

Stats 2 - June 2013

① a) $\bar{x} = \frac{8532}{9} = 948$

$$s^2 = \frac{38538}{8} = 4817.25$$

$$v = n - 1 = 9 - 1 = 8$$

$$t \text{ value } (8), 2 \text{ tailed}, 98\% = 2.896$$

$$\begin{aligned} \text{Confidence Interval} &= 948 \pm 2.896 \times \sqrt{\frac{4817.25}{9}} \\ &= 948 \pm 67.00 \\ &= (881.0, 1015.0) \end{aligned}$$

b) i) $\mu = \text{midpoint of interval} = (927 + 1063) \times 1/2 = 995$

ii) Because the two confidence intervals overlap, we do not have enough evidence to suggest mean weights have changed.

② a) Expected

	Apply	Not Apply
Letter	35.2	124.8
Phone	8.8	31.2

Yates' Correction

$$\frac{(|O - E| - 0.5)^2}{E}$$

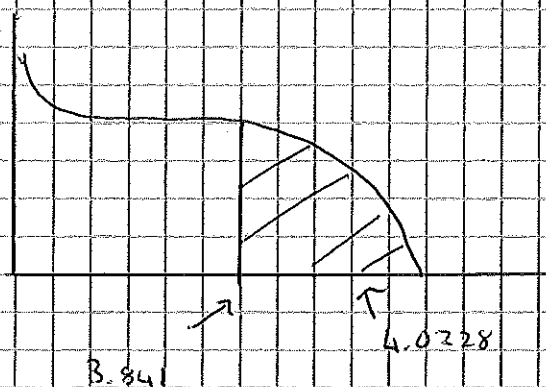
	Apply	Not Apply
Letter	0.6276	0.1770
Phone	2.5102	0.1080

H_0 : No association between how they communicated and the outcome

H_1 : There is an association

$$\text{Test Statistic: } \sum X^2 = 4.0228$$

$$\text{Critical Value: } \chi^2, v=1, 5\% = 3.841$$



$$4.0228 > 3.841$$

\therefore Reject H_0

Evidence suggests that phone calls' effects were higher than expected, which supports the Council's beliefs.

b) Type I error
 H_0 has been rejected when it was true.

③ a) i) Quickest she can get there:

$$2 + 0 + 20 + 5 = 27$$

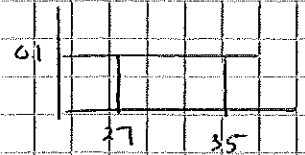
ii) Waits 10 mins per train \rightarrow 37

b) Rectangular Distribution:

$$E(T) = \frac{1}{2}(27 + 37) = 32$$

$$\text{Var}(T) = \frac{1}{12}(37 - 27)^2 = \frac{100}{12} = 8\frac{1}{3}$$

$$\begin{aligned} \text{c) } P(25 \leq T \leq 35) &= P(27 \leq T \leq 35) \\ &= 8 \times 0.1 = 0.8 \end{aligned}$$



$$\text{④ a) i) } P(G=4) \rightarrow \frac{e^{-3.5} \times 3.5^4}{4!} = 0.189$$

ii) Each day $\rightarrow G \sim P_0(0.5)$ $3.5 \div 7$

$$\begin{aligned} P(G \geq 2) &= 1 - P(G \leq 1) \\ &= 1 - 0.9098 = 0.0902 \end{aligned}$$

iii) 28 days $\rightarrow G \sim P_0(14)$ 28×0.5

$$\begin{aligned} P(10 < G < 20) &= P(G \leq 19) - P(G \leq 10) \\ &= 0.9235 - 0.1757 = 0.7478 \end{aligned}$$

b) Explosions should be random.

⑤ a) i)

x	1	2	3	4	5
$P(X=x)$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	k

$$k = 1 - \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = \frac{1}{20}$$

$$\begin{aligned} \text{ii) } E(X) &= 1 \times \frac{1}{3} + 2 \times \frac{1}{4} + 3 \times \frac{1}{5} + 4 \times \frac{1}{6} + 5 \times \frac{1}{20} \\ &= 2.35 \end{aligned}$$

$$\begin{aligned} E(X^2) &= 1^2 \times \frac{1}{3} + 2^2 \times \frac{1}{4} + 3^2 \times \frac{1}{5} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{20} \\ &= 7.05 \end{aligned}$$

$$\text{Var}(X) = 7.05 - (2.35)^2$$

$$= 1.5275$$

$$\text{iv) } P(X \geq 2) = 1/5 + 1/6 + 1/20 = 5/12$$

$$\text{b) } E(Y) = 100 \times E(X) - 50$$

$$= 100 \times 2.35 - 50 = 185$$

$$\text{sd}(Y) = 100 \times \text{sd}(X) = 100 \times \sqrt{1.5275}$$

$$= 5\sqrt{611} = 123.59$$

⑥ a) $H_0: \mu = 175$
 $H_1: \mu < 175$ (1 tailed test)

$$\bar{x} = 168.1$$

$$\sigma = 9.4 \leftarrow \text{we know } \sigma, \text{ so must use } Z$$

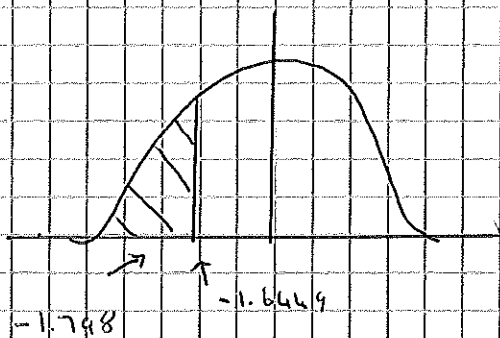
$$\text{Test Statistic} = \frac{168.1 - 175}{\frac{9.4}{\sqrt{6}}} = -1.798$$

$$\text{Critical Value: } Z, 5\%, 1 \text{ tailed test} = -1.6449$$

$$-1.798 < -1.6449$$

\therefore Reject H_0

There is evidence at 5% level that mean weight of the pears has dropped.



$$\text{b) } \bar{x} = 169.4$$

$$n = 20$$

$$H_0: \mu = 175$$

$$H_1: \mu < 175$$

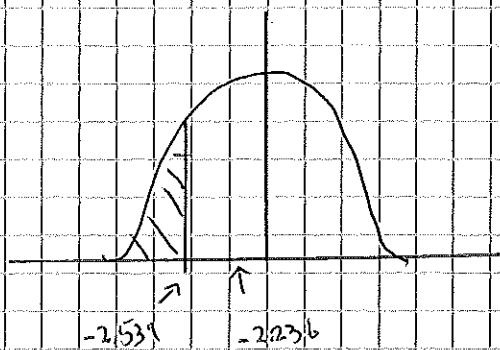
$$s = 11.2$$

$$v = 19$$

\leftarrow must use t as don't know σ

$$\text{Test Statistic} = \frac{169.4 - 175}{\frac{11.2}{\sqrt{20}}} = -2.236$$

$$\text{Critical Value, } t, (19), 1\%, 1 \text{ tailed test} = -2.539$$



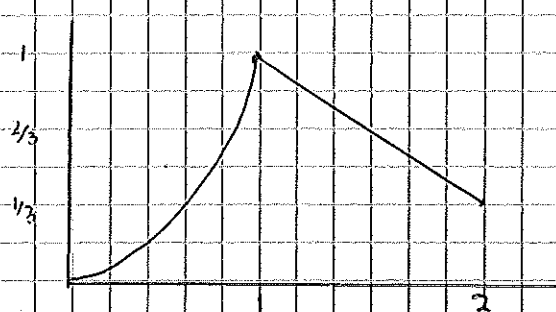
$$-2.236 > -2.539$$

\therefore Accept H_0

Not enough evidence at 1% level to support claim that the mean weight has fallen.

c) Because significance level is smaller.

7) a)



$$b) i) F(x) = \int_0^x f(x) = \int_0^x x^2 = \frac{x^3}{3} \text{ or } \frac{1}{3} x^3$$

$$ii) F(x) = 0.25 \rightarrow \frac{1}{3} x^3 = 0.25$$

$$\rightarrow x^3 = 0.75 \rightarrow x = 0.9085$$

$$c) i) \frac{1}{3} \int_1^x 5 - 2x = \frac{1}{3} [5x - x^2]$$

$$= \frac{1}{3} [(5x - x^2) - (5 - 1)]$$

$$= \frac{1}{3} [5x - x^2 - 4]$$

Need to add on $F(1) = \frac{1}{3} (1)^3 = \frac{1}{3}$

$$\rightarrow \frac{1}{3} [5x - x^2 - 4] + \frac{1}{3}$$

$$= \frac{1}{3} [5x - x^2 - 3]$$

$$ii) F(x) = 0.75 \rightarrow \frac{1}{3} [5x - x^2 - 3] = 0.75$$

$$\rightarrow 5x - x^2 - 3 = 2.25$$

$$\rightarrow x^2 - 5x + 5.25 = 0$$

$$\Rightarrow 4x^2 - 20x + 21 = 0$$

$$(2x - 3)(2x - 7) = 0$$

↓

$$x = 3/2$$

↓

$$x = 7/2$$

UG must be $3/2$ as $7/2$ is out of range