Stats 2 - January 2011

(1) a) We know σ^2, so use Z

Z value for 95%, 2 tailed = 1.96

$\therefore$ 95% CI = $450 \pm 1.96 \times \sqrt{\frac{4}{16}}$

$= 450 \pm 0.98$

$= (449.02, 450.98)$

b) i) Don't know variance, so must use t

t value, degrees of freedom 8, 90%, 2 tailed = 1.86

$\therefore$ $\bar{x} = \frac{4950}{9} = 550$

$s^2 = \frac{334}{8} = 41.75$

$\therefore$ 90% CI for $\mu = 550 \pm 1.86 \times \sqrt{\frac{41.75}{9}}$

$= 550 \pm 4.0$

$= (546, 554)$

ii) 545 not in confidence interval

$\therefore$ Evidence to reject Molly's claim at the 10% significance level.

(2) a) "Observed"

	C	L	LD	Other	Total
Male	156	144	120	60	480
Female	216	135	108	81	540
Total	372	279	228	141	1020

b) H_0: No association between gender & voting preference

H_1: Association between gender & voting preference

Expected

	C	L	LD	Other
Male	175.06	131.29	107.29	66.35
Female	196.94	147.71	120.71	74.65

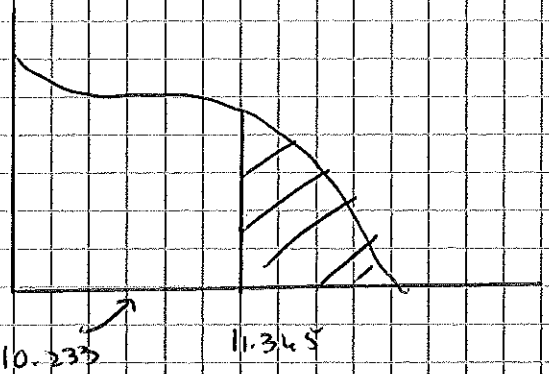
χ^2 values $\quad \frac{(O-E)^2}{E}$

	C	L	LD	Other
Male	2.075	1.230	1.505	0.608
Female	1.844	1.093	1.337	0.541

Test statistic : $\Sigma \chi^2 = 10.233$

Critical value : Degrees of freedom : $(4-1) \times (2-1) = 3$

1% s.g $\rightarrow$ 11.345



$10.233 < 11.345$

$\therefore$ Accept H_0

Evidence at 1% level suggests voter preference is independent of gender.

(3) a) i) $P(X \leq 1) = 0.8781$ (from tables)

ii) $P(Y \geq 1) = 1 - P(Y = 0)$

$= 1 - \frac{e^{-0.15} \times 0.15^0}{0!}$

$= 1 - 0.8607 = 0.1393$

$\therefore P(X \leq 1 \text{ AND } Y \geq 1) = 0.8781 \times 0.1393$

$= 0.122$

b) catches (X) + run-outs (Y) are independent of each other.

c) i) $S_L \sim Po(16 \times 0.6)$

$\rightarrow S_L \sim Po(9.6)$

$P(S_L = 10) = \frac{e^{-9.6} \times 9.6^{10}}{10!} = 0.124$

ii) $S_R \sim Po(16 \times 0.15)$

$\rightarrow S_R \sim P(2.4)$

$\therefore S_T \sim Po(9.6 + 2.4) = Po(12)$

$P(T \geq 15) = 1 - P(T \leq 14)$

$= 1 - 0.7720$ (from tables)

$= 0.228$

(4) a) i) $E(X) = 1 \times 0.2 + 2 \times 0.1 + 3 \times 0.4 + 4 \times 0.3$

$= 2.8$

$E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.1 + 3^2 \times 0.4 + 4^2 \times 0.3$

$= 9$

$\therefore Var(X) = 9 - 2.8^2 = 1.16$

ii) $E(S) = 3E(X) = 3 \times 2.8 = 8.4$

$Var(S)$ INVALID Q - see Examiners Report

b)

T	3	6	9	12
$P(T=t)$	$1/20$	$2/20$	$3/20$	$14/20$

$E(T) = 3 \times 1/20 + 6 \times 2/20 + 9 \times 3/20 + 12 \times 14/20$

$= 10.5$

$E(T^2) = 3^2 \times 1/20 + 6^2 \times 2/20 + 9^2 \times 3/20 + 12^2 \times 14/20$

$= 117$

$\therefore Var(T) = 117 - 10.5^2 = 6.75$

c) i) $P(X > 1) = 1 - 0.2 = 0.8$

ii) $P(X + T \leq 9 \text{ AND } X > 1)$

List ways this could work:

X	T	Prob		
2	3	0.1 × 1/20	=	0.005
2	6	0.1 × 2/20	=	0.010
3	3	0.4 × 1/20	=	0.020
3	6	0.4 × 2/20	=	0.040
4	3	0.3 × 1/20	=	0.015
		Total Prob =		0.09

iii) $P(X+T \leq 9 \mid X > 1) = \frac{P(A \cap B)}{P(B)} = \frac{0.09}{0.80} = 9/80$

(5) a) i) $H_0: \mu = 165$ (H_0)

$H_1: \mu > 165$ (H_1)

ii) $n > 30$, so use Z for both.

$\bar{x} = 161.7$

$s^2 = 101.2$

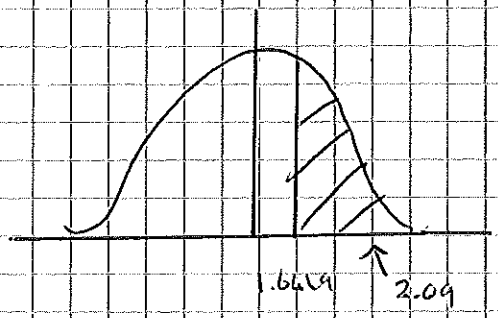
$n = 100$

Test statistic: $Z = \frac{161.7 - 165}{\sqrt{\frac{101.2}{100}}} = 2.09$

DAVID

Critical value, 5%, 1TT

$Z = 1.6449$



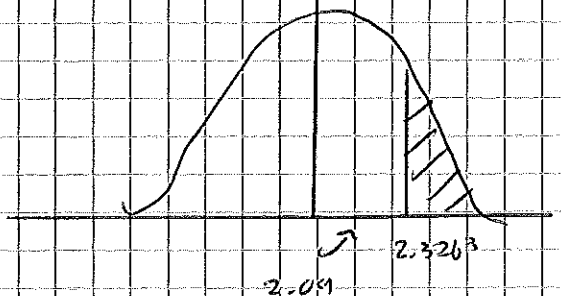
Reject H_0

Evidence at 5% level to suggest mean height has increased

JAMES

Critical value: 1%, 1TT

$Z = 2.3263$



Accept H_0

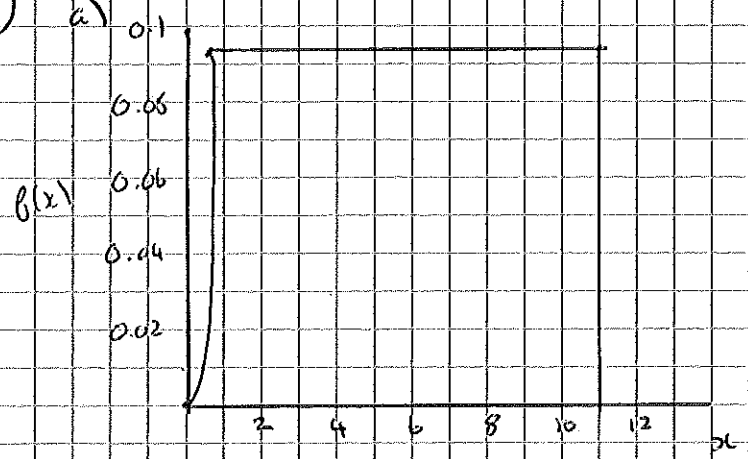
No evidence at 1% level to suggest increase in mean height

(iii) Population distribution not known to be normal

b) i) David, Rejected H_0 when H_0 was correct
= Type I error

ii) James, Accepted H_0 when H_0 correct
= No error.

(6) a)



Height = 3/32 = 0.09375

b) i) $P(X \geq 8\frac{1}{3}) = [11 - 8\frac{1}{3}] \times 3/32 = 1/4$

ii) $P(X \geq 3) = [11 - 3] \times 3/32 = 3/4$

c) i) IQR = UQ − LQ

$= 8\frac{1}{3} - 3 = 5\frac{1}{3}$

ii) Median $= 1/2\,[8\frac{1}{3} + 3] = 5\frac{2}{3}$ (as rect distribution)

d) $P(X < m \mid X \geq 3) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(X < 5\frac{2}{3} \text{ AND } X \geq 3)}{P(X \geq 3)}$

$= \dfrac{3/32 \times [5\frac{2}{3} - 3]}{3/4}$

$= \dfrac{1/4}{3/4} = 1/3$