Stats 2 - January 2013

① a) From calc:

$\sum x = 318.36$ $\qquad$ $\bar{x} = 53.06$

$\sum x^2 = 16898.6816$ $\qquad$ $s = 1.140175...$

$n = 6$, $v = 5$ $\qquad$ need t as don't know σ

2 tailed, 95% for t : ± 2.571

$\therefore$ 95% CI : $53.06 \pm 2.571 \times \frac{1.140175}{\sqrt{6}}$

$= 53.06 \pm 1.1965...$

$= (51.86, 54.26)$

b) 1, Sample mean (53.06) is lower than last years (53.41), so claim may be true

2, BUT, 53.41 lies with confidence interval, so no strong evidence that the mean is different.

② a) H_0: no association between property type and time to sell

H_1: there is an association between type and time

Expected

	Flat	T	S-D	D
< 3	8.736	34.944	24.192	16.128
> 3	4.264	17.056	11.808	7.872

Flat < 5, so combine Flat + Terrace

New Expected

	F + T	S-D	D
< 3	43.68	24.192	16.128
> 3	21.32	11.808	7.872

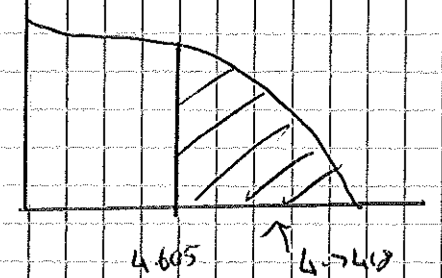
$X^2 \leftarrow \frac{(O-E)^2}{E}$ $\qquad$ $\sum X^2 = 4.7418$

	F + T	S-D	D
< 3	0.7386	0.5946	0.2173
> 3	1.5132	1.2281	0.4452

Test Statistic: $\Sigma X^2 = 4.7418$

$v = (3-2) \times (2-1) = 2$

Critical Value: $10\%\ X^2(2) = 4.605$



$4.7418 > 4.605$

$\therefore$ Reject H_0

Evidence at 10% level suggest association between property type & time to sell

b) i) More terraces than any other so may have big effect

ii) Very far away from their expected values for selling time.

③ a) $WD \sim Po(1.5)$

i) $P(WD = 3) = \dfrac{e^{-1.5} \times 1.5^3}{3!} = 0.126$

ii) $WE \sim Po(0.5)$

Over both days, $Both \sim Po(1)$

$P(Both > 1) = 1 - P(Both \leq 1)$

$= 1 - 0.7358 = 0.264$

iii) $Week \sim Po(8.5)$

$P(Week < 10) = P(Week \leq 9)$

$= 0.653$ (from tables)

b) $Week \sim Po(8.5)$

From tables, $P(Week \leq 15) = 0.9862$

$P(Week \leq 16) = 0.9934$

$\therefore$ He should keep 16 tubes

c) Not consistent use of light (and $\therefore$ failure) over the course of a day

(4) a)

b) $\int_0^3 kx^2\,dx = \left[\frac{kx^3}{3}\right]_0^3$

$= 27k/3 - 0 = 9k$

Area between 3 + 4 $= 1 \times 9k$
$= 9k$

$\therefore$ Total Area $= 9k + 9k = 18k$

Area must $= 1 \rightarrow k = 1/18$

c) i) 1/2 area is to left of 3 $\rightarrow$ median = 3

ii) $\int_0^Q kx^2\,dx = \left[kx^3/3\right]_0^Q = \left[x^3/54\right]_0^Q = 0.25$

$Q^3/54 - 0 = 0.25$

$\rightarrow Q^3 = 13.5 \rightarrow Q = \sqrt[3]{13.5}$

$= 2.3811..$

(5) a) $E(X) = 0 \times 0.1 + 1 \times 0.35 + 2 \times 0.25 + 3 \times 0.2 + 4 \times 0.1$

$= 1.85$ = mean

$E(X^2) = 0^2 \times 0.1 + 1^2 \times 0.35 + 2^2 \times 0.25 + 3^2 \times 0.2 + 4^2 \times 0.1$

$= 4.75$

$\therefore Var(X) = E(X^2) - [E(X)]^2$

$= 4.75 - 1.85^2 = 1.3275$

b) i) $T = c + nX$

ii) $E(T) = c + nE(X)$

$= c + 1.85n$

$Var(T) = n^2\,Var(X)$

$= 1.3275\,n^2$

(6) a) $P(t \leq x) = 0.9 \Rightarrow F(x) = 0.9$

$\rightarrow \frac{t^3}{216} = 0.9$

$\rightarrow t^3 = 194.4$

$\rightarrow t = 5.7929..$ weeks

$5.7929 \times 7 = 40.55 \rightarrow 41$ days

b) Must differentiate $F(t)$

$$\frac{t^3}{216} \rightarrow \frac{3t^2}{216} = \frac{t^2}{72}$$

$$\therefore \quad f(t) = \begin{cases} \frac{t^2}{72} & 0 \le t < 6 \\ 0 & \text{otherwise} \end{cases}$$

c) $$E(T) = \int_0^6 t\left(\frac{t^2}{72}\right) dx = \int_0^t \frac{t^3}{72}\, dx$$

$$= \left[\frac{t^4}{288}\right]_0^6 = \frac{6^4}{288} - 0 = 4.5$$

$$E(T^2) = \int_0^6 t^2\left(\frac{t^2}{72}\right) dx = \int \frac{t^4}{72}\, dx$$

$$= \left[\frac{t^5}{360}\right]_0^6 = \frac{6^5}{360} - 0 = 21.6$$

$$\therefore \ Var(T) = E(T^2) - [E(T)]^2$$

$$= 21.6 - 4.5^2 = 1.35$$

d) $sd(T) = \sqrt{Var(T)} = \sqrt{1.35} = 1.16189\ldots$

$$P(T > (4.5 + 1.16189)) = P(T > 5.662)$$

$$= 1 - P(T < 5.662)$$

$$= 1 - F(5.662)$$

$$= 1 - \frac{5.662^3}{216} = 0.160 \ (3sf)$$

7) a) $H_0: \mu = 20 \leftarrow (3020 - 20)$

$H_1: \mu \neq 20$ (2 tailed test)

$$\bar{y} = \frac{1847}{100} = 18.47$$

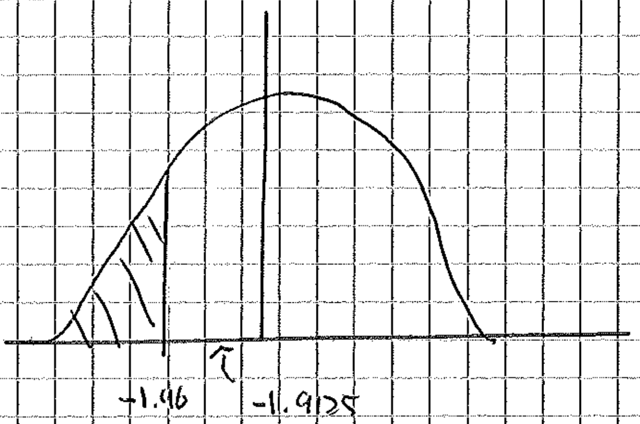
$$s^2 = \frac{6336}{99} = 64$$

$$n = 100$$

As ~~var~~ $n > 30$, we can use Z because of CLT.

Test Statistic: $Z = \dfrac{18.47 - 20}{\sqrt{64/100}} = -1.9125$

Critical Value: 5%, 2 tailed $\Rightarrow$ $Z = \pm 1.96$



$-1.9125 > -1.96$

$\therefore$ Accept H_0

No significant evidence at 5% level that the mean value has changed

b) No error as we accepted H_0!