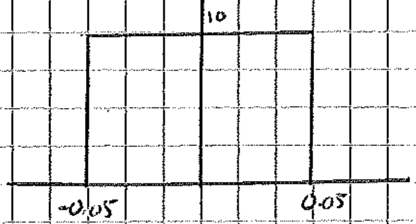
Stats 2 - January 2012

① a) nearest 0.1cm $\rightarrow$ 21.05 to 21.15

b)



b) $E(X) = 0$ (by symmetry)

$Var(X) = \frac{1}{12}(0.05 - (-0.05))^2$

$= \frac{1}{12}(\frac{1}{100}) = \frac{1}{1200}$

$\Rightarrow SD(x) = \sqrt{\frac{1}{1200}} = \frac{1}{20\sqrt{3}}$

c) From diagram : $0.04 \times 10 = 0.4$

② a) i) $H_0: \mu = 61.4$

$H_1: \mu \neq 61.4$

$n = 16$

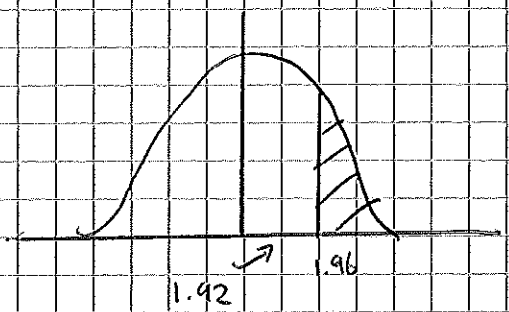
$\bar{x} = 65$

$\sigma = 7.5$

we know σ, so use Z

Test Statistic : $Z = \frac{65 - 61.4}{7.5/\sqrt{16}} = 1.92$

Critical Value: Z, 5%, 2 tailed $\rightarrow$ $Z = \pm 1.96$



$1.92 < 1.96$

$\therefore$ Accept H_0

Not enough evidence at 5% level to suggest mean age has changed

ii) Try 3 SDs from mean $\rightarrow$ $61.4 - 3(7.5) = 38.9$

It's unlikely anyone is < 38.9 years old

$\therefore$ likely to be 0 people < 25 years old

b) i) $n = 12$

$\bar{y} = 702/12 = 58.5$

$s^2 = 88.25/11 = 353/43$

don't know σ and $n < 30$

$\therefore$ use t

$V = 12 - 1 = 11$

t value: $\nu = 11$, 90% CI, 2 tailed = 1.796

$\therefore$ 90% CI $= 58.5 \pm 1.796 \times \dfrac{8.02...}{\sqrt{12}}$

$= 58.5 \pm 1.4685..$

$= (57.03, 59.97)$

$= (57.0, 60.0)$ (1dp)

ii) upper limit of CI $\Rightarrow$ < 61.4 years old $\rightarrow$ $\therefore$ lowered age

(3) a) i) (a) $\dfrac{mp}{N}$ (b) $\dfrac{mq}{N}$ (c) $\dfrac{np}{N}$ (d) $\dfrac{nq}{N}$

ii) $\sum E = \dfrac{mp + mq + np + nq}{N}$

$= \dfrac{m(p+q)}{N} + \dfrac{n(p+q)}{N}$

$p + q = N \rightarrow \dfrac{mN}{N} + \dfrac{nN}{N} = m + n$

$m + n = N \rightarrow\ = N$

b) Expected (*)

	Wind	No wind
W	17.82	15.18
L	9.18	7.82

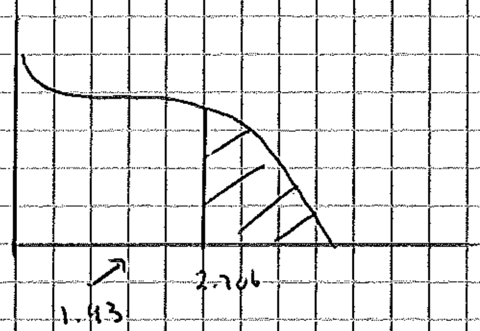
As 2x2, need Yates' Correction:

$X^2 = \dfrac{(|O - E| - 0.5)^2}{E}$

	Wind	No wind
W	0.3020	0.3546
L	0.5863	0.6883

$\sum X^2 = 1.93$ = Test Statistic

Critical Value: $X^2_{(10\%)}$ $(\nu = 1)$ = 2.706



$1.93 < 2.706$

$\therefore$ Accept H_0

No Association between Andy's results and the weather conditions

(*) H_0: No Association between weather & results
H_1: Association between weather & results

(4) a) i) Poisson

ii) $E(3X-1) = 3E(X) - 1 = 3\lambda - 1$

$Var(3X-1) = 9\,Var(X) = 9\lambda$

iii) using formula: $P(X = x+1) = \dfrac{e^{-\lambda} \times \lambda^{x+1}}{(x+1)!}$

$$= \frac{e^{-\lambda} \times \lambda^{x} \times \lambda^{1}}{x!\,(x+1)} = \frac{\lambda}{x+1} \times \frac{e^{-\lambda} \times \lambda^{x}}{x!}$$

$$= \frac{\lambda}{x+1} \times P(X = x)$$

b) i) Total vehicles per hour = $500 + 10 = 510$

$\Rightarrow$ vehicles per min = $510/60 = 8.5$

$\therefore V \sim P(8.5)$

$P(V \geq 10) = 1 - P(V \leq 9)$

$= 1 - 0.6530 = 0.347$

ii) Total vehicles per hour = $836 + 22 = 858$

$\Rightarrow$ vehicles per min = $858/60 = 14.3$

$P(V \leq 3) = P(V=0) + P(V=1) + P(V=2) + P(V=3)$

$$= e^{-14.3}\left[\frac{14.3^0}{0!} + \frac{14.3^1}{1!} + \frac{14.3^2}{2!} + \frac{14.3^3}{3!}\right]$$

$= e^{-14.3} \times 604.912...$

$= 0.000371$ (3sf)

(5) a)

n	1	2	3	4	5
$P(N=n)$	$1/2$	$1/4$	$1/8$	$1/16$	$1/16$

$E(n) = 1 \times 1/2 + 2 \times 1/4 + 3 \times 1/8 + 4 \times 1/16 + 5 \times 1/16$

$= 31/16$

b)

m	1	2	3	4	5
$P(M=m)$	$1/4$	$3/16$	$9/64$	$27/256$	$81/256$
		$\frac{3}{4} \times \frac{1}{4}$	$\left(\frac{3}{4}\right)^2 \times \frac{1}{4}$	$\left(\frac{3}{4}\right)^3 \times \frac{1}{4}$	$\left(\frac{3}{4}\right)^4 \times \frac{1}{4}$

c) i) Both 1 $\rightarrow$ $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Both 2 $\rightarrow$ $\frac{1}{4} \times \frac{3}{16} = \frac{3}{64}$

Both 3 $\rightarrow$ $\frac{1}{8} \times \frac{9}{64} = \frac{9}{512}$

Both 4 $\rightarrow$ $\frac{1}{16} \times \frac{27}{256} = \frac{27}{4096}$

Both 5 $\rightarrow$ $\frac{1}{16} \times \frac{81}{256} = \frac{81}{4096}$

P(equal tosses) = sum = $\underline{\frac{221}{1024}}$

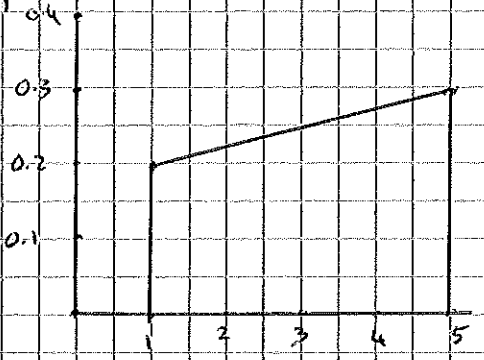
ii) Use Binomial, prob success = $\frac{221}{1024}$

$P(X \geq 2) = P(X=2) + P(X=3)$

$= {}^3C_2 \times \left(\frac{221}{1024}\right)^2 \times \left(\frac{803}{1024}\right) + {}^3C_3 \times \left(\frac{221}{1024}\right)^3$

$= 0.120$ (3dp)

(6) a)



b) $E(X) = \int x f(x)$

$= \int_1^5 x\left[\frac{1}{40}(x+7)\right]$

$= \frac{1}{40}\int_1^5 x^2 + 7x$

$= \frac{1}{40}\left[\frac{x^3}{3} + \frac{7x^2}{2}\right]_1^5$

$= \frac{1}{40}\left[\left(\frac{5^3}{3} + \frac{7(5^2)}{2}\right) - \left(\frac{1}{3} + \frac{7}{2}\right)\right]$

$= 3\frac{2}{15}$ or $\frac{47}{15}$

c) $F(x) = \int_1^x f(x)\,dx$

$= \frac{1}{40}\int_1^x x + 7\,dx$

$= \frac{1}{40}\left[\frac{x^2}{2} + 7x\right]_1^x$

$= \frac{1}{40}\left[\frac{x^2}{2} + 7x - \frac{1}{2} - 7\right]$

$= \frac{1}{80}\left[x^2 + 14x - 15\right]$

$= \frac{1}{80}(x+15)(x-1)$

d) i) $P(2.5 \le X \le 4.5) = F(4.5) - F(2.5)$

$= \frac{1}{80}(4.5+15)(4.5-1) - \frac{1}{80}(2.5+15)(2.5-1)$

$= \frac{21}{40}$

ii) $F(m) = 0.5$

$\rightarrow \frac{1}{80}(m+15)(m-1) = 0.5$

$\rightarrow (m+15)(m-1) = 40$

$\rightarrow m^2 + 14m - 15 = 40$

$\rightarrow m^2 + 14m - 55 = 0$

e) Use quadratic formula:

$$m = \frac{-14 \pm \sqrt{14^2 - 4 \times 1 \times (-55)}}{2}$$

$$= \frac{-14 \pm \sqrt{416}}{2}$$

$$= \frac{-14 \pm 20.396}{2}$$

As $m > 1 \rightarrow \frac{-14 + 20.396}{2} = 3.198$ (3dp)