.2)$$

$$\rightarrow T \sim P_0(8.2)$$

$$P(T=9) = e^{-8.2} \times \frac{8.2^9}{9!} = 0.12686\dots$$

$$P(T=10) = e^{-8.2} \times \frac{8.2^{10}}{10!} = 0.10403\dots$$

$$\begin{aligned} \text{TOTAL} &= 0.12686\dots + 0.10403\dots \\ &= 0.23089\dots \end{aligned}$$

$$(b) a) H_0: \mu = 20$$

$$H_1: \mu \neq 20 \quad (\text{2 tailed test})$$

$$\cdot \text{From calc: } \bar{x} = 22.625, \quad s = 4.5650066\dots$$

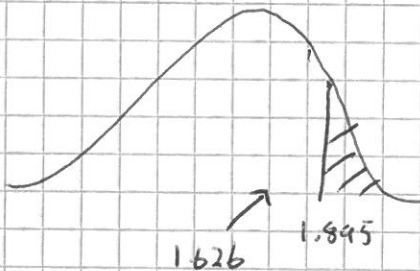
Need t distribution as $n < 30$ & don't know σ

$$\text{TEST STATISTIC} = \frac{22.625 - 20}{\frac{4.565\dots}{\sqrt{8}}} = 1.626\dots$$

$$\text{CRITICAL VALUE} \quad v = n - 1 = 8 - 1 = 7$$

2 tailed, 10% sig level

$$t_7 (10\%) = \pm 1.895$$

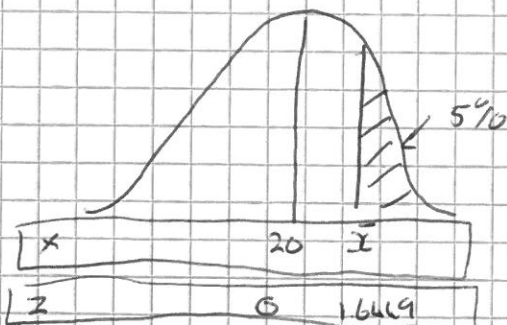


$$1.626 < 1.895$$

\therefore Accept H_0

There is not enough evidence at the 10% significance level to support the belief that Gary does not take an average of 20 mins

b) $n > 30 \rightarrow$ use Z distribution



1 tailed as testing if it is longer

$$Z \text{ value for } 5\% = 1.6449$$

$$\text{Standardize: } \frac{\bar{x} - 20}{\frac{4.6}{\sqrt{100}}} = 1.6449$$

$$\rightarrow \bar{x} - 20 = 1.6449 \times \frac{4.6}{10}$$

$$\begin{aligned}\rightarrow \bar{x} &= 1.6449 \times 4.6/10 + 20 \\ &= 20.75665\dots\end{aligned}$$

\therefore to ~~supp~~ not support Rajul's suspicion

$$\bar{x} \leq 20.75$$

$$\begin{aligned}\textcircled{7} \text{ a) } P(X < 1) &= \int_0^1 \frac{4}{5} x \\ &= \frac{4}{5} \left[\frac{x^2}{2} \right]_0^1 \\ &= \frac{4}{5} \left[\frac{1}{2} - 0 \right] = \frac{2}{5}\end{aligned}$$

$$\begin{aligned}\text{b) i) Need } &\int_1^x \frac{1}{20}(x-3)(3x-11) \\ &= \frac{1}{20} \int_1^x (3x^2 - 20x + 33) dx \\ &= \frac{1}{20} \left[x^3 - 10x^2 + 33x \right]_1^x \\ &= \frac{1}{20} \left[x^3 - 10x^2 + 33x - 1 + 10 - 33 \right] \\ &= \frac{1}{20} \left[x^3 - 10x^2 + 33x - 24 \right]\end{aligned}$$

Now need $F(1) = \frac{2}{5}$

$$\begin{aligned}\therefore F(x) &= \frac{1}{20}(x^3 - 10x^2 + 33x - 24) + \frac{2}{5} \\ &= \frac{1}{20}(x^3 - 10x^2 + 33x - 24) + \frac{1}{20}(8) \\ &= \frac{1}{20}(x^3 - 10x^2 + 33x - 24 + 8) \\ &= \frac{1}{20}(x^3 - 10x^2 + 33x - 16) \quad \text{QED}\end{aligned}$$

ii) For median, $F(x) = 0.5$

$$\begin{aligned}F(1.13) &= \frac{1}{20}(1.13^3 - 10(1.13)^2 - 33(1.13) - 16) \\ &= 0.49819\dots\end{aligned}$$

$$\begin{aligned}F(1.14) &= \frac{1}{20}(1.14^3 - 10(1.14)^2 - 33(1.14) - 16) \\ &= 0.50527\dots\end{aligned}$$

$\therefore 1.13 < \text{Median } 1.14$