

## Stats 2 - June 2011

① a) i)  $2.6 \times 5 = 13 \rightarrow X \sim P_0(13)$

ii)  $P(X=20) = \frac{e^{-13} \times 13^{20}}{20!} = 0.0177$

iii)  $P(6 \leq X \leq 18) = P(X \leq 18) - P(X \leq 5)$   
 $= 0.9302 - 0.0107 = 0.9195$

b) Number of cars passing by B not random  
(cars are not independent of each other)  
Mean likely to change at rush hour

a)

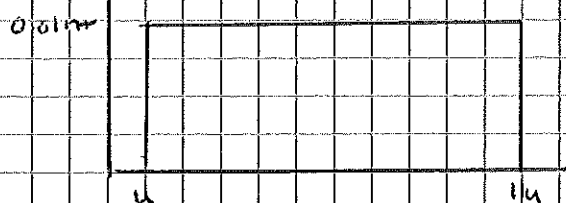
c)  $Y \sim B(20, 0.2)$

$P(Y \geq 5) = 1 - P(Y \leq 4)$  (from table)  
 $= 1 - 0.6296 = 0.3704$

d) Assumption:  $X = Y$  are independent

prob =  $0.0177 \times 0.3704 = 0.0065608$

② a)



i) Area = 1

$\rightarrow 11u \times 0.01\pi r = 1$

$\rightarrow 1/10 u \pi r = 1$

$\rightarrow u^2 = 10/\pi r$

ii)  $E(X) = 1/2(11u + u) = 6u = 6(10/\pi r) = 60/\pi r$

$Var(X) = 1/12(11u - u)^2 = 1/12(100u^2)$   
 $= 100/12(10^2/\pi r^2) = 10,000/12\pi r^2$

iii) ~~width~~  $C = \pi \times d$

length:  $\rightarrow C = \pi(X + \frac{10}{\pi r}) = \pi X + 10$

$\therefore E(C) = \pi E(X) + 10$

$= \pi(60/\pi r) + 10 = 70$

$Var(C) = \pi^2 Var(X)$

$= \pi^2(10,000/12\pi r^2) = 10,000/12 = 833 \frac{1}{3}$

b)  $n = 100$

$\bar{y} = 40.5$

$\sigma^2 = 25$

$n > 100$ , so use  $Z$

95%, 2 tailed  $\Rightarrow Z = 1.96$

$$\begin{aligned} \therefore 95\% \text{ CI} &= 40.5 \pm 1.96 \times \sqrt{\frac{25}{100}} \\ &= 40.5 \pm 0.98 \\ &= (39.52, 41.48) \end{aligned}$$

③ a)  $H_0$ : no association between type of school and performance at GCSE

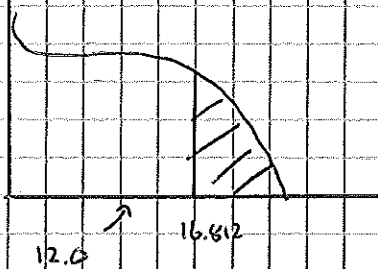
b) Test stat: Need  $\frac{(O - E)^2}{E}$

$\rightarrow$ 0.19581...	0.0623...	0.54185...
0.48216...	1.2694...	0.06601...
0.00356...	0.78549...	3.27419...
1.0805...	0.18380...	4.09642...

$\sum X^2 = 12.0197... = 12.0$  (1dp)

d)  $v = (4 - 1) \times (3 - 1) = 6$

critical value:  $X^2_{(6)} 1\% = 16.812$



$12.0 < 16.812$

$\therefore$  Accept  $H_0$

No significant evidence to support Emily's belief of an association

d) Fewer than expected gained no GCSEs

e)  $X^2_{(6)} 10\%$  critical value = 10.645

$10.645 < 12.0$

$\therefore$  Reject  $H_0$

Evidence to support Emily's belief of an association

4) a)  $X \begin{cases} 1 & 2 & 3 & 4 & 5 \\ P(X=x) & \frac{3}{40} & \frac{6}{40} & \frac{9}{40} & \frac{12}{40} & \frac{5}{20} \end{cases}$

$$E(X) = 1 \times \frac{3}{40} + 2 \times \frac{6}{40} + 3 \times \frac{9}{40} + 4 \times \frac{12}{40} + 5 \times \frac{5}{20} = 3.5$$

b) i)  $E(1/X) = 1 \times \frac{3}{40} + \frac{1}{2} \times \frac{6}{40} + \frac{1}{3} \times \frac{9}{40} + \frac{1}{4} \times \frac{12}{40} + \frac{1}{5} \times \frac{5}{20} = \frac{7}{20}$

ii)  $E(1/X^2) = 1^2 \times \frac{3}{40} + (\frac{1}{2})^2 \times \frac{6}{40} + (\frac{1}{3})^2 \times \frac{9}{40} + (\frac{1}{4})^2 \times \frac{12}{40} + (\frac{1}{5})^2 \times \frac{5}{20} = \frac{133}{800}$

$$\therefore \text{Var}(1/X) = \frac{133}{800} - (\frac{7}{20})^2 = \frac{7}{160}$$

c)

$X$	1	2	3	4	5
$Y$	40	20	40/3	10	8
prob	$\frac{3}{40}$	$\frac{6}{40}$	$\frac{9}{40}$	$\frac{12}{40}$	$\frac{5}{20}$

i)  $P(Y < 20) = P(X = 3, 4, 5) = \frac{9}{40} + \frac{12}{40} + \frac{5}{20} = \frac{31}{40}$

ii)  $P(X < 4 / Y < 20) = \frac{P(X < 4 \text{ AND } Y < 20)}{P(Y < 20)}$

Only happens when  $X=3$   $\Rightarrow \frac{\frac{9}{40}}{\frac{31}{40}} = \frac{9}{31}$

5) a)  $H_0: \mu = 20,000$

$H_1: \mu \neq 20,000$  (2 tailed)

$n = 25$

$\bar{y} = 19,700$

$\sigma = 640$

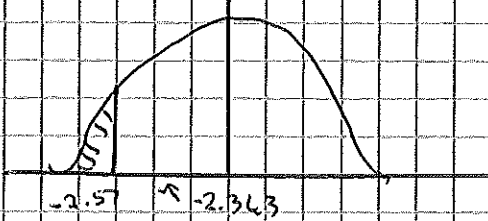
we know  $\sigma$  so can use Z

Test Statistic:  $Z = \frac{19700 - 20000}{\frac{640}{\sqrt{25}}} = -2.34375$

Critical Value: 1%, 2 tailed  $\Rightarrow Z = \pm 2.5758$

$-2.343 > -2.575$

$\therefore$  Accept  $H_0$



Not enough evidence to suggest mean life of lightbulb has changed.

b) i)  $H_1: \mu < 10,000$  (1-tailed)

ii)  $n = 16$

$s = 500$

don't know  $\sigma$ , so must use  $t$

$v = 16 - 1 = 15$

critical value:  $t$ , 5%,  $v = 15$ , 1-tailed

$= 1.753$

would not reject  $H_0$ :  $\bar{x} \pm 1.753 \times \frac{500}{\sqrt{16}}$

$\rightarrow 10,000 \pm 1.753 \times \frac{500}{\sqrt{16}}$

$= 10,000 \pm 219.125$

$= (9,780.875, 10,219.125)$

would not reject if  $\bar{x}$  was  $> 9780$  ( $>$  1-tailed test)

iii) No error as  $H_0$  is  $\mu = 10,000$

6) a)  $F(x) = \int_0^x f(x) dx$   
 $= \int_0^x \frac{3}{8}(x^2 + 1) dx = \frac{3}{8} \int_0^x (x^2 + 1) dx$   
 $= \frac{3}{8} \left[ \frac{x^3}{3} + x \right]_0^x$   
 $= \frac{3}{8} \left[ \left( \frac{x^3}{3} + x \right) - 0 \right] = \frac{1}{8} x^3 + \frac{3}{8} x$   
 $= \frac{1}{8} x (x^2 + 3)$

b)  $F(m) = \frac{1}{2}$

$F(1) = \frac{1}{8}(1)(1^2 + 3) = \frac{1}{8}(4) = \frac{1}{2} \therefore 1 = \text{median}$

c) UQ must lie in  $1 \leq x \leq 2$

Need  $F(\text{UQ}) = 0.75$

$\int_1^a \frac{1}{4}(5 - 2x) dx = \frac{1}{4} \int_1^a (5 - 2x) dx$   
 $= \frac{1}{4} \left[ 5x - x^2 \right]_1^a = \frac{1}{4} \left[ (5a - a^2) - (5 - 1) \right]$   
 $= \frac{1}{4} \left[ 5a - a^2 - 4 \right]$

must add on  $F(1) = \frac{1}{2}$

$$\rightarrow \frac{1}{2} + \frac{1}{4} [5q - q^2 - 4] = 0.75$$

$$\rightarrow \frac{1}{4} [-q^2 + 5q - 4] = 0.25$$

$$\rightarrow -q^2 + 5q - 4 = 1$$

$$\rightarrow q^2 - 5q + 5 = 0$$

Solve using formula:

$$\rightarrow q = \frac{5 \pm \sqrt{5^2 - 4 \times 1 \times 5}}{2}$$

$$= \frac{5 \pm \sqrt{5}}{2}$$

but  $q$  must be between 1 & 2

$$\rightarrow q = \frac{5 - \sqrt{5}}{2} \approx \frac{1}{2} (5 - \sqrt{5})$$

$$d) P(q < X < 1.5)$$

$$= P\left(\frac{1}{2}(5 - \sqrt{5}) < X < 1.5\right)$$

$$\textcircled{\text{~~0.25 < X < 1.5~~}}$$

$$\text{For } P(X < 1.5) \text{ use } F(x) = \frac{1}{2} + \frac{1}{4} [-x^2 + 5x - 4]$$

$$\rightarrow F(1.5) = \frac{1}{2} + \frac{1}{4} [-1.5^2 + 5(1.5) - 4] \\ = \frac{13}{16}$$

$$\text{For } P(X < q) = \frac{3}{4} \quad (\text{as it's u.a.)}$$

$$\therefore P(q < X < 1.5) = \frac{13}{16} - \frac{3}{4} = \frac{1}{16}$$