General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS **MS2B**
Unit Statistics 2B

Thursday 29 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Fortune High School gave its students a wider choice of subjects to study. The table shows the number of students, of each gender, who chose to study each of the additional subjects during the school year 2007/08.

	Bulgarian	**Climate Change**	**Finance**	**Polish**
Male	7	31	25	40
Female	2	24	22	19

Assuming that these data form a random sample, use a χ^2 test, at the 10% level of significance, to test whether the choice of these subjects is independent of gender. *(11 marks)*

2 A group of estate agents in a particular area claimed that, after the introduction of a new search procedure at the Land Registry, the mean completion time for the purchase of a house in the area had not changed from 8 weeks.

(a) A random sample of 9 house purchases in the area revealed that their completion times, in weeks, were as follows.

6 7 10 12 9 11 7 8 14

Assuming that completion times in the area are normally distributed with standard deviation 2.5 weeks, test, at the 5% level of significance, the group's claim. *(7 marks)*

(b) It was subsequently discovered that, after the introduction of the new search procedure at the Land Registry, the mean completion time for the purchase of a house in the area remained at 8 weeks.

Indicate whether a Type I error, a Type II error or neither has occurred in carrying out your hypothesis test in part (a). Give a reason for your answer. *(2 marks)*

3 Joe owns two garages, Acefit and Bestjob, each specialising in the fitting of the latest satellite navigation device.

The daily demand, X, for the device at Acefit garage may be modelled by a Poisson distribution with mean 3.6.

The daily demand, Y, for the device at Bestjob garage may be modelled by a Poisson distribution with mean 4.4.

(a) Calculate:

(i) $\mathrm{P}(X \leqslant 3)$; *(1 mark)*

(ii) $\mathrm{P}(Y = 5)$. *(2 marks)*

(b) The total daily demand for the device at Joe's two garages is denoted by T.

(i) Write down the distribution of T, stating any assumption that you make. *(2 marks)*

(ii) Determine $\mathrm{P}(6 < T < 12)$. *(3 marks)*

(iii) Calculate the probability that the total demand for the device will exceed 14 on each of two consecutive days. Give your answer to one significant figure. *(4 marks)*

(iv) Determine the minimum number of devices that Joe should have in stock if he is to meet his total demand on at least 99% of days. *(2 marks)*

Turn over for the next question

4 The continuous random variable X has the cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < -c \\ \dfrac{x+c}{4c} & -c \leqslant x \leqslant 3c \\ 1 & x > 3c \end{cases}$$

where c is a positive constant.

(a) Determine $\mathrm{P}\left(-\dfrac{3c}{4} < X < \dfrac{3c}{4}\right)$. *(2 marks)*

(b) Show that the probability density function, $\mathrm{f}(x)$, of X is

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{4c} & -c \leqslant x \leqslant 3c \\ 0 & \text{otherwise} \end{cases}$$

(2 marks)

(c) Hence, or otherwise, find expressions, in terms of c, for:

(i) $\mathrm{E}(X)$; *(1 mark)*

(ii) $\mathrm{Var}(X)$. *(1 mark)*

5 Jane, who supplies fruit to a jam manufacturer, knows that the weight of fruit in boxes that she sends to the manufacturer can be modelled by a normal distribution with unknown mean, μ grams, and unknown standard deviation, σ grams.

Jane selects a random sample of 16 boxes and, using the t-distribution, calculates correctly that a 98% confidence interval for μ is (70.65, 80.35).

(a) (i) Show that the sample mean is 75.5 grams. *(1 mark)*

(ii) Find the width of the confidence interval. *(1 mark)*

(iii) Calculate an estimate of the standard error of the mean. *(3 marks)*

(iv) Hence, or otherwise, show that an unbiased estimate of σ^2 is 55.6, correct to three significant figures. *(2 marks)*

(b) Jane decides that the width of the 98% confidence interval is too large.

Construct a 95% confidence interval for μ, based on her sample of 16 boxes. *(2 marks)*

(c) Jane is informed that the manufacturer would prefer the confidence interval to have a width of at most 5 grams.

(i) Write down a confidence interval for μ, again based on Jane's sample of 16 boxes, which has a width of 5 grams. *(1 mark)*

(ii) Determine the percentage confidence level for your interval in part (c)(i). *(3 marks)*

Turn over for the next question

6 A small supermarket has a total of four checkouts, at least one of which is always staffed. The probability distribution for R, the number of checkouts that are staffed at any given time, is

$$\mathrm{P}(R=r)=\begin{cases}\frac{2}{3}\left(\frac{1}{3}\right)^{r-1} & r=1,\,2,\,3\\ k & r=4\end{cases}$$

(a) Show that $k=\frac{1}{27}$. *(2 marks)*

(b) Find the probability that, at any given time, there will be at least 3 checkouts that are staffed. *(1 mark)*

(c) It is suggested that the total number of customers, C, that can be served at the checkouts per hour may be modelled by

$$C=27R+5$$

Find:

(i) $\mathrm{E}(C)$; *(3 marks)*

(ii) the standard deviation of C. *(4 marks)*

7 The continuous random variable X has the probability density function given by

$$\mathrm{f}(x)=\begin{cases}\frac{1}{16}x^3 & 0\leqslant x\leqslant 2\\ \frac{1}{6}(5-x) & 2\leqslant x\leqslant 5\\ 0 & \text{otherwise}\end{cases}$$

(a) Sketch the graph of f. *(3 marks)*

(b) Prove that the cumulative distribution function of X for $2\leqslant x\leqslant 5$ can be written in the form

$$\mathrm{F}(x)=1-\frac{1}{12}(5-x)^2$$

(4 marks)

(c) Hence, or otherwise, determine $\mathrm{P}(X\geqslant 3\mid X\leqslant 4)$. *(5 marks)*

END OF QUESTIONS

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