Stats 2 - January 2009

① H_0: Subject choice is Independent of Gender

H_1: Subject choice is ASSOCIATED with gender

② Expected

	B	C	F	P
M	5.45	33.32	28.48	35.75
F	3.55	21.68	18.52	23.25

① Observed

	B	C	F	P	
M	7	31	25	40	103
F	2	24	22	19	67
	9	55	47	59	170

④ X^2 $\frac{(O-E)^2}{E}$

	B	C	F	P + B
M		0.1615	0.4252	0.8165
F		0.2483	0.6539	1.2552

Must combine cells as expected < 5

Makes sense to combine Polish with Bulgarian

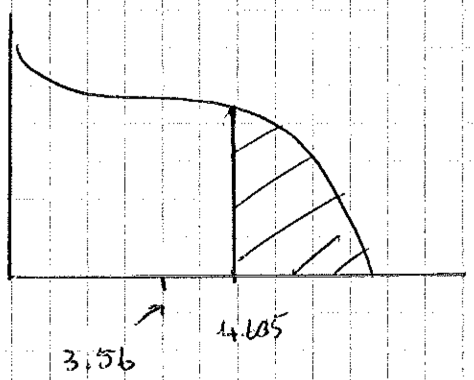
③ New Combined Expected

	B	C	F	P + B
M		33.32	28.48	41.20
F		21.68	18.52	26.80

Total X_2 = 3.56 = TEST STATISTIC

CRITICAL VALUE: $X_{2(2)}(10\%)$ = 4.605

$V = (3-1) \times (2-1) = 2$



4.605 > 3.56

∴ Accept H_0

Insufficient evidence at 10% level to suggest subject choice is associated with gender.

② a) H_0: $\mu = 8$

H_1: $\mu \neq 8$ (2 tailed test)

As we know σ, we use Z-distribution

Data (from calc):

$n = 9$

$\Sigma x = 84$

$\Sigma x^2 = 840$

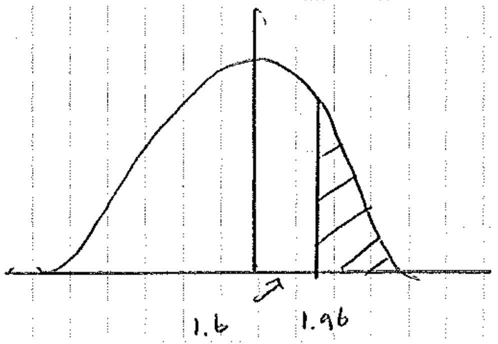
$\bar{x} = 9\frac{1}{3}$

$\sigma = 2.5$

$s^2 = 2.5^2$

TEST STATISTIC: $z = \frac{9\frac{1}{3} - 8}{2.5/\sqrt{9}} = 1.6$

CRITICAL VALUE: 5% sig, 2 tailed $= \pm 1.96$



$1.6 < 1.96$

$\therefore$ Accept H_0

Insufficient evidence at 5% level to suggest mean completion time has changed.

b) Neither error has occurred as we accepted $H_0: \mu = 8$ and $\mu = 8$!

3 a) i) $X \sim Po(3.6)$

$P(X \le 3) = 0.5152$ (from tables)

ii) $Y \sim Po(4.4)$

$P(Y = 5) = \frac{e^{-4.4} \times (4.4)^5}{5!} = 0.1687...$

b) i) $T \sim Po(8)$

Assumption: X & Y are INDEPENDENT

ii) $P(6 < T < 12) = P(T \le 11) - P(T \le 6)$

$= 0.8881 - 0.3134 = 0.575$

iii) $P(T > 14) = 1 - P(T \le 14)$

$= 1 - 0.9827 = 0.0173$

For 2 consecutive days $= 0.0173^2 = 0.00029...$

$= 0.0003$ (1sf)

iv) Need number so $P(T \le n) > 0.99$

$P(T \le 14) = 0.9827$ (from tables)

$P(T \le 15) = 0.9918$

$\therefore$ Minimum number $= 15$.

④ a) $P(-3c/4 < X < 3c/4) = F(3c/4) - F(-3c/4)$

$$= \frac{3c/4 + c}{4c} - \frac{-3c/4 + c}{4c}$$

$$= \frac{3c/2}{4c} = \frac{3/2}{4} = 3/8$$

b) for $f(x)$ need to differentiate $F(x)$

$$\rightarrow \text{if } y = \frac{x + c}{4c} \rightarrow y = \frac{x}{4c} + \frac{c}{4c}$$

For $x > 3c$ & $x < -c$

so $F'(x) = 0$

$$dy/dx = \frac{1}{4c} + 0 = f(x)$$

$$\rightarrow f(x) = \begin{cases} 1/4c & -c \le x \le 3c \\ 0 & \text{otherwise} \end{cases}$$

c) i) $E(X) = \frac{-c + 3c}{2} = c$

ii) $Var(X) = 1/12\,(3c - (-c))^2 = 1/12\,(4c)^2 = 4c^2/3$

⑤ a) i) $\bar{x} = 1/2\,(70.65 + 80.35) = 75.5$

ii) width $=$ MAX $-$ MIN $= 80.35 - 70.65 = 9.7$

iii) t distribution, $v = 16 - 1 = 15$, critical value$_{(98\%)}$ $= 2.602$

Standard error of mean $= \frac{s}{\sqrt{n}}$

$\boxed{98\%\ CI}$ $75.5 \pm 2.602\,\frac{s}{\sqrt{n}}$

we know width $= 9.7$

$$\therefore 2.602\,\frac{s}{\sqrt{n}} = \frac{9.7}{2}$$

$$\rightarrow \frac{s}{\sqrt{n}} = \frac{9.7}{2 \times 2.602} = 1.8639\ldots$$

iv) $\frac{s}{\sqrt{n}} = 1.8639\ldots$ $n = 16$

$\rightarrow s = 4 \times 1.8639$

$\rightarrow s^2 = (4 \times 1.8639)^2 = 55.5890\ldots = 55.6$ (3sf)

= unbiased estimator of σ^2

b) 95% CI, t value: 2.131

$\rightarrow \quad 75.5 \pm 2.131 \times 1.8639 \swarrow \quad \frac{s}{\sqrt{n}}$

$= 75.5 \pm 3.9719\ldots$

$= (71.5,\ 79.5)$

c) i) mean $= 75.5$, so must be 2.5 g each side

$\rightarrow \quad (73,\ 78)$

ii) $75.5 \pm p \times 1.8639$

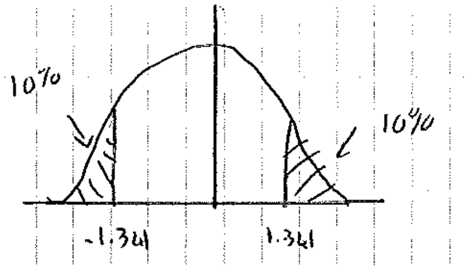
As width must be 5, $\quad p \times 1.8639 = 2.5$

$\rightarrow \quad p = \dfrac{2.5}{1.8639} = 1.341$

$n = 16, \quad v = 16 - 1 = 15$

From tables, $P(X \leq 1.341) = 0.9$

As 2 tailed test (confidence interval), % must be 80%



6) a)

r	1	2	3	4
$P(R=r)$	$2/3$	$2/9$	$2/27$	k

As probs $\Sigma 1 \rightarrow \quad 2/3 + 2/9 + 2/27 + k = 1$

$\rightarrow \quad k = 1/27$

b) $P(R \geq 3) = P(R=3) + P(R=4)$

$= 2/27 + 1/27 = 1/9$

c) i) $E(C) = 27 \times E(R) + 5$

$E(R) = 1 \times 2/3 + 2 \times 2/9 + 3 \times 2/27 + 4 \times 1/27$

$= 40/27$

$\rightarrow \quad E(C) = 27 \times 40/27 + 5$

$= 45$

ii) $E(R^2) = 1^2 \times 2/3 + 2^2 \times 3/9 + 3^2 \times 2/27 + 4^2 \times 1/27$

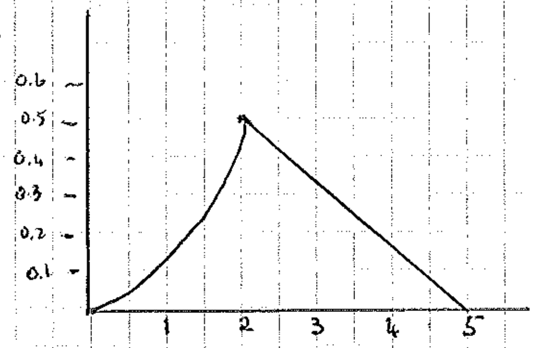
$= 76/27$

$\therefore Var(R) = 76/27 - (40/27)^2 = 452/729$

$sd(R) = \sqrt{452/729}$

$\therefore sd(C) = 27 \times \sqrt{\frac{452}{729}} = 21.2602..$

(7) a)



b) First need $F(2) = P(X \leq 2)$

$= 1 - P(X \geq 2)$

$= 1 - (1/2 \times 3 \times 0.5) = 0.25$

↑ Triangle on diagram

For $2 < X < 5$

$F(x) = F(2) + \int_2^x 1/6(5-x)$

$= 0.25 + 1/6 \int_2^x 5 - x$

$= 0.25 + 1/6 \left[5x - x^2/2\right]_2^x$

$= 0.25 + 1/6 \left[5x - x^2/2 - 10 + 2\right]$

$= 0.25 + 5x/6 - x^2/12 - 8/6$

$= \frac{-x^2}{12} + \frac{5x}{6} - \frac{13}{12}$

$= -1/12\,(x^2 - 10x + 13)$

$= -1/12\,(x^2 - 10x + 25) + 1$

$= 1 - 1/12\,(x-5)^2$

$= 1 - 1/12\,(5-x)^2$

c) $P(X \geq 3 / X \leq 4) = \dfrac{P(A \cap B)}{P(B)}$ From S1!

$P(B) = P(X \leq 4) = F(4) = 1 - \frac{1}{12}(5-4)^2 = \frac{11}{12}$

$$\begin{aligned}
P(A \cap B) &= P(X \geq 3 \text{ AND } X \leq 4) \\
&= P(3 \leq x \leq 4) \\
&= F(4) - F(3) \\
&= \tfrac{11}{12} - \left[1 - \tfrac{1}{12}(2)^2\right] \\
&= \tfrac{11}{12} - \tfrac{2}{3} = \tfrac{1}{4}
\end{aligned}$$

$\therefore P(x \geq 3 / X \leq 4) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1/4}{11/12} = 3/11$