

Stats 2 - January 2010

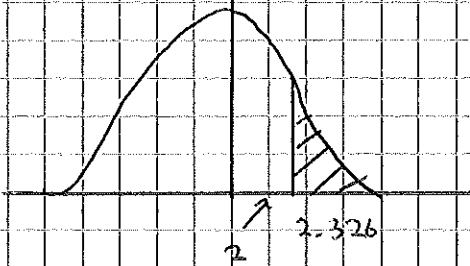
① $H_0: \mu = 45$

$H_1: \mu > 45$ (1 tailed test)

$n = 30$, ^{so use} ~~var known~~ Z

Test Statistic: $Z = \frac{45.8 - 45}{\sqrt{\frac{4.8}{30}}} = 2$

Critical Value, 1%, 1 tailed = 2.3263



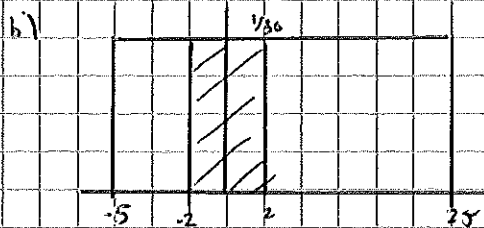
$2 < 2.326$

∴ Accept H_0

Not enough evidence at 1% level to support Roger's claim

② a) i) $E(T) = \frac{1}{2} (25 + (-5)) = 10$

ii) $\text{Var}(T) = \frac{1}{2} (25 - (-5))^2 = 75$



$P(-2 < C < 2) = 4 \times \frac{1}{30} = \frac{4}{30}$

∴ $P(|C| > 2) = 1 - \frac{4}{30} = \frac{13}{15}$

③ $H_0: \mu = 190$

$H_1: \mu \neq 190$ (2 tailed test)

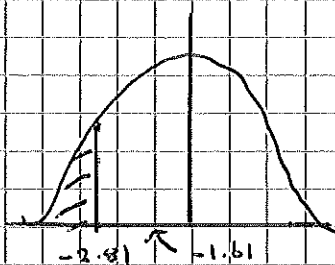
$\sigma^2 = \frac{1840}{10} = 184$

$s^2 = \frac{240}{9} = 26.7$

$v = 10 - 1 = 9$

Test Statistic = $\frac{184 - 190}{\sqrt{\frac{137.7}{10}}} = -1.6164$

Critical Value, $t_{(9)}$ 2%, 2 tailed = ± 2.821



$-2.81 < -1.61$

∴ Accept H_0

Evidence suggests Lorraine is correct that

there has been no significant change.

Assumption: The length of shots are normally distributed.

- (4) a) H_0 : No association between age & performance (Independent)
 H_1 : Association between age & performance (Non-Independent)

Observed

	Pass	Fail	
17-18	28	20	(48)
14-30	2	14	(16)
31-39	12	33	(45)
40-60	6	5	(11)
	(48)	(72)	(120)

Expected

	Pass	Fail
17-18	19.2	28.8
14-30	6.4	9.6
31-39	18	27
40-60	4.4	6.6

$$X^2 = \frac{(O - E)^2}{E}$$

	Pass	Fail
17-18	4.0333	2.6889
14-30	3.0250	2.0167
31-39	0.8643	0.5762
40-60	///	///

As Expected < 5
 need to combine class 2 rows

31-60	22.4	33.6
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Test statistic (Total X^2) = 13.20

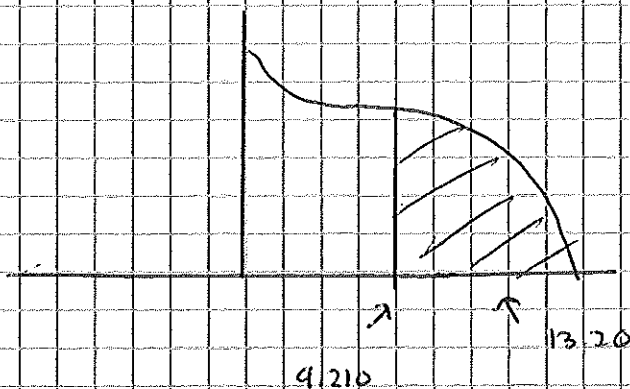
Critical value: $\nu = (3-1) \times (2-1) = 2$

$X^2_{(2)} 1\% = 9.210$

$13.20 > 9.210$

\therefore Reject H_0 .

Evidence at 1% level to support Juris' beliefs about association between age and passes.



b) Observed: 28 passed, Expected: 19.2 passed

\therefore More students passed than expected

5) a) Let X = number with disorder

$$X \sim B(25, 0.7)$$

$$P(X \geq 15) = P(X \geq 16)$$

Best to think at Y = number that don't have it

$$Y \sim B(25, 0.3)$$

$$P(X \geq 16) = P(Y \leq 9) = 0.8106 \text{ from tables}$$

b) i) $X \sim P_0(2.6)$

$$P(X \leq 5) = 0.951 \text{ from tables}$$

ii) 4.9 per million = 4.9 per 100,000

$$Y \sim P(4.9)$$

$$P(Y = 10) = \frac{e^{-4.9} \times (4.9)^{10}}{10!} = 0.0164$$

iii) $T \sim P_0(7.5)$

$$P(T > 16) = 1 - P(T \leq 16)$$

$$= 1 - 0.9980 = 0.002$$

6) a) i) $a = 25/6$ (prob sum to 1)

ii) $E(X) = 2.5$ (as distribution is symmetric)

$$\begin{aligned} \text{iii) } E(X^2) &= 0^2 \times \frac{1}{252} + 1^2 \times \frac{25}{252} + 2^2 \times \frac{100}{252} \\ &\quad + 3^2 \times \frac{100}{252} + 4^2 \times \frac{25}{252} + 5^2 \times \frac{1}{252} \\ &= \frac{25}{18} \end{aligned}$$

$$\therefore \text{Var}(X) = \frac{25}{18} - 2.5^2 = \frac{25}{36}$$

$$\therefore \text{sd}(X) = \sqrt{\frac{25}{36}} = \frac{5}{6}$$

b) i) $E(\text{money}) = 40p \times \frac{3}{4} + (-50p) \times \frac{5}{4} + 60p \times \frac{2}{4}$
 $= 10p \text{ loss per game!}$

ii) After 100 games, expected loss = $100 \times 10p = \pounds 10$

$$(7) \text{ a) i) } s = \sqrt{\frac{93}{12}} = \sqrt{7.75}$$

$$\therefore s^2 = 7.75$$

$$\text{ii) } n = 12, \text{ so } v = 11$$

$$t \text{ value, } v = 11, 90\% \text{ CI, } 2 \text{ tails} = 1.363$$

$$\text{so CI} = 64.8 \pm 1.363 \times \sqrt{7.75}$$

$$= 64.8 \pm 3.74$$

$$= (61.0, 68.6) \quad (366)$$

$$\text{b) i) Symmetrical about mean} = (59.8, 69.8)$$

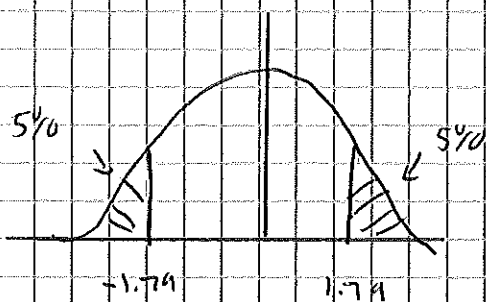
$$\text{ii) } 64.8 + t \sqrt{7.75} = 69.8$$

$$t \sqrt{7.75} = 5$$

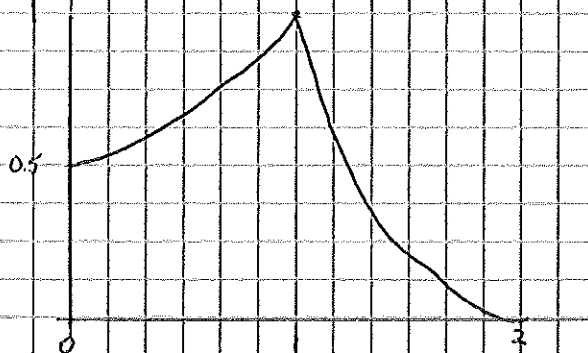
$$t = \frac{5}{\sqrt{7.75}} = 1.7960\dots$$

$$\text{From tables, } P(X > 1.7960) = 0.05$$

\therefore 90% confidence interval.



(8)



$$\text{b) } P(X \leq 1) = \int_0^1 \frac{1}{2}(x^2+1) dx$$

$$= \left[\frac{x^3}{6} + \frac{x}{2} \right]_0^1$$

$$= \frac{1}{6} + \frac{1}{2} - 0 = \frac{2}{3}$$

$$\text{a) } E(X^2) = \int_0^1 x^2 \cdot \frac{1}{2}(x^2+1) dx + \int_1^2 x^2 (x-2)^2 dx$$

$$= \int_0^1 \frac{1}{2} x^4 + \frac{1}{2} x^2 dx + \int_1^2 x^4 - 4x^3 + 4x^2 dx$$

$$= \left[\frac{x^5}{10} + \frac{x^3}{6} \right]_0^1 + \left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3} \right]_1^2$$

$$= \frac{1}{10} + \frac{1}{6} + \left(\frac{3^2}{5} - 16 + \frac{3^2}{3} \right) - \left(\frac{1}{5} - 1 + \frac{4}{3} \right)$$
$$= \frac{4}{5}$$

d) i) $\text{Var}(X) = E(X^2) - E(X)^2$

$$\frac{499}{K} = \frac{4}{5} - \left(\frac{19}{24} \right)^2$$

$$\frac{499}{K} = \frac{499}{2880} \rightarrow K = 2880$$

ii) $E(5X^2 + 24X - 3)$

$$= 5E(X^2) + 24E(X) - 3$$

$$= 5 \times \frac{4}{5} + 24 \times \frac{19}{24} - 3 = 20$$

iii) $\text{Var}(12X - 5)$

$$= 12^2 \text{Var}(X) = 144 \times \frac{499}{2880} = \frac{499}{20}$$