Stats 2 - June 2009

(1) $H_0\ \mu = 768$
$H_1\ \mu \neq 768$ (2 tailed test)

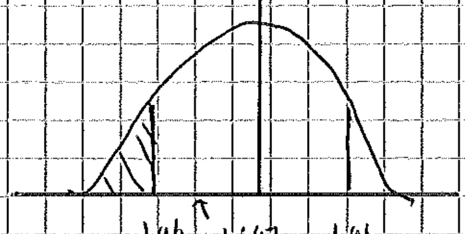
$\bar{x} = 764.8$

$\sigma = 8$ we know σ, so can use Z distribution

$n = 18$

Test Statistic: $z = \dfrac{764.8 - 768}{8/\sqrt{18}} = -1.697...$

Critical Value: 5%, 2 tailed: ± 1.96



$-1.697 > -1.96$

$\therefore$ Accept H_0

Not enough evidence at 5% level to support claim that value has changed so not enough evidence to deny Yvonne's claim

(2) a) i) $P(X < 4) = P(X \leq 3) = 0.2650$ (from tables)

ii) $P(X = 4) = \dfrac{e^{-1.5} \times (1.5)^4}{4!} = 0.0471$

b) i) Let T = Total Letters

$T \sim Po(6.5)$

$P(T > 5) = 1 - P(T \leq 5)$

$= 1 - 0.3690 = 0.631$

ii) Use Binomial:

$\boxed{7\text{ Days}}$ $\ ^8C_7 \times (0.631)^7 \times (0.369)^1 = 0.11758$

$\boxed{8\text{ days}}$ $(0.631)^8 = 0.02513$

Total Prob $= 0.143$ (3dp)

c) i) From calc: mean $= 8$ SD $= 4.0986$

variance $= 16.8$

ii) Poisson not a good model as mean and variance are very different.

(3) H_0: no association between age and attitude (Independent)

H_1: there is an association (non-Independent)

Observed

	Against	Not Against	
16-17	9	2	11
18-21	17	10	27
22-49	115	90	205
50-65	41	34	75
Over 65	3	4	7
	185	140	$\boxed{325}$

Expected

	Against	Not Against
16-17	$6\frac{17}{65}$	$4\frac{48}{65}$
18-21	$15\frac{24}{65}$	$11\frac{41}{65}$
22-49	$116\frac{9}{13}$	$88\frac{4}{13}$
50-65	$42\frac{9}{13}$	$32\frac{4}{13}$
Over 65	$3\frac{64}{65}$	$3\frac{1}{65}$

Some Expected values < 5, so must combine cells:

New Expected

16-21	21.63	16.37
22-49	116.69	88.31
50+	46.68	35.32

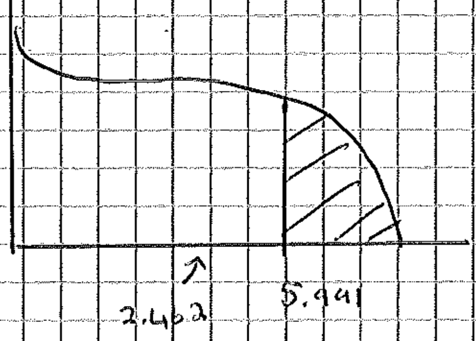
X^2 $\frac{(O-E)^2}{E}$

16-21	0.8825	1.1662
22-49	0.0245	0.0324
50+	0.1535	0.2029

Test Statistic (sum of X_2) = 2.462

$v = (3-1) \times (2-1) = 2$

Critical Value $_{(2)}$ 5% = 5.991

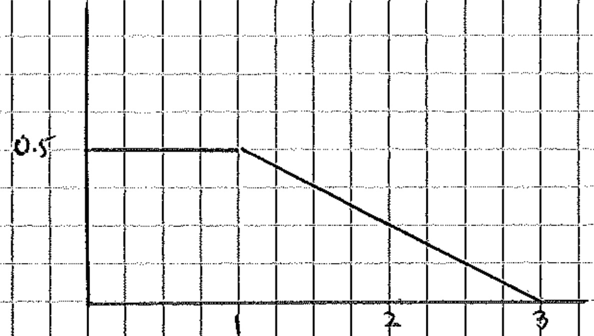


2.462 < 5.991

Accept H_0

No evidence at 5% level of an association between age and attitude to school reorganisation.

(4) a)



b) From graph we can see that $P(X < 1) = 0.5$

$\therefore$ median $= 1$

c) $E(X) = \int x f(x)$

$= \int_0^1 x \tfrac{1}{2} + \int_1^3 x \tfrac{3-x}{4}$

$= \left[\tfrac{x^2}{4}\right]_0^1 + \left[\tfrac{3x^2}{8} - \tfrac{x^3}{12}\right]_1^3$

$= \left[\tfrac{1}{4} - 0\right] + \left[\tfrac{27}{8} - \tfrac{27}{12} - \tfrac{3}{8} + \tfrac{1}{12}\right]$

$= \tfrac{1}{4} + \tfrac{5}{6} = \tfrac{13}{12}$

d) $P(X < 3\mu - 1) = P(X < 3(\tfrac{13}{12}) - 1)$

$= P(X < 2\tfrac{1}{4})$

From graph, $P(X > 2\tfrac{1}{4}) =$ Area of Δ

$= \tfrac{1}{2} \times \tfrac{3}{4} \times \dfrac{3 - 2\tfrac{1}{4}}{4} = \tfrac{9}{128}$

$\therefore P(X < 2\tfrac{1}{4}) = 1 - \tfrac{9}{128} = \tfrac{119}{128}$

(5) a) i) $P(B,B) = 0$, $P(G,G) = \tfrac{2}{10} \times \tfrac{1}{9} = \tfrac{2}{90}$

$P(Y,Y) = \tfrac{3}{10} \times \tfrac{2}{9} = \tfrac{6}{90}$ $P(R,R) = \tfrac{4}{10} \times \tfrac{3}{9} = \tfrac{12}{90}$

$\therefore P(\text{2 same}) = \tfrac{2}{90} + \tfrac{6}{90} + \tfrac{12}{90} = \tfrac{20}{90} = \tfrac{2}{9}$

ii) $P(\text{1st blue, 2nd not blue}) = \tfrac{1}{10} \times \tfrac{9}{9} = \tfrac{1}{10}$

$P(\text{1st not blue, 2nd blue}) = \tfrac{9}{10} \times \tfrac{1}{9} = \tfrac{1}{10}$

Total Prob $= \tfrac{1}{10} + \tfrac{1}{10} = \tfrac{1}{5}$

b) i)

x	135	145	-45
$P(X = x)$	$\tfrac{2}{9}$	$\tfrac{1}{5}$	$\tfrac{26}{45}$

ii) $E(X) = 135 \times \tfrac{2}{9} + 145 \times \tfrac{1}{5} + (-45) \times \tfrac{26}{45} = 33p$

c) i) $E(Y) = 104 - 3E(X) = 104 - 3 \times 33 = 5p$

Playing 100 times $\rightarrow$ $100 \times 5p = £5$ (expected win)

ii) First need $Var(X)$

$E(X^2) = 135^2 \times 2/9 + 145^2 \times 1/5 + (-45)^2 \times 26/45$

$= 9425$

$Var(X) = 9425 - 33^2 = 8336$

$Var(Y) = 3^2 \times Var(X) = 9 \times 8336 = 75{,}024$

$\therefore sd(Y) = \sqrt{75{,}024} = 273.9p$ (1dp)

$= 274p$ (nearest p)

(6) a) i) From calc:

$\cancel{\bar{x}} \; \Sigma x = 348 \qquad \bar{x} = 43.5$

$\Sigma x^2 = 15166 \qquad s = 2$

$n = 8 \qquad v = 7$

t value$_{(7)}$ 95% (2 tailed) $= 2.365$

Confidence Interval $= 43.5 \pm 2.365 \times 2/\sqrt{8}$

$= 43.5 \pm 1.6723$

$= (41.8, 45.2)$ (1dp)

Assumption: Weights are normally distributed

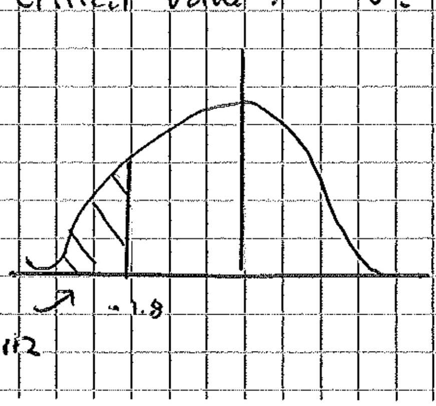
ii) Bishen believes mean = 45g. This is just about in the CI, so his belief is justified at 5% level

b) i) $H_0: \mu = 45$

$H_1: \mu < 45$ (1 tailed test)

Test Statistic: $\dfrac{43.5 - 45}{2/\sqrt{8}} = -2.1213..$

Critical value: $v = 7$, 5%, 1 tailed: -1.895



$-2.12 < -1.895$

$\therefore$ Reject H_0

Evidence at 5% level to support Ashby's claim that mean content is less than 45g.

iii) Type 1 Error

Rejected H_0 when H_0 was in fact true.