Stats 2 - June 2012

① a) $\bar{x} = \frac{546}{15} = 36.4$

$s^2 = \frac{1407.6}{14} = 100.5428...$

98%, $v = 14$, two tailed $\Rightarrow$ critical value: $t = \pm 2.624$

$\Rightarrow$ 98% CI: $36.4 \pm 2.624 \times \frac{\sqrt{100.5428...}}{\sqrt{15}}$

$\Rightarrow 36.4 \pm 6.8..$

$= (29.6, 43.2)$

b) 40 lies <u>inside</u> the confidence interval
$\rightarrow$ mean age has <u>not</u> changed

② a) $H_0: \mu = 4$
$H_1: \mu > 4$ (1 tailed)

$\bar{x} = 4.2$

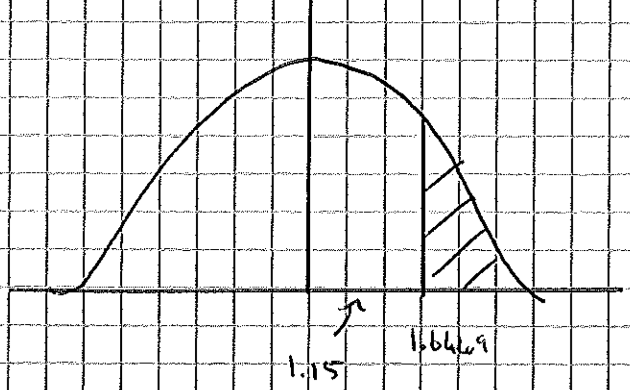
$s = 1.1$

$n = 40$

40 people, so can use
Z dist due to CLT

Test statistic: $z = \frac{4.2 - 4}{1.1/\sqrt{40}} = 1.15$

Critical value: 5%, 1 tailed $\Rightarrow$ 1.6449



~~As~~ $1.15 < 1.6449$

Accept H_0

Not enough evidence at 5% level to support Julian's claims that times > 4 hours

b) Type II Error
$\rightarrow$ Accepted H_0 when H_0 is false.

(3) a) Find $f(x)$ by differentiating $F(x)$

$$F(x) = x/20 + 5/20$$

$$F'(x) = 1/20 = f(x)$$

b) i) $P(X \geq 7) = 1 - P(X \leq 7)$

$$= 1 - F(7)$$

$$= 1 - (7/20 + 5/20) = 8/20$$

ii) $P(X \neq 7) = 1$ [as $P(X = 7) = 0$]

iii) Symmetrical distribution $\rightarrow E(X) = \frac{1}{2}(-5 + 15) = 5$

iv) $E(3X^2)$ Need $\int_{-5}^{15} (3X^2) f(x)\, dx$

$$\rightarrow \int_{-5}^{15} \frac{3x^2}{20}\, dx$$

$$\rightarrow \left[x^3/20 \right]_{-5}^{15} = 15^3/20 - (-5^3/20)$$

$$= 175$$

(4) a)

r	1	2	3	4	5
$P(R=r)$	0.5	0.24	0.144	0.0864	0.0296

b) $P(R < 3) = P(R=1) + P(R=2)$

$\therefore P(R \text{ Not } < 3) = 1 - [P(R=1) + P(R=2)]$

$$= 1 - [0.5 + 0.24] = 0.26$$

c) i) $E(R) = 1 \times 0.5 + 2 \times 0.24 + 3 \times 0.144 + 4 \times 0.0864$

$$+ 5 \times 0.0296 = 1.9056$$

ii) $E(R^2) = 1^2 \times 0.5 + 2^2 \times 0.24 + 3^2 \times 0.144 +$

$$4^2 \times 0.0864 + 5^2 \times 0.0296 = 4.8784$$

$\therefore Var(R) = E(R^2) - [E(R)]^2$

$$= 4.8784 - 1.9056^2 = 1.2470\ldots$$

d) $E(M) = 1250 \times E(R) - 282$

$$= 1250 \times 1.9056 - 282 = 2100$$

$Var(m) = 1250^2 \times Var(R)$

$= 1250^2 \times 1.2470 = 1,948,437.5$

$\therefore SD(m) = \sqrt{1948437.5} = 1395.86...$

⑤ a) $X \sim Po(8.5)$

i) $P(X \geq 9) = 1 - P(X \leq 8)$ (from table!)

$= 1 - 0.5231 = 0.4796$

ii) $P(5 < X < 10) = P(X \leq 9) - P(X \leq 5)$

$= 0.653 - 0.1496 = 0.5034$

b) $Y \sim Po(1.5)$

$P(Y < 2) = P(Y=0) + P(Y=1)$

$= e^{-1.5} \times \frac{1.5^0}{0!} + e^{-1.5} \times \frac{1.5}{1!} = 0.55782...$

c) i) $\lambda = 8.5 + 1.5 = 10$

ii) $P(T > 16) = 1 - P(T \leq 16)$

$= 1 - 0.9730 = 0.027$

iii) Use binomial!

$P(success) = 0.027$

$P(failure) = 1 - 0.027 = 0.973$

$P(2 \text{ years}) \rightarrow {}^3C_2 \times 0.027^2 \times 0.973^1 = 0.002128$

$P(3 \text{ years}) \rightarrow 0.027^3 = 0.00001968$

$\therefore P(\text{Accidents} > 16) = 0.002128 + 0.00001968 = 0.0021$ (4dp)

(6) a) H_0: No association between A Level & degree

H_1: There is an association.

~~Observed~~ Expected

	1	2(i)	2(ii)	3
A	11.6	36.4	28	4
B	17.4	54.6	42	6

Fewer than 5 in (A,3), so need to combine 2ii and 3rd.

New Expected

	1	2(i)	2(ii) & 3
A	11.6	36.4	32
B	17.4	54.6	48

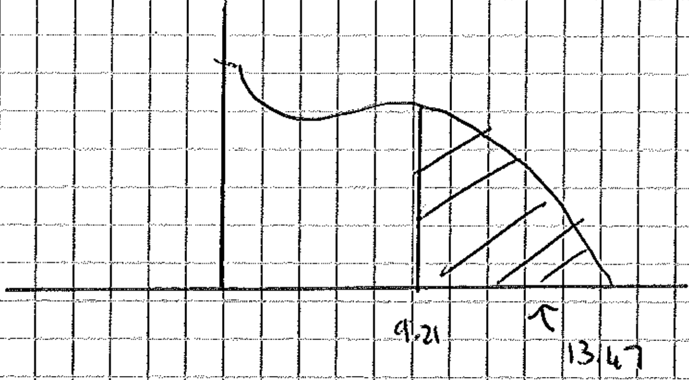
X^2

	1	2(i)	2(ii) & 3
A	6.0827	0.0044	2
B	4.0552	0.0029	1.333

Test Statistic: $\sum X^2 = 13.47$

$v = (3-1) \times (2-1) = 2$

Critical Value: $X^2_{1\%}(2) = 9.210$



$13.47 > 9.210$

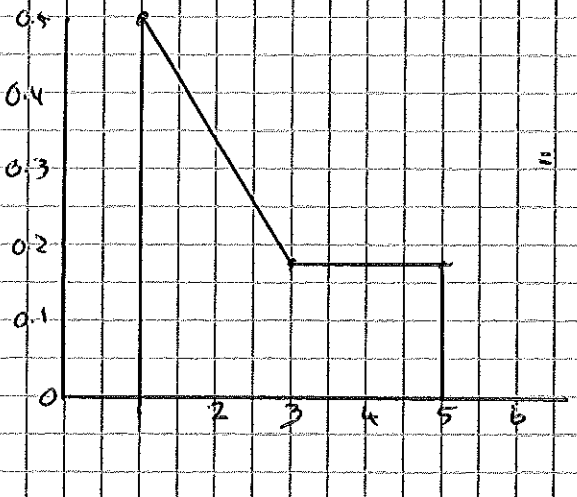
$\therefore$ Reject H_0

Fiona's belief about the association is justified at 1% level

b) Fewer than expected gained a 1st Class Degree (9 compared to 17.4)

More than expected gained a 2:2 degree (48 compared to 42)

(7) a)



b) $E(x) = \int_1^3 x \tfrac{1}{6}(4-x) + \int_3^5 x \tfrac{1}{6}$

$= \tfrac{1}{6}\int_1^3 4x - x^2 + \tfrac{1}{6}\int_3^5 x$

$= \tfrac{1}{6}\left[2x^2 - \tfrac{x^3}{3}\right]_1^3 + \tfrac{1}{6}\left[\tfrac{x^2}{2}\right]_3^5$

$= \tfrac{1}{6}\left[\left(2(3)^2 - \tfrac{3^3}{3}\right) - \left(2(1)^2 - \tfrac{1}{3}\right)\right]$

$+ \tfrac{1}{6}\left[\left(\tfrac{5^2}{2} - \tfrac{3^2}{2}\right)\right]$

$= \frac{1}{6}\left(\frac{22}{3}\right) + \frac{1}{6}(8) = 2\frac{5}{9}$

$p(2.5 < x < 3)$ $p(x > 3)$

Trapezium

c) i) $P(X > 2.5)$ From diagram = +

$= \cancel{\tfrac{1}{2} \times} \left(\frac{0.25 + \frac{1}{6}}{2}\right) \times \frac{1}{2} + 2\left(\frac{1}{6}\right)$

$\frac{5}{48} + \frac{1}{3} = \frac{7}{16}$

ii) $P(1.5 < x < 4.5) = P(1.5 < x < 3) + P(3 < x < 4.5)$

$= \left(\frac{\frac{5}{12} + \frac{1}{6}}{2}\right) \times 1.5 + 1.5\left(\frac{1}{6}\right)$

$= \frac{7}{16} + \frac{1}{4} = \frac{11}{16}$

iii) $p(X > 2.5)$ AND $1.5 < X < 4.5$

$= P(2.5 < X < 4.5)$

$= P(2.5 < X < 3) + P(3 < x < 4.5)$

$= \frac{5}{48} + \frac{1}{4} = \frac{17}{48}$

iv) $P(X > 2.5 / 1.5 < x < 4.5)$

$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{17}{48}}{\frac{11}{16}} = \frac{17}{33}$