Stats 2 - June 2010

(1) $H_0: \mu = 79$

$H_1: \mu > 79$ (1 tailed test)

From calculator:

$\Sigma x = 984$ $\bar{x} = 82$

$\Sigma x^2 = 81,030$ $s = 5.5759$

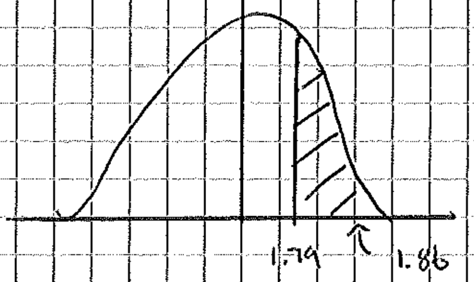
$n = 12 \Rightarrow v = 11$

Don't know σ, and $n < 30$, so use t.

Test statistic: $t = \dfrac{82 - 79}{5.5759/\sqrt{12}} = 1.86$

Assumption: Number of customers served is normally distributed

Critical Value: $t_{(11)}$, 5%, 1 tailed $= 1.796$



$1.86 > 1.796$

$\therefore$ Reject H_0

Evidence at 5% to support belief that number of customers has increased

(2) H_0: No association between Drug & sickness

H_1: Association between Drug & sickness

2 by 2, so need Yates Correction [degrees of freedom = 1]

Expected

	Sick	No sick
Drug	28	52
No Drug	7	13

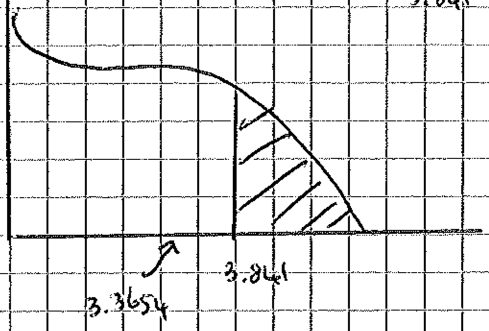
Yates $\dfrac{\left[|O - E| - 0.5\right]^2}{E}$

	Sick	No sick
Drug	0.4375	0.2356
No Drug	1.7500	0.9423

Test statistic: $\Sigma X^2 = 3.3654$

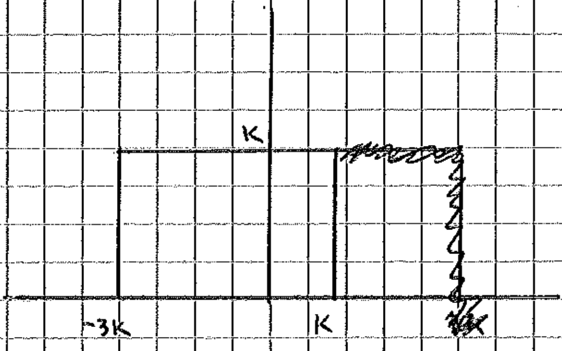
Critical Value: 5%, $v = 1$ $= 3.841$

$3.841 > 3.365 \therefore$ Accept H_0



No evidence at 5% level of an association between drug and prevention of sickness

(3) a) i)



ii) Area must $= 1$

$\therefore 4k(k) = 1$

$\rightarrow 4k^2 = 1$

$\rightarrow k^2 = 1/4$

$\rightarrow k = 1/2$ as $k > 0$

b) Rectangular Distribution

$\therefore E(X) = 1/2\,(-3k + k) = 1/2\,(-2k) = -k = -1/2$

$Var(X) = 1/12\,(k - (-3k))^2$

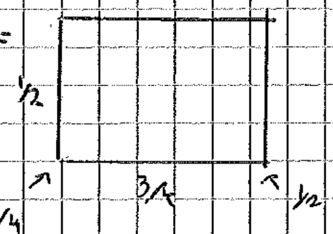
$= 1/12\,(4k)^2 = 1/12\,(16k^2) = 4k^2/3 = 1/3$

$\frac{4(1/2)^2}{3}$

$\rightarrow sd(X) = \sqrt{1/3}$ or $1/\sqrt{3}$

c) i) $P(X \geq -1/4)$

From diagram $=$



$= 1/2 \times 3/4$

$= 3/8$

ii) $P(X \neq -1/4) = 1$

as $P(X = -1/4) = 0$

(4) $\bar{x} = \frac{0.35}{10} = 0.035$

Don't know σ^2 and $n < 30$ so need t

$s^2 = \frac{0.12705}{9} = 0.014116...$

$n = 10, \quad v = 9$

$t_{(9)}$, 99%, 2 tailed $\rightarrow$ t value of 3.250

$$99\% \text{ CI} = 0.035 \pm 3.25 \times \sqrt{\frac{0.0411b}{10}}$$

$$= 0.035 \pm 0.1221$$

$$= (-0.087, 0.157)$$

(5) a) i $X \sim Po(7)$

i) $P(X \leq 5) = 0.301$ (from tables)

ii) $P(X = 7) = \frac{e^{-7} \times 7^7}{7!} = 0.149$

iii) $P(5 \leq X < 10) = P(X \leq 9) - P(X \leq 4)$

$= 0.8305 - 0.1730 = 0.6575$

b) $7/8 = 0.875 \rightarrow Y \sim Po(0.875)$

c) i) 4 engineers $\Rightarrow$ 4 available

ii) $Y \sim Po(0.875)$

$P(Y < 4) = P(Y=0) + P(Y=1) + P(Y=2) + P(Y=3)$

$$= e^{-0.875} \times \left[\frac{0.875^0}{0!} + \frac{0.875^1}{1!} + \frac{0.875^2}{2!} + \frac{0.875^3}{3!} \right]$$

$$= 0.4169 \left[1 + 7/8 + 49/128 + 343/3072 \right]$$

$= 0.987740$ = Probability that at least 1 engineer ~~available~~ has not been called out

$\therefore$ Probability all 4 are Not available $= 1 - 0.987740$
$= 0.0123$

d) λ probably not constant as number of calls per day not likely to be evenly spread out.

(6) a) i) $P(R \geq 5) = 0.3 + 0.25 + 0.1 + 0.05 = 0.7$

ii) $E(R) = 3 \times 0.1 + 4 \times 0.2 + 5 \times 0.3 + 6 \times 0.25 + 7 \times 0.1$

$+ 8 \times 0.05 = 5.2$

iii) $E(R^2) = 3^2 \times 0.1 + 4^2 \times 0.2 + 5^2 \times 0.3 + 6^2 \times 0.25$

$+ 7^2 \times 0.1 + 8^2 \times 0.05 = 28.7$

$\therefore Var(R) = 28.7 - 5.2^2 = 1.66$

b) i) $P(R+S=6) = 0.1 \times 0.15 = 0.015$

$P(R+S=7) = 0.1 \times 0.4 = 0.04$
or $0.2 \times 0.15 = 0.03$ $\Big\}$ 0.07

$P(R+S=8) = 0.1 \times 0.3 = 0.03$
or $0.2 \times 0.4 = 0.08$
or $0.3 \times 0.15 = 0.045$ $\Big\}$ 0.155

$P(R+S \le 8) = P(=6) + P(=7) + P(=8) \rightarrow$ ~~0.0~~ 0.24

ii) $P(\text{sucess}) = 0.24$. $P(T \geq 4) = P(T=4) + P(T=5)$

$P(T=4) \rightarrow p = {}^5C_4 \times 0.24^4 \times 0.76^1 = 0.0126$

$P(T=5) \rightarrow p = {}^5C_5 \times 0.25^5 \times 0.76^0 = 0.000796$

$\therefore P(T \geq 4) = 0.034$

iii) $P(R=4 / R+S \le 8) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(R=4 \text{ and } R+S \le 8)}{P(R+S \le 8)}$

A B

$= \dfrac{0.03 + 0.08}{0.24}$ (4×4)

(4×3)

$= 11/24$

(7) a) Median $= 1$ [as $P(0 \le x \le 1) = 1/2$]

LQ $= 1/2$ [as rect distribution]

b) For $0 \le x \le 1$, $F(1) = 1/2$

For $1 \le x \le 4$ $F(x) = \int f(x)$

need x as upper limit!

$= \int_1^x 1/18 (x-4)^2 \, dx$

$= \left[1/54 (x-4)^3 \right]_1^x$

$= 1/54 (x+4)^3 - 1/54 (-3)^3$

$= 1/54 (x-4)^3 + 1/2$

So $F(x) = 1/2 + 1/54 (x-4)^3 + 1/2 = 1 + 1/54 (x-4)^3$

c) $P(2 \leq X \leq 3) = F(3) - F(2)$

$= 1 + \frac{1}{54}(3-4)^3 - \left[1 + \frac{1}{54}(2-4)^3\right]$

$= \frac{53}{54} - \frac{46}{54} = \frac{7}{54}$

d) i) For UQ, $F(q) = 0.75$

$\rightarrow 1 + \frac{1}{54}(x-4)^3 = 0.75$

$\rightarrow \frac{1}{54}(x-4)^3 = -0.25$

$\rightarrow (x-4)^3 = -13.5 \quad \rightarrow (q-4)^3 = -13.5$

ii) $\rightarrow x - 4 = \sqrt[3]{-13.5}$

$\rightarrow x = \sqrt[3]{-13.5} + 4$

$= 1.619$ (3dp)